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# 0 Introduction

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Welcome to computer activities for Calculus. We hope the manual will be useful to you in several ways:

- as a **lab assignment guide**;

Activity sections are included in almost all the chapters. Your professor will assign problems from these sections to be solved and formally turned in. Other problems may be worked through during in-class computer activities. Nearly all the activities of this manual involve Mathcad (version 5.0 for Windows). Several activities also require Maple V (release 3 for Windows).

- as a **self study guide**;

Homework Help sections are designed to allow you to use technology to aid you with your solution of homework problems from the text. For example, a graph obtained on the computer screen can frequently *verify* that your solution (obtained with a pencil and paper) is correct. A graph can also help you *understand* the nature of the problem that you have solved (all too frequently, such understanding does not automatically accompany algebraic skills).



Some chapters also include a Questions section, intended to help you assess the insight you gained into the visual and numerical aspects of the calculus concepts discussed.

- as a **reference**;

While this guide is not intended to be a software manual, you will hopefully find it provides useful information to aid in the use of the software during your calculus course(s), as well as afterwards. Software-related information, most of which is given in the course of the activities, is fully indexed for convenience. Appendix A contains a summary of Mathcad operations and functions. Appendix B includes a list of common problems encountered by students while working with Mathcad.

## Key to the symbols

The abbreviation "LHE" denotes a reference to the "Calculus with Analytic Geometry" text by Larson, Hostetler and Edwards (D.C. Heath, 5th Edition).

Whenever the symbol  appears, a Mathcad template file is used; similarly,  denotes a

Maple V template file. Your professor should give you further information about accessing these files.

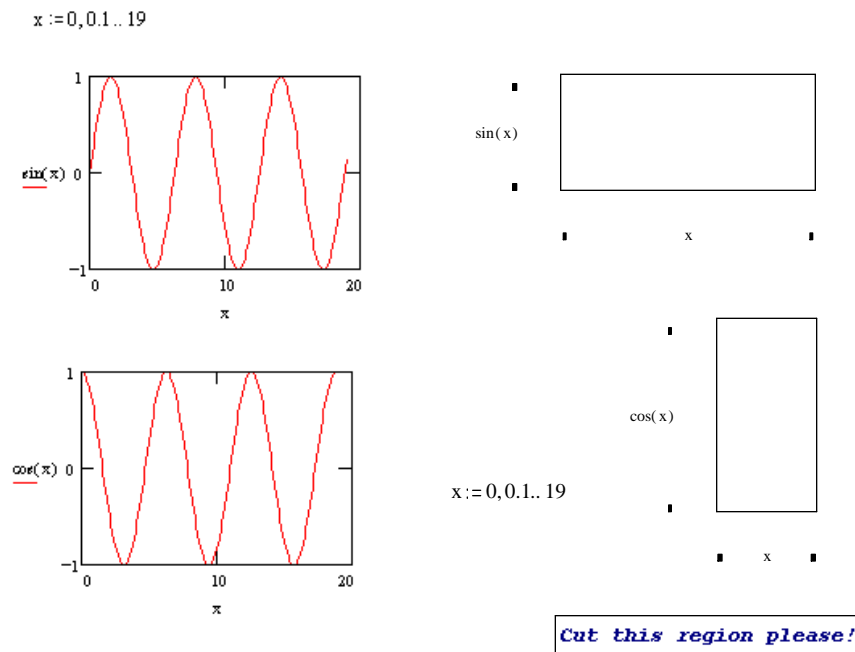


Figure 1: Make the right side look like the left!

## 0.1 Activity: Mathcad Regions

Every Mathcad document is made up of a number of *regions*, which may involve algebraic expressions, graphs, tables, text, or other items. As a Mathcad user, you have the ability to create regions of various kinds, and to lay them out in your document in a flexible manner.

In the next activity, "Mathcad Basics", you learn how to create Mathcad regions to perform calculations, graph functions, solve equations, etc. Before that, however, this brief activity will help you

- understand the basics of the layout of Mathcad documents and
- learn how you can affect your document's layout, in particular:
  - move regions around the document,
  - delete regions, and
  - resize Mathcad graphs.

