MATH 206
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QUIZ 8

SOLUTION KEY

Summer 2001

1. Solve the differential equation $xy\dfrac{dy}{dx} = \sec y$

   Separate the variables: $y\cos y\,dy = \dfrac{1}{x}dx$
   Integrate: $\int y\cos y\,dy = \int \dfrac{1}{x}dx$

   The left hand side can be integrated by parts with $u = y,\ dv = \cos y\,dy;\ du = dy,\ v = \sin y$

   This leads to: $y\sin y - \int \sin y\,dy = y\sin y + \cos y + c_1$

   Therefore

   $y\sin y + \cos y = \ln|x| + C$

2. Solve the differential equation $\dfrac{dy}{dx} = e^x - y$

   Rewrite in the standard form of a linear equation: $\dfrac{dy}{dx} + y = e^x$

   Integrating factor: $I(x) = e^{\int dx} = e^x$

   Multiply both sides of the differential equation by $I(x)$:

   $e^x(\dfrac{dy}{dx} + y) = e^x e^x$ yields

   $(e^x y)' = e^{2x}$

   Integrate both sides

   $e^x y = \int e^{2x}dx = \dfrac{1}{2}e^{2x} + C$

   The general solution: $y = \dfrac{1}{2}e^x + Ce^{-x}$