Arc Length Problems with Answers

Find the length of the parametric curve:

- $x = t^2,\ y = 4t^3 - 1$ for $-1 \le t \le 1$.

Answer: $\int_{-1}^{1} \sqrt{(2t)^2 + (12t^2)^2}dt = \frac{37}{27}\sqrt{37} - \frac{1}{27}$
Hints:

- $\sqrt{t^2} = |t|$;
- Use substitution: $u = 1 + 36t^2$

- $x = t^2$, $y = 2t$ for $0 \le t \le 2$.

Answer: $\int_{0}^{2} \sqrt{(2t)^2 + (2)^2}dt = 2\sqrt{5} + \ln(2 + \sqrt{5})$
Hint: Trig. substitution: $t = \tan\theta$ should lead to $\int \sec^3\theta d\theta$ - see Example 8 p. 481

- $x = \arcsin t,\ y = \ln\sqrt{1 - t^2}$ for $0 \le t \le \frac{1}{2}$.

Answer: $\int_{0}^{1/2} \sqrt{\left(\frac{1}{\sqrt{1-t^2}}\right)^2 + \left(\frac{t}{1-t^2}\right)^2}dt = \frac{1}{2}\ln 3$
Hint: Simplify the integrand, then use the trig. substitution $t = \sin\theta$

- $x = \sqrt{t},\ y = 3t - 1$ for $0 \le t \le 1$.

Answer: $\int_{0}^{1} \sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2 + 3^2}dt = \frac{1}{12}(6\sqrt{37} + \ln(6 + \sqrt{37}))$
Hints:

- Substitution $t = u^2$ leads to $\int_{0}^{1} \sqrt{1 + 36u^2}du$
- Trig. substitution $u = \frac{1}{6}\tan\theta$ yields an integral of $\sec^3\theta$ - see Example 8 p. 481