### Instructions

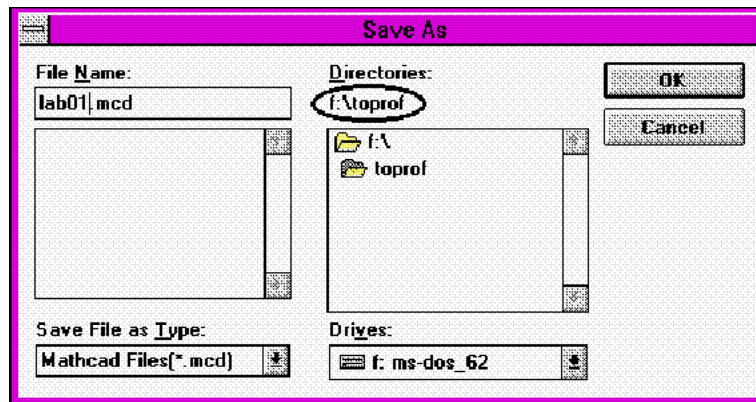


Figure 2: The "Save As" dialog box lets you specify the location and name of the file before it is saved.

A "template" document has been created for this assignment. Unless your professor instructs you otherwise, load the document **REGIONS.MCD**. The document should appear as shown in Figure 1.

In order to load the "template" document into Mathcad, select **Open** from the **File** menu. This brings up the "Open" dialog box. The following shows how to load the document **REGIONS.MCD** from the **G:** drive



- select the "G Drive" from the "Drives" box (change the selected drive by clicking on the box and typing the letter G);
- select the directory G:\ from the "Directories:" box;
- select (or type) the file name, **REGIONS.MCD** under the "File Name:" box;
- click on the "OK" button.

*Save your work frequently.*

Mathcad has two "save" items on the **File** menu:

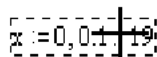
- \* **Save As...** - opens up the "Save As" dialog box which lets you specify the location (drive and directory) and name for the file before it's saved (see Figure 2). "Save As" allows you to create a new version of a file while allowing you to keep the previous version.
- \* **Save** - saves the current file without changing its location or name. The effect of "Save" is to overwrite your old version of a file with the current state of the document.

If your template is loaded from a read-only network drive (G:), you must do your first "save" using **Save As...** so that you can change the location of the file to your home directory. We assume your home directory is on drive "F:". The **Save** command can now be used until the assignment is completed. When you are ready to submit your document for grading, use **Save As ...** to change the destination to directory "F:\TOPROF". Figure 2 shows an example where the directory has been changed to "F:\TOPROF". Note the (circled) "F:\TOPROF" in the line directly underneath "Directories:" - this is the indicator of the file destination.


The left side of the document contains a picture of what the right side should become. However, at the moment, the regions on the right side are badly out-of-place and out-of-shape. Follow these steps to set things straight:

1. Begin by typing your team's name, and the team members' names at the top of the document. Start saving the document at regular intervals (see Comment 1).
2. Follow the procedure described below to move the region "x:=0,0.1.19" to its proper position. As a result, the sine and cosine graphs should appear. (The explanation for this is that the region you just moved gives Mathcad a valid domain for these graphs - also see Comment 2). In order to *move* a region in the Mathcad document, follow these steps:

- (a) Select the region. To select a region, move the pointer close to (but outside) the region you want to move. Holding down the left mouse button, drag the pointer into the region. Release the left mouse button once the pointer is within the region. The region should now be enclosed in a dotted frame (referred to as *selection rectangle* in Mathcad manual) and the pointer should turn into a big black cross:



- (b) Drag the region to the desired location.
3. Follow the procedure described below to delete the text region which says "Cut this region please!". (Be careful not to delete any other regions in the process!) In order to *delete* an entire region, follow step 2(a) to select the region, then use **Cut** from the **Edit** menu.
4. Follow the procedure described below to resize each of the to graphs to make them look like the graphs on the left side. Finally, move both graphs into the proper position. In order to *resize a graph region*:

- (a) Follow step 2(a) to select the region.
- (b) With the left mouse button released, move the pointer towards the *bottom right* corner of the frame, until the black cross changes into a double-headed arrow (.
- (c) Drag the bottom right corner to the desired position (the top left corner remains fixed).



If you want to change the width of the graph only, move it to the right side of the frame. A different double-headed arrow will appear ( $\longleftrightarrow$ ), allowing you to alter the width.

Likewise, moving the pointer towards the bottom side of the graph frame, produces the arrow ( $\updownarrow$ ) that can be used to change the graph's height.

5. Visually verify that the right side of the screen looks like the left. Ask for help if you have trouble accomplishing this.

You are now ready to tackle the "Mathcad Basics" activity!

### Comments

1. Multiple regions can be moved or deleted by initially dragging the pointer across more than one region. Alternatively, individual regions can be added to the selection (or omitted from it) by using the <Shift> key while clicking on the region.
2. Mathcad reads its documents just like we do: from top to bottom, left-to-right. If a region requires information from another, then it must be positioned to the right, or below that region. To see exactly where the regions are, and in which order Mathcad processes them, select **View Regions** from the **Edit** menu. As a result, the screen areas not occupied by regions are marked gray. The black dot at the left side of each region is called its *anchor* - it is the position of the region's anchor, relative to other anchors, that determines the ordering of regions within the document.

