MATH 205
Spring 2002

SAMPLE QUESTIONS FOR QUIZ 4
SOLUTION KEY FOR PROBLEMS 7-10

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7. $\frac{d}{dx}\left[(2x)^3\right] = \lim_{h \to 0} \frac{[2\cdot(x+h)]^3 - (2x)^3}{h}$ — The definition of derivative

$= \lim_{h \to 0} \frac{8\cdot\left(x^3 + 3x^2 h + 3x\cdot h^2 + h^3\right) - 8\cdot x^3}{h}$ — Expand in the numerator

$= \lim_{h \to 0} \frac{8x^3 + 24\cdot x^2\cdot h + 24\cdot x\cdot h^2 + 8\cdot h^3 - 8\cdot x^3}{h}$

$= \lim_{h \to 0} \frac{24\cdot x^2\cdot h + 24\cdot x\cdot h^2 + 8\cdot h^3}{h}$ — Simplify in the numerator

$= \lim_{h \to 0} \frac{h\cdot\left(24\cdot x^2 + 24\cdot x\cdot h + 8\cdot h^2\right)}{h}$ — Factor in the numerator

$= \lim_{h \to 0} \left(24\cdot x^2 + 24\cdot x\cdot h + 8\cdot h^2\right)$ — As h approaches 0, $h \neq 0$. Consequently, both the numerator and the denominator can be divided by h.

$= 24\cdot x^2$ — As h approaches 0, both $24\cdot x\cdot h$ and $8\cdot h^2$ approach 0.

8. $$\frac{d}{dx}\left(\frac{-4}{x^2}\right) = \lim_{h \to 0} \frac{\frac{-4}{(x+h)^2} - \frac{-4}{x^2}}{h}$$ The definition of derivative

$$= \lim_{h \to 0} \frac{\frac{-4 \cdot x^2 + 4 \cdot (x+h)^2}{(x+h)^2 \cdot x^2}}{h}$$

The common denominator of the two fractions in the numerator is $(x+h)^2 \cdot x^2$.

$$= \lim_{h \to 0} \frac{-4 \cdot x^2 + 4 \cdot x^2 + 8 \cdot x \cdot h + 4 \cdot h^2}{(x+h)^2 \cdot x^2 \cdot h}$$

Dividing: $\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{b \cdot c}$

Also, expand in the numerator

$$= \lim_{h \to 0} \frac{8 \cdot x \cdot h + 4 \cdot h^2}{(x+h)^2 \cdot x^2 \cdot h}$$

Collect like terms in the numerator

$$= \lim_{h \to 0} \frac{h \cdot (8x + 4h)}{(x+h)^2 \cdot x^2 \cdot h}$$

Factor in the numerator

$$= \lim_{h \to 0} \frac{(8x + 4h)}{(x+h)^2 \cdot x^2}$$

As h approaches 0, $h \neq 0$. Consequently, both the numerator and the denominator can be divided by h.

$$= \frac{8x}{\left(x^2\right)\left(x^2\right)}$$

$$= \frac{8}{x^3}$$

9. $\frac{d}{dx}\sqrt{x+2} = \lim_{h \to 0} \frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h}$ The definition of derivative

$= \lim_{h \to 0} \frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{(x+h)+2} + \sqrt{x+2}}{\sqrt{(x+h)+2} + \sqrt{x+2}}$ Multiply both the numerator and the denominator by the conjugate of $\sqrt{(x+h)+2} - \sqrt{x+2}$

$= \lim_{h \to 0} \frac{(x+h+2) - (x+2)}{h \cdot \left[\sqrt{(x+h)+2} + \sqrt{x+2}\right]}$ Multiply in the numerator

$= \lim_{h \to 0} \frac{x+h+2-x-2}{h \cdot \left[\sqrt{(x+h)+2} + \sqrt{x+2}\right]}$

$= \lim_{h \to 0} \frac{h}{h \cdot \left[\sqrt{(x+h)+2} + \sqrt{x+2}\right]}$ Expand in the numerator

$= \lim_{h \to 0} \frac{1}{\left[\sqrt{(x+h)+2} + \sqrt{x+2}\right]}$ As h approaches 0, $h \neq 0$. Consequently, both the numerator and the denominator can be divided by h.

$= \frac{1}{2 \cdot \sqrt{x+2}}$

10. $$\frac{d}{dx}\left(\frac{2\cdot x+3}{3x-1}\right) = \lim_{h \to 0} \frac{\frac{2\cdot(x+h)+3}{3\cdot(x+h)-1} - \frac{2\cdot x+3}{3x-1}}{h}$$ The definition of derivative

$$= \lim_{h \to 0} \frac{\left[\frac{(2\cdot x+2\cdot h+3)\cdot(3x-1)-(2x+3)\cdot(3x+3h-1)}{(3x+3h-1)\cdot(3x-1)}\right]}{h}$$ Common denominator

$$= \lim_{h \to 0} \frac{6x^2-2x+6x\cdot h-2h+9x-3-6x^2-6x\cdot h+2x-9x-9h+3}{(3x+3h-1)\cdot(3x-1)\cdot h}$$

$$= \lim_{h \to 0} \frac{-11h}{(3x+3h-1)\cdot(3x-1)\cdot h}$$

$$= \lim_{h \to 0} \frac{-11}{[(3x+3h-1)\cdot(3x-1)]}$$

$$= \frac{-11}{(3x-1)^2}$$