SECTION 5.5 (SUBSTITUTION) PRACTICE QUESTION SOLUTIONS

1. $$\int \frac{x^2 - 4x}{x^3 - 6x^2 + 7}\,dx = \frac{1}{3}\cdot\int \frac{1}{u}\,du$$

$$= \frac{1}{3}\cdot\ln(|u|) + C = \frac{1}{3}\cdot\ln\left(\left|x^3 - 6x^2 + 7\right|\right) + C$$

Substitute
$u = x^3 - 6x^2 + 7$
$du = \left(3x^2 - 12x\right)dx$
$\frac{du}{3} = \left(x^2 - 4x\right)\cdot dx$

2. $$\int x^2\cdot e^{x^3}\,dx = \frac{1}{3}\cdot\int e^u\,du$$

$$= \frac{1}{3}\cdot e^u + C = \frac{1}{3}\cdot e^{x^3} + C$$

Substitution:
$u = x^3$
$du = 3\,x^2dx$
$\frac{1}{3}\cdot du = x^2dx$

3. $$\int \frac{(\ln(x))^2}{x}\,dx = \int u^2\,du$$

$$= \frac{u^3}{3} + C = \frac{(\ln(x))^3}{3} + C$$

Substitution:
$u = \ln(x)$
$du = \frac{1}{x}dx$

4. $$\int \frac{\sin(x)}{\sqrt{\cos(x)}}\,dx = -\int u^{\frac{-1}{2}}\,du$$

$$= -2u^{\frac{1}{2}} + C = -2\cdot\sqrt{\cos(x)} + C$$

Substitution:
$u = \cos(x)$
$du = -\sin(x)\cdot dx$
$-du = \cos(x)\,dx$

5. $$\int \sec(2x + 1)\cdot\tan(2x + 1)\,dx = \frac{1}{2}\cdot\int \sec(u)\cdot\tan(u)\,du$$

$$= \frac{1}{2}\cdot\sec(u) + C = \frac{1}{2}\cdot\sec(2x + 1) + C$$

Substitution:
$u = 2x + 1$
$du = 2dx$
$\frac{1}{2}du = dx$

6. $\displaystyle\int \frac{x^3}{\sqrt{x^2+3}}\,dx = \int \frac{x^2\cdot x}{\sqrt{x^2+3}}\,dx$

Substitution: $u = x^2 + 3 \qquad x^2 = u - 3$

$du = 2x\cdot dx \qquad \frac{1}{2}du = x\cdot dx$

$$= \frac{1}{2}\cdot\int \frac{u-3}{\sqrt{u}}\,du = \frac{1}{2}\cdot\int u^{\frac{1}{2}} - 3u^{\frac{-1}{2}}\,du = \frac{1}{2}\cdot\left(\frac{2}{3}\cdot u^{\frac{3}{2}} - 3\cdot 2\cdot u^{\frac{1}{2}}\right) + C$$

$$= \frac{1}{3}\cdot\left(x^2+3\right)^{\frac{3}{2}} - 3\cdot\sqrt{x^2+3} + C$$

7. $\displaystyle\int \frac{\sin\left(\sqrt{x-3}\right) + e^{\sqrt{x-3}}}{\sqrt{x-3}}\,dx = 2\cdot\int \sin(u) + e^u\,du$

Substitution: $u = \sqrt{x-3}$

$du = \frac{1}{2\cdot\sqrt{x-3}}dx$

$2du = \frac{1}{\sqrt{x-3}}\cdot dx$

$$= 2\cdot\left(-\cos(u) + e^u\right) + C = 2\cdot\left(-\cos\left(\sqrt{x-3}\right) + e^{\sqrt{x-3}}\right) + C$$

8. $\displaystyle\int_0^2 (3x-2)\cdot\sqrt{3x^2 - 4x + 1}\,dx$

Substitution: $u = 3x^2 - 4x + 1$

$x = 0 \quad ==> \quad u = 1$

$x = 2 \quad ==> \quad u = 12 - 8 + 1 = 5$

$du = (6x-4)dx = 2\cdot(3x-2)\cdot dx \qquad \frac{1}{2}\cdot du = (3x-2)dx$

$$= \frac{1}{2}\cdot\int_1^5 \sqrt{u}\,du$$

$$= \frac{1}{2}\cdot\left(\frac{2}{3}\cdot u^{\frac{3}{2}}\right)\Bigg|_{u=1}^{u=5}$$

$$= \frac{1}{3}\cdot\left(5^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{1}{3}\cdot\left(5\cdot\sqrt{5} - 1\right)$$

9. $\displaystyle\int_{-1}^{0} \left(\sec\left(e^{x}\right)\right)^{2} \cdot e^{x}\,dx$

Substitution: $u = e^{x}$ $\quad x = -1 \implies u = \frac{1}{e}$

$du = e^{x}dx$ $\quad x = 0 \implies u = 1$

$$= \int_{\frac{1}{e}}^{1} (\sec(u))^{2}\,du$$

$$= (\tan(u))\Big|_{u=\frac{1}{e}}^{u=1}$$

$$= \tan(1) - \tan\left(\frac{1}{e}\right)$$

10. The following evaluation is incorrect

$$\int_{2}^{e} \frac{1}{x\cdot\sqrt{\ln(x)}}\,dx \overset{\text{A}}{=} \int_{2}^{e} \frac{1}{\sqrt{u}}\,du \overset{\text{B}}{=} \int_{2}^{e} u^{\frac{-1}{2}}\,du \overset{\text{C}}{=} \left(2u^{\frac{1}{2}}\right)\Big|_{u=2}^{u=e} \overset{\text{D}}{=} 2\cdot\sqrt{e} - 2\cdot\sqrt{2}$$

since in Step A, the limits of integration were not properly changed.

Here is the correct evaluation:

$$\int_{2}^{e} \frac{1}{x\cdot\sqrt{\ln(x)}}\,dx$$

Substitution: $u = \ln(x)$ $\quad x = 2 \implies u = \ln(2)$

$du = \frac{1}{x}dx$ $\quad x = e \implies u = 1$

$$= \int_{\ln(2)}^{1} \frac{1}{\sqrt{u}}\,du$$

$$= \int_{\ln(2)}^{1} u^{\frac{-1}{2}}\,du = \left(2u^{\frac{1}{2}}\right)\Big|_{u=\ln(2)}^{u=1} = 2 - 2\cdot\sqrt{\ln(2)}$$