MATH 205
Spring 2002

SAMPLE QUESTIONS FOR QUIZ 3
SOLUTION KEY FOR PROBLEMS 7-10

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7. $$\lim_{x \to \infty} \frac{2\cdot x^3 + x - 1}{-3x^3 - \left(x^2 + 4\right)}$$

As x approaches ∞, both the numerator and the denominator contain terms approaching ∞ or $-\infty$.

$$= \lim_{x \to \infty} \frac{\frac{2\cdot x^3}{x^3} + \frac{x}{x^3} - \frac{1}{x^3}}{\frac{-3x^3}{x^3} - \frac{x^2}{x^3} - \frac{4}{x^3}}$$

Divide each term in the numerator and in the denominator by the highest power of x in the denominator: x^3

$$= \lim_{x \to \infty} \frac{2 + \frac{1}{x^2} - \frac{1}{x^3}}{-3 - \frac{1}{x} - \frac{4}{x^3}}$$

Simplify

$$= \frac{-2}{3}$$

As x approaches ∞, the terms: $\frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x}$ and $\frac{4}{x^3}$ all approac

0. Consequently, the numerator approaches 2, and the denominator approaches - 3.

(Note that this means the curve $y = \frac{2\cdot x^3 + x - 1}{-3x^3 - x^2 + 4}$ has a horizontal asymptote $y = \frac{-2}{3}$.)

8. $$\lim_{x \to -\infty} \frac{4x^2 + 5x - 5}{2x^2 \cdot \left(2 + x^2\right)}$$

First of all, expand the denominator...

$$= \lim_{x \to -\infty} \frac{4x^2 + 5x - 5}{4x^2 + 2x^4}$$

As x approaches ∞, both the numerator and the denominator contain terms approaching ∞ or $-\infty$.

$$= \lim_{x \to -\infty} \frac{\dfrac{4x^2}{x^4} + \dfrac{5x}{x^4} - \dfrac{5}{x^4}}{\dfrac{4x^2}{x^4} + \dfrac{2x^4}{x^4}}$$

Divide each term in the numerator and in the denominator by the highest power of x in the denominator: x^4

$$= \lim_{x \to -\infty} \frac{\dfrac{4}{x^2} + \dfrac{5}{x^3} - \dfrac{5}{x^4}}{\dfrac{4}{x^2} + 2}$$

Simplify

$$= 0$$

Except for 2 in the denominator, all the other terms in both the numerator and the denominator approach 0 as x approaches $-\infty$. Consequently, the numerator approaches 0, and the denominator approaches 2.

(Note that this means the curve $y = \dfrac{4x^2 + 5x - 5}{4x^2 + 2x^4}$ has a horizontal asymptote $y = 0$.)

9. $$\lim_{x \to \infty} \frac{\sqrt{x^5 + 2x^4 + 1}}{2x^2 + 3}$$

As x approaches ∞, both the numerator and the denominator approach ∞.

$$= \lim_{x \to \infty} \frac{\sqrt{\frac{x^5}{x^4} + \frac{2x^4}{x^4} + \frac{1}{x^4}}}{\frac{2x^2}{x^2} + \frac{3}{x^2}}$$

Divide each term in the numerator and in the denominator by the highest power of x in the denominator: x^2

In the numerator, use the form $\sqrt{x^4} = x^2$.

$$= \lim_{x \to \infty} \frac{\sqrt{x + 2 + \frac{1}{x^4}}}{2 + \frac{3}{x^2}}$$

Simplify

$$= \infty$$

As x approaches ∞ , the terms $\frac{1}{x^4}$ and $\frac{3}{x^2}$ both approach 0.

Consequently, the numerator approaches ∞, and the denominator approaches 2.

10. $$\lim_{x \to -\infty} \frac{3 + x}{\sqrt{x^2 + 3}}$$

As x approaches $-\infty$, the numerator approaches $-\infty$, while the denominator approaches ∞.

$$= \lim_{x \to -\infty} \frac{\frac{3}{-x} + \frac{x}{-x}}{\sqrt{\frac{x^2}{x^2} + \frac{3}{x^2}}}$$

Divide each term in the numerator and in the denominator by the highest power of x in the denominator: $\sqrt{x^2}$

In the numerator, use the form $\sqrt{x^2} = -x$.

(Note that as x approaches $-\infty$, we can assume $x < 0$, so that the general identity $\sqrt{x^2} = |x|$ results in $\sqrt{x^2} = -x$.)

$$= \lim_{x \to -\infty} \frac{\frac{3}{-x} - 1}{\sqrt{1 + \frac{3}{x^2}}}$$

Simplify.

$$= \quad -1$$

As x approaches $-\infty$, the terms $\frac{3}{-x}$ and $\frac{3}{x^2}$ both approach 0.

Consequently, the numerator approaches -1 , and the denominator approaches 1.

(Note that this means the curve $y = \frac{3 + x}{\sqrt{x^2 + 3}}$ has a horizontal asymptote $y = -1$.)