1. \( y = x \cos y \)
   \( y' = \cos y - x (\sin y) y' \)
   \( y'(1 + x \sin y) = \cos y \)
   \( y' = \frac{\cos y}{1 + x \sin y} \)
   \( y'' = \frac{-\sin y y' (1 + x \sin y) - \cos (y' (1 + x \sin y) - \cos y (y' (1 + x \sin y)) y')}{(1 + x \sin y)^2} \)

2. \( \frac{dx}{dt} = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t - 1)(t - 3) \)
   \( \frac{dx}{dt} \bigg|_{t=0} = 9 > 0 \) therefore for \( t < 1 \), \( x \) is moving to the right;
   \( \frac{dx}{dt} \bigg|_{t=2} = -3 < 0 \) therefore for \( 1 < t < 3 \), \( x \) is moving to the left;
   \( \frac{dx}{dt} \bigg|_{t=4} = 9 > 0 \) therefore for \( t > 3 \), \( x \) is moving to the right;
   \( \frac{d^2x}{dt^2} = 6t - 12 = 6(t - 2) \)
   \( \frac{d^2x}{dt^2} \bigg|_{t=0} = -12 < 0 \) therefore for \( t < 2 \), \( x \) is accelerating to the left;
   \( \frac{d^2x}{dt^2} \bigg|_{t=3} = 6 > 0 \) therefore for \( t > 2 \), \( x \) is accelerating to the right;
   (a) the point is moving to the right for \( t < 1 \) and for \( t > 3 \);
   (b) the point is accelerating to the right for \( t > 2 \);
   (c) the point is speeding up whenever the signs of \( \frac{dx}{dt} \) and \( \frac{d^2x}{dt^2} \) match, i.e. for \( 1 < t < 2 \) and for \( t > 3 \).

3. \( V = \frac{4}{3} \pi r^3; \frac{dV}{dt} = \frac{4}{3} \pi (3r^2) \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt} \).
   Since \( \frac{dV}{dt} = -20 \ \text{ft}^3/\text{min}; r = 2 \ \text{ft}, \) we have
   \( -20 = 4\pi (2)^2 \frac{dr}{dt} \) therefore \( \frac{dr}{dt} = \frac{-20}{16\pi} = \frac{-5}{4\pi} \).
   Answer: the radius is decreasing at the rate of \( 1.25/\pi \) feet per minute.

4. \( f(x) = 2x^3 - 9x^2 - 3; f'(x) = 6x^2 - 18x = 6x(x - 3) \)
   - Critical points: Set \( f'(x) = 0; \) thus \( x = 0, f(0) = -3 \) and \( x = 3, f(3) = -30 \)
     are the two critical points. However only the first point is inside the interval \([-1, 1]\).
   - Singular points ( \( f' \) is undefined while \( f \) is defined)
     Since \( f' \) is a polynomial, it is defined for all real \( x \), therefore there are no singular points.
   - Endpoints: \( f(-1) = -14; f(1) = -10 \).

The absolute maximum is \( f(0) = -3 \). The absolute minimum is \( f(-1) = -14 \).
5. 

Length of the fence: \( 2000 = 6x + 4y \).

Maximize the area: \( A = 3xy \).

Solve the fence length equation for \( y \): \( y = 500 - \frac{3}{2}x \) and substitute into the area formula: \( A(x) = 3x(500 - \frac{3}{2}x) = 1500x - \frac{9}{4}x^2 \) defined on \( [0, \frac{2000}{3}] \).

- Critical points: \( A'(x) = 1500 - 9x = 0 \Rightarrow x = \frac{1500}{9} = \frac{500}{3} \) is inside the interval; \( A\left(\frac{500}{3}\right) = 500(500 - 250) = (500)(250) = 125,000 \).
- No singular points (derivative defined for all \( x \))
- Endpoints \( A(0) = 0; A\left(\frac{2000}{3}\right) = 0 \).

Absolute maximum at \( x = \frac{500}{3} \).

Answer: the largest rectangular area that can be enclosed is 125,000 sq ft.

6. 

Volume of the box: \( V(x) = (12 - 2x)^2(x) \) for \( x \) in \( [0, 6] \).

- Critical points: \( V'(x) = 2(12 - 2x)(-2)(x) + (12 - 2x)^2 = (12 - 2x)(-4x + 12 - 2x) = (12 - 2x)(-6x + 12) = 0 \)
  \( x = 6 \Rightarrow V(6) = 0 \)
  \( x = 2 \Rightarrow V(2) = 64(2) = 128 \)
- Singular points: none
- Endpoints: \( V(0) = 0, V(6) = 0 \) (already listed as a critical point)

Absolute maximum at \( x = 2 \).

Answer: The dimensions of the box with maximum volume are 8 in \( \times \) 8 in \( \times \) 2 in.

7. (a) \( \frac{dy}{dx} = e^{2x}(2) + \frac{1}{x^2}(2x) = 2e^{2x} + \frac{2}{x} \)
(b) \( \frac{dy}{dx} = e^{\sin^2 x}(2 \sin x)(\cos x) \)
(c) \( \frac{dy}{dx} = (\sec^2(\ln x)) \frac{1}{x} \)
(d) \( \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{5x^2 + 2x + 1}\right) \left(\frac{(10x + 2)(x^2 - x + 1) - (5x^2 + 2x + 1)(2x - 1)}{(x^2 - x + 1)^2}\right) \)
(e) \( y = \frac{\ln(3x + 1)}{\ln 2}; \frac{dy}{dx} = \left(\frac{1}{\ln 2}\right) \left(\frac{3}{3x + 1}\right) \)
(f) \( y = e^{(\ln 4)(x - \cos x)}; \frac{dy}{dx} = e^{(\ln 4)(x - \cos x)}(\ln 4)(1 + \sin x) = 4x - \cos x(\ln 4)(1 + \sin x) \)
(g) \( y = e^{(\ln x)x}; \frac{dy}{dx} = e^{(\ln x)x}(\frac{1}{x}x + \ln x) = x^x(1 + \ln x) \)
(h) \[ y = e^{(\ln(\sin x)) \cos x}, \quad \frac{dy}{dx} = e^{(\ln(\sin x)) \cos x} \left( \frac{\cos x}{\sin x} \cos x + \ln(\sin x)(-\sin x) \right) \]
\[ = (\sin x)^{\cos x} \left( \frac{\cos^2 x}{\sin x} - \sin x(\ln(\sin x)) \right) \]

8. (a) \[ \frac{dy}{dx} = \frac{1}{\cos x} (-\sin x) = -\tan x \]
(b) \[ \frac{dy}{dx} = \frac{1}{\csc x + \cot x} (-\csc x \cot x - \csc^2 x) = \frac{-\csc x(\cot x + \csc x)}{\csc x + \cot x} = -\csc x \]