1. \[ I = \int \frac{-1}{x^2 - x} \, dx = \int \frac{1}{(x-1)x} \, dx \] Integrand is a proper rational function - no division necessary.

Partial fraction expansion:

\[ \frac{1}{(x-1)x} = \frac{A}{x} + \frac{B}{x-1} \]

Multiply both sides by \((x-1)x\):

\[ 1 = A(x-1) + Bx \]

when \(x = 1\), \(1 = B\)

when \(x = 0\), \(1 = -A \Rightarrow A = -1\)

\[ I = \int \left( \frac{-1}{x} + \frac{1}{x-1} \right) \, dx = -\ln |x| + \ln |x-1| + C \]

2. \[ \frac{x^4 + 1}{x^2 (x^2 + 1)} = \frac{x^4 + 1}{x^4 + x^2} \] is an improper rational function - must divide first

\[
\begin{array}{c|cccc}
 & x^4 & +1 & -x^2 & -x^2 +1 \\
\hline
x^4+x^2 & - & - & - & - \\
-x^2 & - & - & - & - \\
\end{array}
\]

From long division:

\[ \frac{x^4 + 1}{x^4 + x^2} = 1 + \frac{-x^2 + 1}{x^4 + x^2} \]

Partial fraction expansion:

\[ \frac{-x^2 + 1}{x^4 + x^2} = \frac{A}{x^2} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1} \]

\[ -x^2 + 1 = Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2 \]

\[ -x^2 + 1 = Ax^3 + Ax^2 + Bx + Cx^3 + Dx^2 \]

Compare coefficients of like terms on both sides:

\[
\begin{align*}
x^3: & \quad 0 = A + C \\
x^2: & \quad -1 = B + D \\
x: & \quad 0 = A \\
1: & \quad 1 = B
\end{align*}
\]

Answer:

\[ \frac{x^4 + 1}{x^4 + x^2} = 1 + \frac{-x^2 + 1}{x^4 + x^2} = 1 + \frac{1}{x^2} + \frac{-2}{x^2 + 1} \]

3. \[ \int_{-\infty}^{0} e^{-x} \, dx = \lim_{a \to -\infty} \int_{a}^{0} e^{-x} \, dx = \lim_{a \to -\infty} [-e^{-x}]_{a}^{0} = \lim_{a \to -\infty} (-e^{0} + e^{-a}) = -1 + \infty \quad \text{(Diverges)} \]

4. \[ \int_{0}^{1} \frac{1}{\sqrt{x}} \, dx = \lim_{a \to 0^+} \int_{a}^{1} x^{-1/2} \, dx = \lim_{a \to 0^+} \left[ \frac{3}{2} x^{2/3} \right]_{a}^{1} = \lim_{a \to 0^+} \left( \frac{3}{2} - \frac{3}{2} a^{2/3} \right) = \frac{3}{2} - 0 = \frac{3}{2} \quad \text{(converges)} \]