1.1. Linear Systems

Linear Equations ($\rightarrow$ p. 1)

Examples:

$$5x = 6$$

or

$$x_1 + 3x_2 - x_3 = 7$$

or

$$0x = 1$$

Generally,

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

Typically,

- $a_1, \ldots, a_n, b$ - known constants,
- $x_1, \ldots, x_n$ - unknowns.

A **solution** - a sequence $s_1, \ldots, s_n$ such that when $x_1 = s_1, \ldots, x_n = s_n$, the equation is satisfied.

In our examples:

$$5x = 6$$

has

$$x = \frac{6}{5}$$

as the **only solution**.

$$x_1 + 3x_2 - x_3 = 7$$

has

$$x_1 = 1, x_2 = 2, x_3 = 0$$

and

$$x_1 = 8, x_2 = 0, x_3 = 1$$

and **infinitely many** other **solutions**.

$$0x = 1$$

has **no solutions**.

System of m linear equations in n unknowns:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}
$$

where a_{ij} is the coefficient (known number) associated with the j-th unknown (x_j) in the i-th equation.

A **solution** - a sequence $s_1, \ldots, s_n$ such that when $x_1 = s_1, \ldots, x_n = s_n$, each of the m equations is satisfied.

EXAMPLE 1 Solve the system

$$
\begin{aligned}
2x - y &= -4 \\
-3x - 2y &= -1
\end{aligned}
$$

using the method of elimination.

Divide eq_1 by 2:

$$\begin{aligned} x - \frac{1}{2}y &= -2 \\ -3x - 2y &= -1 \end{aligned}$$

Add $3 \bullet \text{eq}_1$ to eq_2:

$$\begin{aligned} x - \frac{1}{2}y &= -2 \\ -\frac{7}{2}y &= -7 \end{aligned}$$

Divide eq_2 by $-\frac{7}{2}$:

$$\begin{aligned} x - \frac{1}{2}y &= -2 \\ y &= 2 \end{aligned}$$

Add $\frac{1}{2} \bullet \text{eq}_2$ to eq_1:

$$\begin{aligned} x &= -1 \\ y &= 2 \end{aligned}$$

One solution.

→ Read Section 1.1. In particular, see:

- Example 2 p. 3 (no solutions)
- Example 4 p. 4 (infinitely many solutions).