



# Mathematical Methods for Partial Differential Equations

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## PREFACE

This is an introductory text on mathematical methods used in the study of partial differential equations and boundary value problems. The text contains material suitable for a two semester upper division undergraduate or beginning graduate course in fundamental concepts associated with partial differential equations. It is assumed that the reader has background knowledge from the subject areas of calculus, differential equations, and Laplace transforms. The material emphasizes mathematical methods and qualitative theory rather than numerical methods. The material is suitable for mathematicians, engineers, physicist and scientist who desire an understanding of the basics associated with partial differential equations and boundary value problems.

Chapter one reviews concepts from the study of linear ordinary differential equations (ODE's) and introduces some elementary ideas associated with first and second order partial differential equations (PDE's). Chapter two introduces orthogonal functions and Sturm-Liouville systems. Chapter three uses orthogonal functions to develop Fourier series. Knowledge of the Fourier series leads to the Fourier integral representation of functions. Chapter four introduces the heat equation and applications for both finite and infinite domains. The method of separation of variables is introduced along with applications of Fourier series and Fourier integrals. Chapter five introduces the wave equation with selected applications. Chapter six introduces the Laplace equation, Poisson's equation and the Helmholtz equation along with selected applications. The chapters four, five and six emphasizes separation of variables, Fourier series and Fourier integrals. Chapter seven introduces the Heaviside unit step function and Dirac delta function along with techniques associated with Laplace, Fourier exponential, Fourier sine, Fourier cosine and Hankel transforms together with selected finite transforms associated with Sturm-Liouville systems. Chapter eight develops basic concepts associated with Green's functions for ordinary differential equations. Chapter nine develops Green's functions for partial differential equations.

At the end of each chapter there is a variety of exercises where solution techniques can be applied. In addition, there are many exercise problems where new concepts are introduced and so students should be encouraged to read all the exercises. The example problems are concluded using the blackbox notation represented by the symbol ■.

The Appendix A contains units of measurements from the Système International d'Unités along with some selected physical constants. The Appendix B contains solutions to selected exercises. All footnotes associated with influential scientists whose theories have contributed to the mathematics in this text have been collected into the Appendix C. The Appendix D contains a short table of definite integrals.

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