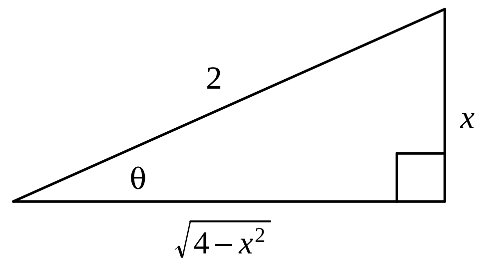
MATH 206
Dr. Bogacki

QUIZ 5
SOLUTION KEY

Summer 2001

1. $I = \int \frac{1}{x^2\sqrt{4-x^2}}dx$

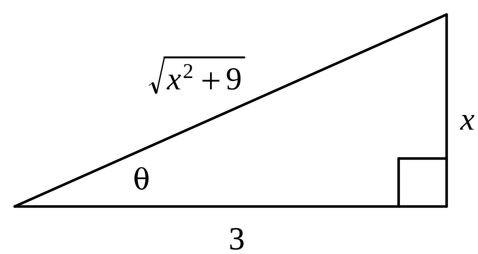


Trig. substitution: $x = 2\sin\theta,\ dx = 2\cos\theta\, d\theta,$
$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$

$$I = \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\,(2)\cos\theta} = \frac{1}{4}\int \frac{dx}{\sin^2\theta}$$

$$= \frac{1}{4}\int \csc^2\theta\, d\theta = \frac{-1}{4}\cot\theta + C = \frac{-1}{4}\frac{\sqrt{4-x^2}}{x} + C$$

2. $I = \int \frac{1}{(x^2+9)^{3/2}}\, dx$



Trig. substitution: $x = 3\tan\theta,\ dx = 3\sec^2\theta\, d\theta,$
$\sqrt{x^2+9} = \sqrt{9\tan^2\theta+9} = 3\sec\theta$

$$I = \int \frac{3\sec^2\theta\, d\theta}{(3\sec\theta)^3} = \frac{1}{9}\int \frac{d\theta}{\sec\theta}$$

$$= \frac{1}{9}\int \cos\theta\, d\theta = \frac{1}{9}\sin\theta + C = \frac{1}{9}\frac{x}{\sqrt{x^2+9}} + C$$

3. For each of the following integrals, rewrite it using the appropriate trigonometric substitution. Do not evaluate.

(a) $\int \frac{x^2}{\sqrt{x^2-1}}\, dx = \int \frac{\sec^2\theta}{\tan\theta}\sec\theta\tan\theta d\theta = \int \sec^3\theta d\theta$

(trig. subst.: $x = \sec\theta$, $dx = \sec x \tan x dx$, $\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \tan\theta$)

(b) $\int \frac{x^2}{x^2+2}\, dx = \int \frac{2\tan^2\theta}{(\sqrt{2}\ \sec\theta)^2}\sqrt{2}\ \sec^2\theta\, d\theta = \sqrt{2}\int \tan^2\theta\, d\theta$

(trig. subst.: $x = \sqrt{2}\ \tan\theta$, $dx = \sqrt{2}\ \sec^2\theta d\theta$, $\sqrt{x^2+2} = \sqrt{2\tan^2\theta + 2} = \sqrt{2}\ \sec\theta$)