MATH 205
Spring 2002

SAMPLE QUESTIONS FOR QUIZ 7
SOLUTION KEY

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In problems 1-3, find $\frac{dy}{dx}$ by implicit differentiation.

1. $x^3 + y^2 = \sin(x) \cdot y$

$$3x^2 + 2y \cdot \frac{dy}{dx} = \cos(x) \cdot y + \sin(x) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - \sin(x)) = -3x^2 + \cos(x) \cdot y$$

$$\frac{dy}{dx} = \frac{-3x^2 + \cos(x) \cdot y}{2y - \sin(x)}$$

2. $y \cdot e^y = x + y$

$$\frac{dy}{dx} \cdot e^y + y \cdot e^y \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(e^y + y \cdot e^y - 1\right) = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y + y \cdot e^y - 1}$$

3. $y^x = x \cdot y$

Rewrite as $e^{\ln(y) \cdot x} = x \cdot y$

$$e^{\ln(y) \cdot x} \cdot \left(\frac{1}{y} \cdot \frac{dy}{dx} \cdot x + \ln(y) \right) = y + x \cdot \frac{dy}{dx}$$

$$y^x \left(\frac{x}{y} \cdot \frac{dy}{dx} + \ln(y) \right) = y + x \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} \cdot \left(\frac{x}{y} \cdot y^x - x \right) = -y^x \cdot \ln(y) + y$$

$$\frac{dy}{dx} = \frac{-y^x \cdot \ln(y) + y}{\frac{x}{y} \cdot y^x - x}$$

In problems 4-6, find an equation of the tangent line to the curve at the given point if possible.

4. $x^2 + y^3 = 2 \cdot x \cdot y + 1$

Implicit Differentiation:

$$2x + 3 \cdot y^2 \cdot \frac{dy}{dx} = 2y + 2x \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} \cdot \left(3y^2 - 2x\right) = -2x + 2y$$

$$\frac{dy}{dx} = \frac{-2x + 2y}{3y^2 - 2x}$$

(a) $(0, 1)$

Check that the point is on the curve:

LHS $= 0^2 + 1^3 = 1$

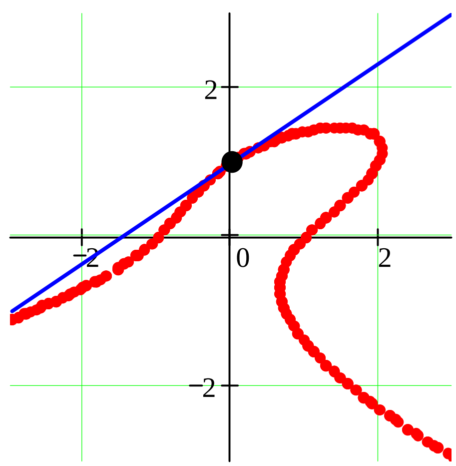
RHS $= 2 \cdot 0 \cdot 1 + 1 = 1$ OK

Slope:

$$m = \frac{-2(0) + 2(1)}{3(1)^2 - 2(0)} = \frac{2}{3}$$

Tangent line equation:

$$y - 1 = \frac{2}{3} \cdot x$$

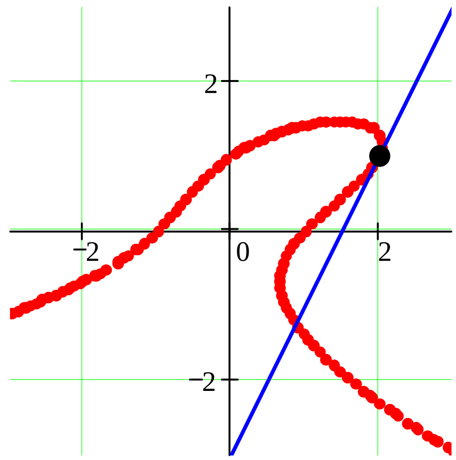


(b) $(2,1)$

Check that the point is on the curve:

