

1. $I = \int x e^{-x} dx$

Integration by parts: $u = x$, $du = dx$; $dv = e^{-x} dx$, $v = -e^{-x}$.

$$I = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$$

2. $I = \int e^x \cos(2x) dx$

Integration by parts: $u = \cos(2x)$, $du = -2 \sin(2x) dx$; $dv = e^x dx$, $v = e^x$

$$I = e^x \cos(2x) + 2 \int e^x \sin(2x) dx$$

Integration by parts: $u_1 = \sin(2x)$, $du_1 = 2 \cos(2x) dx$; $dv_1 = e^x dx$, $v_1 = e^x$

$$I = e^x \cos(2x) + 2(e^x \sin(2x) - 2 \int e^x \cos(2x) dx) = e^x \cos(2x) + 2e^x \sin(2x) - 4I$$

Solve for I :

$$5I = e^x \cos(2x) + 2e^x \sin(2x)$$

$$\text{Answer: } I = \frac{1}{5} e^x \cos(2x) + \frac{2}{5} e^x \sin(2x) + C$$

3. $I = \int x^5 \ln x dx$

Integration by parts: $u = \ln x$, $du = \frac{1}{x} dx$; $dv = x^5 dx$, $v = \frac{x^6}{6}$

$$I = \frac{x^6}{6} \ln x - \int \frac{x^6}{6} \left(\frac{1}{x} \right) dx = \frac{x^6}{6} \ln x - \frac{1}{6} \int x^5 dx = \frac{x^6}{6} \ln x - \frac{x^6}{36} + C$$

4. $I = \int x^2 \sin x dx$

Integration by parts: $u = x^2$, $du = 2x dx$; $dv = \sin x dx$, $v = -\cos x$

$$I = -x^2 \cos x + 2 \int x \cos x dx$$

Integration by parts: $u_1 = x$, $du_1 = dx$; $dv_1 = \cos x dx$, $v_1 = \sin x$

$$I = -x^2 \cos x + 2(x \sin x - \int \sin x dx) = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

5. $I = \int \ln(2x) dx$

Integration by parts: $u = \ln(2x)$, $du = \frac{1}{2x}(2)dx = \frac{1}{x}dx$; $dv = dx$, $v = x$

$$I = x \ln(2x) - \int x \left(\frac{1}{x} \right) dx = x \ln(2x) - \int dx = x \ln(2x) - x + C$$