As $x$ approaches $\infty$, both the numerator and the denominator contain terms approaching $\infty$ or $-\infty$.

Divide each term in the numerator and in the denominator by the highest power of $x$ in the denominator: $x^3$

Simplify

As $x$ approaches $\infty$, the terms: $\frac{1}{x^2}$, $\frac{1}{x^3}$, $\frac{1}{x}$, and $\frac{4}{x^3}$ all approach $0$. Consequently, the numerator approaches $2$, and the denominator approaches $-3$.

(Note that this means the curve $y = \frac{2x^3 + x - 1}{-3x^3 - 2x^2 + 4}$ has a horizontal asymptote $y = \frac{-2}{3}$.)
8. \[ \lim_{x \to -\infty} \frac{4x^2 + 5x - 5}{2x^2 \left(2 + x^2\right)} \]

First of all, expand the denominator...

\[ = \lim_{x \to -\infty} \frac{4x^2 + 5x - 5}{4x^2 + 2x^4} \]

As \( x \) approaches \( \infty \), both the numerator and the denominator contain terms approaching \( \infty \) or \( -\infty \).

\[ = \lim_{x \to -\infty} \frac{\frac{4x^2}{x^4} + \frac{5x}{x^4} - \frac{5}{x^4}}{\frac{4}{x^4} + \frac{2}{x^4}} \]

Divide each term in the numerator and in the denominator by the highest power of \( x \) in the denominator: \( x^4 \)

\[ = \lim_{x \to -\infty} \frac{\frac{4}{x^2} + \frac{5}{x} - \frac{5}{x^4}}{\frac{4}{x^4} + \frac{2}{x^4}} \]

Simplify

\[ = 0 \]

Except for 2 in the denominator, all the other terms in both the numerator and the denominator approach 0 as \( x \) approaches \( -\infty \). Consequently, the numerator approaches 0, and the denominator approaches 2.

(Note that this means the curve \( y = \frac{4x^2 + 5x - 5}{4x^2 + 2x^4} \) has a horizontal asymptote \( y = 0 \).)
9. \[ \lim_{x \to \infty} \frac{\sqrt{\frac{5}{x} + 2x^4 + 1}}{2x^2 + 3} \]

As \( x \) approaches \( \infty \), both the numerator and the denominator approach \( \infty \).

\[
= \lim_{x \to \infty} \frac{\sqrt{\frac{5}{x} + 2x^4 + 1}}{2x^2 + 3}
\]

Divide each term in the numerator and in the denominator by the highest power of \( x \) in the denominator: \( x^2 \).

In the numerator, use the form \( \sqrt{x^4} = x^2 \).

\[
= \lim_{x \to \infty} \frac{\sqrt{\frac{5}{x} + 2x^4 + 1}}{2x^2 + 3}
\]

Simplify

As \( x \) approaches \( \infty \), the terms \( \frac{1}{x^4} \) and \( \frac{3}{x^2} \) both approach 0.

Consequently, the numerator approaches \( \infty \), and the denominator approaches 2.
10. \[ \lim_{x \to -\infty} \frac{3 + x}{\sqrt{x^2 + 3}} \]

As \( x \) approaches \(-\infty\), the numerator approaches \(-\infty\), while the denominator approaches \(\infty\).

Divide each term in the numerator and in the denominator by the highest power of \( x \) in the denominator: \( \sqrt{x^2} \)

In the numerator, use the form \( \sqrt{x^2} = -x \).

(Note that as \( x \) approaches \(-\infty\), we can assume \( x < 0 \), so that the general identity \( \sqrt{x^2} = |x| \) results in \( \sqrt{x^2} = -x \).)

\[ = \lim_{x \to -\infty} \frac{3 - x}{\sqrt{1 + 3}} \]

Simplify.

As \( x \) approaches \(-\infty\), the terms \( \frac{3}{-x} \) and \( \frac{3}{x^2} \) both approach 0.

Consequently, the numerator approaches \(-1\), and the denominator approaches \(1\).

(Note that this means the curve \( y = \frac{3 + x}{\sqrt{x^2 + 3}} \) has a horizontal asymptote \( y = -1 \)).