1.2. Matrices

DEF ($\rightarrow$ p. 10) An $m \times n$ **matrix** A - rectangular array of numbers:

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_{n \text{ columns}} \Bigg\} \, m \text{ rows}$$

a_{ij} - i,jth **element** (or **entry**) of A.

An $n \times 1$ matrix or a $1 \times n$ matrix are called n-**vectors**.

EXAMPLE 1

$$B = \begin{pmatrix} 1 & 0 & -2 & 1 \\ 3 & 1 & 2 & 4 \\ 1 & 2 & 0 & 5 \end{pmatrix}$$

is a 3×4 matrix with

- $b_{11} = 1, b_{12} = 0, b_{21} = 3,$ etc.,
- second row: $\begin{pmatrix} 3 & 1 & 2 & 4 \end{pmatrix}$,
- fourth column: $\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$.

More examples → Example 1 p. 10.

A matrix can be represented by:

- an uppercase letter: $A, B,$ etc.
- a representative element enclosed in brackets: $[a_{ij}], [b_{ij}],$ etc.,
- a rectangular array of numbers enclosed in brackets or parentheses.

EXAMPLE 2

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

is an example of a **square** matrix (generally, $n \times n$ - here: 3×3).
In such a matrix, $a_{11}, a_{22}, \ldots, a_{nn}$ forms the **main diagonal** (here: $(1, 0, 0)$).

Special kinds of square matrices:

	Definition	Example
diagonal m. $\rightarrow$ p. 11	square matrix with $a_{ij} = 0$ for $i \neq j$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
scalar matrix $\rightarrow$ p. 11	diagonal matrix with $a_{ii} = c$ for all i	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
identity matrix	scalar matrix with $a_{ii} = 1$ for all i	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
upper triangular m.	square matrix with $a_{ij} = 0$ for $i > j$	$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$
lower triangular m.	square matrix with $a_{ij} = 0$ for $i < j$	$\begin{pmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 3 & 4 & 0 \end{pmatrix}$

DEF → p. 12

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are **equal** if

- they have the same size, $m \times n$, and
- $a_{ij} = b_{ij}$ for all i and j $(1 \le i \le m, 1 \le j \le n)$.

DEF → p. 12

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m \times n$ matrices then their **sum** is $A + B = [a_{ij} + b_{ij}]$.
The sum of two matrices of *different* sizes is *undefined*.

EXAMPLE 3

$$\begin{pmatrix} 1 & 5 \\ -1 & 0 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 0 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ -1 & 0 \\ 2 & 0 \end{pmatrix}$$

DEF → p. 13

If $A = [a_{ij}]$ is a matrix and c is a scalar, then the **scalar multiple** of A by c is $cA = [ca_{ij}]$.

EXAMPLE 4

$$-2\begin{pmatrix} 2 & -2 & 1 \\ 3 & 4 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 4 & -2 \\ -6 & -8 & 0 \end{pmatrix}$$

DEF → p. 13

If $A_1, A_2, \ldots, A_k$ are $m \times n$ matrices and $c_1, c_2, \ldots, c_k$ are scalars then

$$c_1A_1 + c_2A_2 + \cdots + c_nA_n$$

is called a **linear combination** of $A_1, A_2, \ldots, A_k$.

DEF → p. 14

If $A = [a_{ij}]$ is an $m \times n$ matrix then the **transpose** of A is the $n \times m$ matrix $A^T = [a_{ji}]$.

EXAMPLE 5

If $A = \begin{pmatrix} 2 & 1 \\ -3 & 0 \\ 1 & -1 \end{pmatrix}$ then

$$A^T = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 0 & -1 \end{pmatrix}.$$