

Table of Contents

Introduction to Complex Variables

Chapter 1	Complex Numbers	1
	Preliminary considerations, Stereographic projection, Properties of complex numbers, Algebraic properties of complex numbers, DeMoivre's theorem, Roots, Distances and regions in the complex plane, The triangle inequality, Curves in the z -plane, Dynamics, Definitions and terminology	
Exercises		20
Chapter 2	Functions of a Complex Variable	25
	Basic concepts, The z -plane and ω -plane, Other methods of representing functions, Multiple-valued functions, One-to-one mappings, Elementary mappings, The point at infinity, Limits involving complex variables, Continuity for functions of a complex variable, Derivative of a complex function, Analytic or regular function, Cauchy-Riemann equations, Higher ordered derivatives, Another form for the Cauchy-Riemann equations, Harmonic functions, Chain rule for differentiation, Conformal mapping, Taylor series expansion, The Schwarz function	
Exercises		53
Chapter 3	Elementary Functions	59
	Polynomial functions, Rational algebraic functions, Linear fractional transformation, The exponential function, Periodic function, The trigonometric functions, Trigonometric identities, Addition and subtraction formula, Derivative formula, Hyperbolic functions, Derivatives of hyperbolic functions, Mappings associated with $\omega = \sin z$, Conjugate coordinates and operators, dot and cross product, Gradient operator, Divergence operator, curl operator, Lagrangian operator, Higher derivatives, Singularities, Classification of zero's, L'Hospital's rule, Continuity	
Exercises		80

Table of Contents

Chapter 4	Branch Points and Branch Cuts	87
	Introduction, Branch point at infinity, Riemann surface for n -valued functions, Logarithm function, Riemann surface for $\log z$, Inverse functions, Inverse trigonometric func- tions, Inverse hyperbolic functions, The function z^α , The function α^z , Additional functions and mappings	
Exercises		109
Chapter 5	Contour Integration	113
	Contour integration, Indefinite integration, Definite integrals, Closed curves, Basic properties of line integrals, Green's theorem in the plane, Cauchy integral theorem, Cauchy-Goursat theorem, Extended form of Cauchy's theorem, Principle of path deformation, Line integral independent of integration path, Cauchy's integral formula, Cases for the Cauchy integral formula, Several simple poles, Cauchy integral formula for multiply connected regions, Derivatives of analytic functions, Liouville's theorem, The Gauss mean value theorem, Morera's theorem	
Exercises		141
Chapter 6	Sequences, Series and the Residue Theorem	147
	Cauchy sequence, Taylor series expansion, Differentiation and integration of power series, Analytic continuation, Laurent series, Existence of Laurent series expansion, Singular points, zero of a function, Classification of singular points, Residue theory, Calculation of residues, Residue at infinity	
Exercises		171

Table of Contents

Chapter 7	Evaluation of Integrals	177
	Cauchy residue theorem, Improper integrals, Fourier integrals, Jordan lemma, Integrals of the type $\int_0^{2\pi} F(\cos \theta, \sin \theta) d\theta$, Poles on the x -axis, Integration around branch points, Integrals of the form $\int_0^\infty x^\alpha f(x) dx$ for $0 < \alpha < 1$, Integrals of the form $\int_0^\infty f(x) \log x dx$, Functions defined by integrals, Differentiation and integration of integrals with respect to parameters, Integrals involving the exponential function, Modification of integration path	
Exercises		208
Chapter 8	Introduction to Potential Theory	215
	Vector fields, Complex potential, Circulation of fluid, The Laplace equation, Summary, Laplace's equation and conjugate harmonic functions, Harmonic functions, Boundary value problems, Green's formula, Properties of harmonic functions, Some elementary boundary value problems, Poisson integral formula for the half plane, Poisson integral formula for the disk, More difficult boundary value problems,	
Exercises		246
Chapter 9	Miscellaneous Applications	253
	Summation of series, The argument principle, Two-dimensional fluid flow, Alternate approach to fluid flow, Complex potential, Blasius theorem, Electrostatic potential, Schwarz-Christoffel transformation, Alternative forms, Change of variables, Difficulties, The error function	
Exercises		281

Table of Contents

Chapter 10	Fourier, Laplace and \mathcal{Z}-transforms	289
	Integral transforms, Fourier series, Fourier transforms, Operational properties, Solution of linear partial differential equations, The Laplace transform, Inverse Laplace transform, Laplace inversion theorem, Alternate development for the inverse Laplace transform, The \mathcal{Z} -transform, The inverse \mathcal{Z} -transform, Properties of the \mathcal{Z} -transform	
Exercises	332
Chapter 11	Special Functions	339
	The gamma function, Properties of the gamma function, The beta function, The hypergeometric function, Analysis of the hypergeometric function, Special values for the hypergeometric function, Generalized hypergeometric function, Functions defined by integrals, Elliptic integrals, Jacobi elliptic functions, Differentiation and integration, Special values, Addition formulas, Periodicity, The theta functions	
Exercises	369
Bibliography	374
Appendix A	Units of Measurement	376
Appendix B	Short Table of Mappings	378
Appendix C	Solutions and Hints to Selected Problems	388
Index	426