## 0.2 Activity: Mathcad Basics

The objective of this activity is to introduce the basic features of Mathcad: entering and editing expressions, defining and graphing functions, and numerically solving equations.

### Instructions

A "template" document has been created for this assignment. Before proceeding, read the comments below. Unless your professor instructs you otherwise, load the document `BASICS.MCD`. Follow the instructions on the screen to begin creating your lab report. After the "fill-in the blanks" portion of the assignment answer the problems identified by your instructor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.




### Comments

1. Carefully read all instructions and examples. Ask questions if you need any help. It is expected that all team members will actively participate in solving every problem.

2. The "hand" symbol indicates that you are being asked to perform some task, e.g.:

2. Evaluate  $2^{10} + 1$

Type: 2 ^ 10 Spacebar + 1 = 

You should place your answer to the right of this symbol.

3. After you submit your assignment and your professor grades it, you will be given instructions regarding the retrieval of your assignment. If your assignment is returned to you electronically, you may want to print it for your records (select **Print** from **File** menu).

## Problems

- At the end of the template document, you will find the expression  $\frac{e^{\frac{1+\pi}{4}} - \left| 11 \left( \frac{3}{5+\sqrt{5}} + 7 \right) \right|}{\pi + e + 1}$ . Copy the expression into your document, then cut the numerator and replace it with  $\sqrt{2} + 1$ . Evaluate the new expression.
- Define the function  $f(x) = x + 2 \sin x$  and evaluate  $f(1)$ .
- Graph  $f(x)$  from the previous Problem on  $[-4, 6]$  in steps of 0.1. Chose "points" for the "trace type".
- (a) Use zoom and crosshair to approximate the smallest positive  $x$  such that  $F(x) = G(x)$  where

$$F(x) = 10 \sin \left( \frac{x^2}{15} \right) \quad \text{and} \quad G(x) = -x \cos \frac{x}{2} - 4.$$

Evaluate the difference  $F(x) - G(x)$  at the value of  $x$  you found.

- (b) Use the root function to improve upon the approximation obtained in part (a). (Hint:  $F(x) = G(x)$  is equivalent to  $F(x) - G(x) = 0$ , therefore you are looking for a root of  $F(x) - G(x)$ .) Evaluate the difference  $F(x) - G(x)$  at the new value of  $x$  you just found.

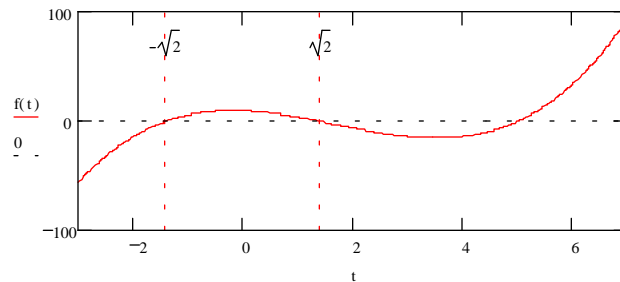
## Recap

Listed below are some of the important topics covered in this laboratory activity.

- Arithmetic operations: The operations of addition, subtraction, multiplication and division are represented by the symbols  $+$ ,  $-$ ,  $*$ , and  $/$ .

**Example**

Let us define a function  $f(t) := (t^2 - 2) \cdot (t - 5)$ . It is clear that  $f$  has three roots:  $-\sqrt{2}, \sqrt{2}$  and 5. Here we have plotted  $f(t)$  for  $t := -3, -2.99..7$



Before we can use the root function, we have to provide Mathcad with an **initial guess** for the root:

$$t := 1.5$$

then we can ask Mathcad for the root:

$$\text{root}(f(t), t) = 1.414$$

Depending on the initial guesses, we may get different roots...

$$t := -2 \quad \text{root}(f(t), t) = -1.414$$

$$t := 4 \quad \text{root}(f(t), t) = 5$$

... or none at all:  $t := 3.5 \quad \text{root}(f(t), t) =$

not converging

As we can see in the Example, if the initial guess is **close to one of the roots** of the function, then the Mathcad's root function will find **that root**.

Figure 3: Example of use of the Mathcad's "root" function.

