1.4. Properties of Matrix Operations

TH 1.1 (→ p. 33) Properties of Matrix Addition

(a) For all $m \times n$ matrices A and B

$$A + B = B + A$$

(b) For all $m \times n$ matrices $A, B,$ and C

$$A + (B + C) = (A + B) + C$$

(c) There exists a unique $m \times n$ matrix O such that for all $m \times n$ matrices A

$$A + O = A$$

(O is called the $m \times n$ **zero matrix**)

(d) For each $m \times n$ matrix A, there exists a unique $m \times n$ matrix D such that

$$A + D = O$$

(D is called the **negative** of A)

TH 1.2 (→ p. 34) Properties of Matrix Multiplication

(a) $A(BC) = (AB)C$

(b) $A(B+C) = AB+AC$

(c) $(A+B)C = AC+BC$

(In each of the equations, A, B, and C are chosen so that both sides are matrices of the same size.)

For any $m \times n$ matrix A, the following property holds:

$$AI_n = I_mA = A$$

where I_m (or I_n) denotes the $m \times m$ (or $n \times n$) identity matrix.

For any $n \times n$ matrix A and a nonnegative integer p, we define the p-th **power** of A as follows:

$$A^p = \underbrace{A \cdots A}_{p \text{ factors}}; \quad A^0 = I_n$$

TH 1.3 (→ p. 38) Properties of Scalar Multiplication

For all matrices A and B and scalars c and d,

(a) $c(dA) = (cd)A$

(b) $(c + d)A = cA + dA$

(c) $c(A + B) = cA + cB$

(d) $A(cB) = c(AB) = (cA)B$

TH 1.4 (→ p. 38) Properties of Transpose

(a) $(A^T)^T = A$

(b) $(A + B)^T = A^T + B^T$

(c) $(AB)^T = B^T A^T$

(d) $(cA)^T = cA^T$

DEF (→ p. 39) A matrix A is called **symmetric** if

$$A^T = A$$