MATH 205
Spring 2002

SAMPLE QUESTIONS FOR QUIZ 2
SOLUTION KEY

Dr. Bogacki

1. $\lim_{x \to 4} \left(\frac{x^2 - 16}{x^2 - 5x + 4} \right)$

As x approaches 4, both the numerator and the denominator approach 0.

$= \lim_{x \to 4} \frac{(x-4)\cdot(x+4)}{(x-4)\cdot(x-1)}$

Factor both the numerator and the denominator.

$= \lim_{x \to 4} \left(\frac{x+4}{x-1} \right)$

Since x approaches 4, then $x \neq 4$. Consequently, $x - 4 \neq 0$, therefore we can divide both the numerator and the denominator by x - 4.

$= \frac{8}{3}$

As x approaches 4, the numerator approaches 8, and the denominator approaches 3.

2. $\lim_{x \to 0} \left(\frac{x^2 - 16}{x^2 - 5x + 4} \right) = -4$

As x approaches 0, the numerator approaches - 16, and the denominator approaches 4.

3. $\lim_{x \to 0} \left(\frac{\sqrt{9+x} - 3}{x} \right)$

As x approaches 0, both the numerator and the denominator approach 0.

$= \lim_{x \to 0} \left(\frac{\sqrt{9+x} - 3}{x} \cdot \frac{\sqrt{9+x} + 3}{\sqrt{9+x} + 3} \right)$

Multiply both the numerator and the denominator by the conjugate expression of $\sqrt{9+x} - 3$

$= \lim_{x \to 0} \left[\frac{9 + x - 9}{x\cdot\left(\sqrt{9+x} + 3\right)} \right]$

Expand the product in the numerator.

$= \lim_{x \to 0} \left[\frac{x}{x\cdot\left(\sqrt{9+x} + 3\right)} \right]$

Collect like terms in the numerator.

$= \lim_{x \to 0} \left(\frac{1}{\sqrt{9+x} + 3} \right)$

Since x approaches 0, then $x \neq 0$. Consequently, we can divide both the numerator and the denominator by x.

$= \frac{1}{6}$

As x approaches 0, the numerator approaches 1, and the denominator approaches 6.

4. $\lim_{x \to 7^+} \left(\dfrac{\sqrt{9+x}-3}{x} \right) = \dfrac{1}{7}$

As x approaches 7 from the right side,
the numerator approaches 1,
and the denominator approaches 7.

5. $\lim_{x \to -2^-} \left(\dfrac{x-2}{x^2-4} \right) = -\infty$

As x approaches - 2 from the left side, the numerator approaches - 4, and the denominator approaches 0 from the right side (since x^2 approaches 4 from the right side).

6. $\lim_{x \to 2} \left(\dfrac{x-2}{x^2-4} \right)$

As x approaches 2, both the numerator and the denominator approach 0.

$= \lim_{x \to 2} \dfrac{x-2}{(x-2)\cdot(x+2)}$

Factor both the numerator and the denominator.

$= \lim_{x \to 2} \left(\dfrac{1}{x+2} \right)$

Since x approaches 2, then $x \neq 2$. Consequently, $x - 2 \neq 0$, therefore we can divide both the numerator and the denominator by x - 2.

$= \dfrac{1}{4}$

As x approaches 2, the numerator approaches 1, and the denominator approaches 4.

7. $\lim_{x \to 4^+} \left(\dfrac{x-2}{x^2-4} \right) = \dfrac{1}{6}$

As x approaches 4, the numerator approaches 2, and the denominator approaches 12.

8. $\lim_{x \to 25^-} \left(\dfrac{x-25}{\sqrt{x}-5} \right)$

As x approaches 25, both the numerator and the denominator approach 0.

$= \lim_{x \to 25^-} \left(\dfrac{x-25}{\sqrt{x}-5} \cdot \dfrac{\sqrt{x}+5}{\sqrt{x}+5} \right)$

Multiply both the numerator and the denominator by the conjugate expression of $\sqrt{x} - 5$

$= \lim_{x \to 25^-} \left[\dfrac{(x-25)\cdot\left(\sqrt{x}+5\right)}{x-25} \right]$

Expand the product in the denominator.

$= \lim_{x \to 25^-} \left(\dfrac{\sqrt{x}+5}{1} \right)$

Since x approaches 25, then $x \neq 25$. Consequently, $x - 25 \neq 0$, therefore we can divide both the numerator and the denominator by x - 25.

$= 10$

As x approaches 25, the numerator approaches 10, and the denominator approaches 1.

9. $\lim_{x \to 1^+} \left(\frac{x}{1 - x} \right) = -\infty$

As x approaches 1 from the right, the numerator approaches 1, and the denominator approaches 0 from the left.

10. $\lim_{x \to 0^-} \left(\frac{x}{1 - x} \right) = 0$

As x approaches 0 from the left, the numerator approaches 0, and the denominator approaches 1.