- To raise an expression to a power, type  $\wedge$  (<shift>-6).
2. To evaluate an expression, follow it by the = (equal) symbol.
  3. To define a variable, i.e. assign a value to it, use the symbol :=, produced when you type : (colon).
  4. Navigating through expressions can be accomplished using the arrow keys and the space bar. Expressions (or portions thereof) can be cut, copied and/or pasted using the system clipboard.
  5. Mathcad has several built-in functions (e.g.,  $\sqrt{x}$ ,  $\sin(x)$ ) and constants (e.g.,  $\pi$ ,  $e$ ) sometimes called predefined variables..
  6. You can define your own functions, e.g.  $f(x) := x+2$ .
  7. A function, say  $f(x)$ , can be graphed by first defining a range variable, e.g.,

$$x := -3, -2.99..2$$

(note that the two dots are obtained by typing the semicolon - ;). You can then choose **Create X-Y Plot** from the **Graphics** menu, and fill in the middle x-axis and y-axis placeholders with  $x$  and  $f(x)$ , respectively.

Graphs of more than one function can be included on the same plot, if you separate the names of functions to be plotted with commas in the middle y-axis placeholder.

8. Graphs can be scaled manually (by changing the extreme placeholders along each axis) or using the "zoom" feature.

To make **Zoom** selection available, click on a graph so that it is enclosed in the blue frame. You will then see the **X-Y Plot** menu appear, from which **Zoom** can be selected. On the same menu, you will also find the **Crosshair** selection, useful for reading coordinates off the graph.

9. The "Graph Format" dialog box can be used to change the appearance of the graph. To invoke the dialog, double-click on the graph.

One possible use for the "Graph Format" dialog is to change the graphs "trace type" from "lines" to "points".

10. When numerically solving an equation, one may use Mathcad's "root" function - Figure 3 contains an example of its use, taken from the template document.

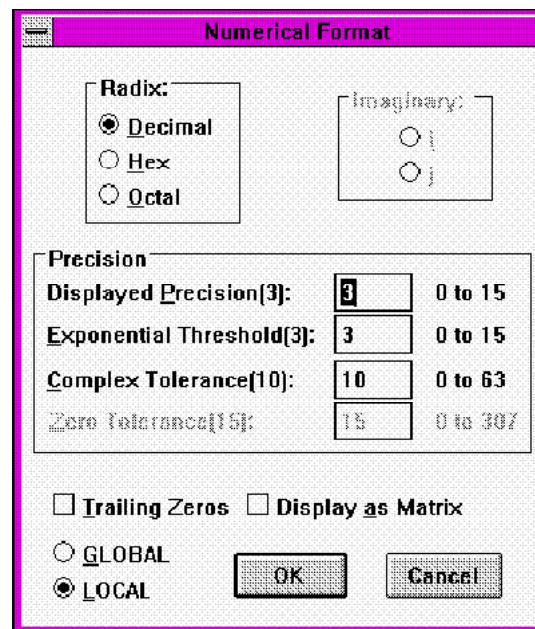


Figure 4: The "Numerical Format" dialog box can be used to control the number of digits displayed by Mathcad.

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## Some Numerical Considerations

In the first activity ("Mathcad Basics") you used Mathcad to evaluate  $\pi$ ; its response: 3.142. Unless instructed otherwise, Mathcad displays all answers to three decimal places. Internally, however, Mathcad keeps approximately 15 decimal digits for each number it uses. Keep in mind that when these numbers are used in calculations, the results are typically less accurate.<sup>1</sup>

In spite of losing accuracy during calculations, we can frequently use more than three decimal places reliably. In order to get Mathcad to display more digits, you can double-click on a number (which resulted from evaluation). This will bring up the "Numerical Format" dialog box shown in Figure 4. In this dialog, you can change the "Displayed Precision" from its default value (3) to any number up to 15. You could also make these changes affect all of your document, by first selecting the "GLOBAL" button. (The dialog box can also be invoked from the **Math** menu.)

We should mention here another situation in which Mathcad's numerical results may differ from the exact values. Whenever Mathcad evaluates a product whose first factor is 0 (or a quotient with 0 in the numerator), it always yields 0, regardless of the rest of the expression. While this "shortcut", taken for efficiency reasons, usually does not create problems, one should note that Mathcad will cheerfully evaluate:  $\frac{0}{0} = 0$  even though the expression has no value.

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<sup>1</sup> Many numerical procedures (e.g., Mathcad's root finder) try to get approximate results within a certain prescribed **tolerance** from the true (unknown) solution. In Mathcad, the tolerance is controlled by the variable TOL. The default value of TOL is 0.001, which roughly means that the procedures influenced by TOL (root finder, as well as numerical differentiation and integration which we will learn about later) keep roughly three digits of accuracy in their answer. The tolerance can be changed, e.g. assigning TOL:=0.000001 would roughly double the number of accurate digits in the answer. This increase in accuracy comes at the expense of slower performance. (You can also access the TOL value by selecting **Built-in Variables** from the **Math** menu.)