6.7 Coordinates

DEF $\rightarrow$ p.294

The coordinate vector of $\overrightarrow{v}$ with respect to the ordered basis $S = \{\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_n}\}$ is

$$\left[\overrightarrow{v}\right]_S = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

where

$$\overrightarrow{v} = c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + \cdots + c_n\overrightarrow{v_n}$$

By Theorem 6.5, $\left[\overrightarrow{v}\right]_S$ is unique.

EXAMPLE 1 Let $V = R^3$ and $S = \{\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}\}$.
Then for any vector $\overrightarrow{x}$ in R^3, $\left[\overrightarrow{x}\right]_S = \overrightarrow{x}$.

EXAMPLE 2 Let $V = R^3$ and
$S = \{(1,0,0),(1,1,0),(1,1,1)\}$. Given $\overrightarrow{x} = (2,-1,3)$, find $\left[\overrightarrow{x}\right]_S$.

$$\begin{pmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 1 & | & -1 \\ 0 & 0 & 1 & | & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & -4 \\ 0 & 0 & 1 & | & 3 \end{pmatrix}$$

Answer: $\left[\overrightarrow{x}\right]_S = \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}$.

EXAMPLE 3

$V = P_2$; $T = \{t^2, t, 1\}$. Given $\overrightarrow{v} = 4t^2 - 2t + 3$, find $\left[\overrightarrow{v}\right]_T$.

Answer: $\left[\overrightarrow{v}\right]_T = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$.

EXAMPLE 4 (Problem 4 p.303) Use V and $\vec{v}$ as in **EXAMPLE 3** and $S = \{t^2 - t + 1, t + 1, t^2 + 1\}$.
Find $\left[\vec{v}\right]_S$.

Set $4t^2 - 2t + 3 =$
$c_1(t^2 - t + 1) + c_2(t + 1) + c_3(t^2 + 1)$.
Compare coefficients of the same power of t:

$$\begin{aligned} 4 &= c_1 + c_3 \\ -2 &= -c_1 + c_2 \\ 3 &= c_1 + c_2 + c_3 \end{aligned}$$

$$\left(\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ -1 & 1 & 0 & -2 \\ 1 & 1 & 1 & 3 \end{array}\right) \to \left(\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Answer: $\left[\vec{v}\right]_S = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

Check:
$1(t^2 - t + 1) - 1(t + 1) + 3(t^2 + 1) = 4t^2 - 2t + 3$.

EXAMPLE 5 (Problem 12 p.304)

Let $V = M_{22}$;

$$S = \left\{ \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}.$$

Given $\left[\vec{v}\right]_S = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}$, find $\vec{v}$.

Answer: $\vec{v} =$

$$1\begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix} + 2\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}.$$