MATH 206
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QUIZ 6
SOLUTION KEY

Summer 2001

1. $I = \int \frac{1}{x^2 - x} dx = \int \frac{1}{(x-1)x} dx$ Integrand is a proper rational function - no division necessary.

   Partial fraction expansion: $\frac{1}{(x-1)x} = \frac{A}{x} + \frac{B}{x-1}$

   Multiply both sides by $(x-1)x$: $\quad 1 = A(x-1) + Bx$

   when $x = 1,\ 1 = B$

   when $x = 0,\ 1 = -A \Rightarrow A = -1$

   $I = \int \left(\frac{-1}{x} + \frac{1}{x-1} \right) dx = -\ln|x| + \ln|x-1| + C$

2. $\frac{x^4+1}{x^2(x^2+1)} = \frac{x^4+1}{x^4+x^2}$ is an improper rational function - must divide first

$$
\begin{array}{rrrr}
 & 1 & & \\
 & - & - & - \\
x^4 + x^2 \;| & x^4 & & +1 \\
 & -x^4 & -x^2 & \\
 & - & - & - \\
 & & -x^2 & +1
\end{array}
$$

From long division: $\frac{x^4+1}{x^4+x^2} = 1 + \frac{-x^2+1}{x^4+x^2}$

Partial fraction expansion: $\frac{-x^2+1}{x^4+x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$-x^2 + 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$

$-x^2 + 1 = Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$

Compare coefficients of like terms on both sides:

$x^3:\ 0 = A + C$

$x^2:\ -1 = B + D$

$x:\ 0 = A$

$1:\ 1 = B$

Answer: $\frac{x^4+1}{x^4+x^2} = 1 + \frac{-x^2+1}{x^4+x^2} = 1 + \frac{1}{x^2} + \frac{-2}{x^2+1}$

3. $\int_{-\infty}^{0} e^{-x}\,dx = \lim_{a \to -\infty} \int_a^0 e^{-x} dx = \lim_{a \to -\infty} [-e^{-x}]_a^0 = \lim_{a \to -\infty} (-e^0 + e^{-a}) = -1 + \infty$ Diverges

4. $\int_0^1 \frac{1}{\sqrt[3]{x}}\,dx = \lim_{a \to 0^+} \int_a^1 x^{-1/3}\,dx = \lim_{a \to 0^+} \left[\frac{3}{2} x^{2/3} \right]_a^1 = \lim_{a \to 0^+} \left(\frac{3}{2} - \frac{3}{2} a^{2/3} \right) = \frac{3}{2} - 0 = \frac{3}{2}$ (converges)