1.1. Linear Systems

Linear Equations (→ p. 1)

Examples:

\[ 5x = 6 \]

or

\[ x_1 + 3x_2 - x_3 = 7 \]

or

\[ 0x = 1 \]

Generally,

\[ a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \]

Typically,

• \( a_1, \ldots, a_n, b \) - known constants,
• \( x_1, \ldots, x_n \) - unknowns.

A solution - a sequence \( s_1, \ldots, s_n \) such that when \( x_1 = s_1, \ldots, x_n = s_n \), the equation is satisfied.
In our examples:

\[ 5x = 6 \]

has

\[ x = \frac{6}{5} \]

as the **only solution**.

\[ x_1 + 3x_2 - x_3 = 7 \]

has

\[ x_1 = 1, x_2 = 2, x_3 = 0 \]

and

\[ x_1 = 8, x_2 = 0, x_3 = 1 \]

and **infinitely many** other solutions.

\[ 0x = 1 \]

has **no solutions**.
System of \( m \) linear equations in \( n \) unknowns:

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
    &\vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{align*}
\]

where \( a_{ij} \) is the coefficient (known number) associated with the \( j \)-th unknown \( (x_j) \) in the \( i \)-th equation.

A **solution** - a sequence \( s_1, \ldots, s_n \) such that when \( x_1 = s_1, \ldots, x_n = s_n \), each of the \( m \) equations is satisfied.

**EXAMPLE 1** Solve the system

\[
\begin{align*}
    2x - y &= -4 \\
    -3x - 2y &= -1
\end{align*}
\]

using the method of elimination.
Divide eq\(_1\) by 2:

\[
x - \frac{1}{2}y = -2
- 3x - 2y = -1
\]

Add 3\(\cdot\)eq\(_1\) to eq\(_2\):

\[
x - \frac{1}{2}y = -2
- \frac{7}{2}y = -7
\]

Divide eq\(_2\) by \(-\frac{7}{2}\):

\[
x - \frac{1}{2}y = -2
y = 2
\]

Add \(\frac{1}{2}\) \(\cdot\)eq\(_2\) to eq\(_1\):

\[
x = -1
y = 2
\]

One solution.

→ Read Section 1.1. In particular, see:

- Example 2 p. 3 (no solutions)
- Example 4 p. 4 (infinitely many solutions).