

1. Find each indefinite integral.

$$\begin{aligned} \text{(a)} \quad & \int 2x(x^2 + 3)^4 dx \quad (\text{Use substitution: } u = x^2 + 3; \, du = 2x dx) \\ & = \int u^4 du = \frac{u^5}{5} + C = \frac{(x^2 + 3)^5}{5} + C \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int \cos(2\theta) d\theta \quad (\text{Use substitution: } u = 2\theta; \, du = 2d\theta; \, \frac{1}{2}du = d\theta) \\ & = \frac{1}{2} \int \cos u \, du = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2\theta + C \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \int \frac{(\ln x)^3}{x} dx \quad (\text{Use substitution: } u = \ln x; \, du = \frac{1}{x} dx) \\ & = \int u^3 du = \frac{u^4}{4} + C = \frac{(\ln x)^4}{4} + C \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & \int e^{\csc x} \csc x \cot x dx \quad (\text{Use substitution: } u = \csc x; \, du = -\csc x \cot x dx) \\ & = -\int e^u du = -e^u + C = -e^{\csc x} + C \end{aligned}$$

2. Find each definite integral. (Simplify your answer.)

$$\begin{aligned} \text{(a)} \quad & \int_0^1 \frac{-x^2}{x^3 + 1} dx \quad (\text{Use substitution: } u = x^3 + 1; \, du = 3x^2 dx; \, -\frac{1}{3}du = -x^2 dx) \\ & \quad \text{Change the limits of integration:} \quad \begin{array}{l} x = 0 \Rightarrow u = 1 \\ x = 1 \Rightarrow u = 2 \end{array} \\ & = -\frac{1}{3} \int_1^2 \frac{1}{u} du = -\frac{1}{3} [\ln |u|]_1^2 = -\frac{1}{3} (\ln 2 - \ln 1) = \frac{-\ln 2}{3} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} \quad (\text{Use substitution: } u = \ln x; \, du = \frac{1}{x} dx) \\ & \quad \text{Change the limits of integration:} \quad \begin{array}{l} x = e \Rightarrow u = \ln e = 1 \\ x = e^4 \Rightarrow u = \ln e^4 = 4 \end{array} \\ & = \int_1^4 u^{-1/2} du = [2u^{1/2}]_1^4 = (2\sqrt{4} - 2\sqrt{1}) = 4 - 2 = 2 \end{aligned}$$