4.3 Intro to Linear Transformations

DEF **Linear Transformation** ($\rightarrow$ p. 201)

A function $L : R^n \rightarrow R^m$ such that

(a) $L(\vec{u} + \vec{v}) = L(\vec{u}) + L(\vec{v}),$ for every $\vec{u}$ and $\vec{v}$ in R^n.

(b) $L(c\vec{u}) = cL(\vec{u}),$ for every vector $\vec{u}$ in R^n and every scalar c.

$L(\vec{u})$ is the **image** of $\vec{u}$

Range of L - set of all images of vectors in R^n.

EXAMPLE 1

Consider $L : R^2 \to R^2$ defined by $L(x,y) = (x,0)$.
Check:

(**a**) $LHS = L(\overrightarrow{u} + \overrightarrow{v})$
$= L((x,y) + (x',y'))$
$= L(x + x', y + y')$
$= (x + x', 0)$
$RHS = L(\overrightarrow{u}) + L(\overrightarrow{v})$
$= L(x,y) + L(x',y')$
$= (x,0) + (x',0)$
Therefore, $LHS = RHS$ for every $\overrightarrow{u}$ and $\overrightarrow{v}$ in R^n.

(**b**) $LHS = L(c\overrightarrow{u}) = L(cx, cy) = (cx, 0)$
$RHS = cL(\overrightarrow{u}) = c(x,0)$
Therefore, $LHS = RHS$ for every vector $\overrightarrow{u}$ in R^n and every scalar c.

Conclude: L is a linear transformation.

Projection of the entire R^2 plane onto the x-axis.
(also $\to$ Example 1 p.201)

EXAMPLE 2

Consider $L : R^2 \rightarrow R^3$ defined by

$$L(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x+y \\ 1 \\ x-y \end{bmatrix}.$$ Check:

(a) $LHS = L(\overrightarrow{u} + \overrightarrow{v})$

$$= L(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix})$$

$$= L(\begin{bmatrix} x+x' \\ y+y' \end{bmatrix})$$

$$= \begin{bmatrix} x+x'+y+y' \\ 1 \\ x+x'-(y+y') \end{bmatrix}$$

$$RHS = L(\overrightarrow{u}) + L(\overrightarrow{v}) =$$

$$= L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) + L\left(\begin{bmatrix} x' \\ y' \end{bmatrix}\right)$$

$$= \begin{bmatrix} x+y \\ 1 \\ x-y \end{bmatrix} + \begin{bmatrix} x'+y' \\ 1 \\ x'-y' \end{bmatrix}$$

$$= \begin{bmatrix} x+y+x'+y' \\ 2 \\ x-y+x'-y' \end{bmatrix}$$

$LHS \neq RHS$ therefore L is not a linear transformation.

($\rightarrow$ Example 2 p. 202)

Other types of linear transformations: dilation, contraction, reflection, rotation. ($\rightarrow$ Examples in the text)

$\rightarrow$ Theorems 4.6, 4.7, 4.8