$LHS = 2^2 + 1^3 = 5$

$RHS = 2\cdot 2\cdot 1 + 1 = 5$ OK



Slope:

$$m = \frac{-2(2) + 2(1)}{3(1)^2 - 2(2)} = \frac{-2}{-1} = 2$$

Tangent line equation:

$$y - 1 = 2\cdot(x - 2)$$

5. $x\cdot e^y + x^2 + y^2 = 0$

Implicit Differentiation:

$$e^y + x\cdot e^y\cdot\frac{dy}{dx} + 2x + 2y\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x\cdot e^y + 2y\right) = -e^y - 2x$$

$$\frac{dy}{dx} = \frac{-e^y - 2x}{x\cdot e^y + 2y}$$

(a) $(-1, 0)$

Check that the point is on the curve:

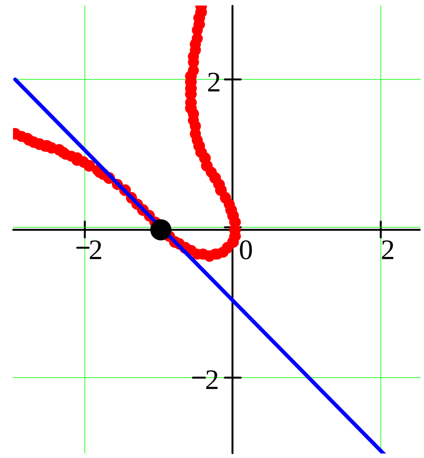
$$\text{LHS} = -1 \cdot e^{0} + (-1)^{2} + 0^{2} = -1 + 1 = 0 \quad \text{OK}$$

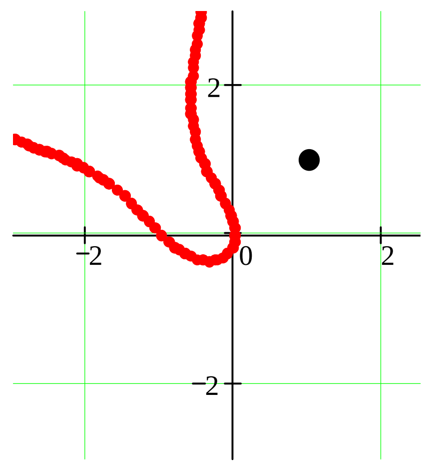
Slope:

$$m = \frac{-e^{0} - 2(-1)}{(-1)e^{0} + 2(0)} = \frac{-1 + 2}{-1} = -1$$

Tangent line equation:

$$y - 0 = -1 \cdot (x + 1)$$





(b) $(1, 1)$

Check that the point is on the curve:

$$\text{LHS} = 1 \cdot e^{1} + 1^{2} + 1^{2} = e + 2 \neq 0$$

The point is not on the curve
- it is not possible to find a tangent line

6. $x^2 + y^2 = x \cdot y + x + y$

Implicit Differentiation:

$$2x + 2y \cdot \frac{dy}{dx} = y + x \cdot \frac{dy}{dx} + 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - x - 1) = -2x + y + 1$$

$$\frac{dy}{dx} = \frac{-2x + y + 1}{2y - x - 1}$$

(a) $(-1, 0)$

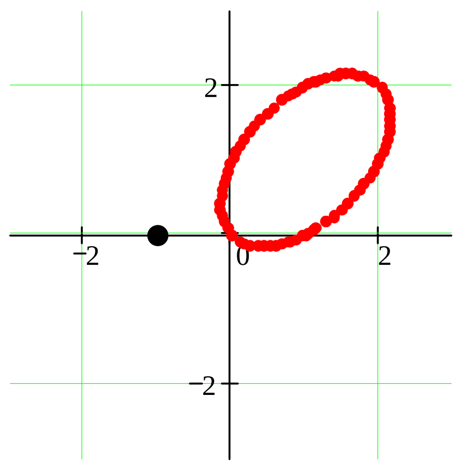
Check that the point is on the curve:

$LHS = (-1)^2 + 0^2 = 1$

$RHS = -1(0) - 1 + 0 = -1$

$LHS \neq RHS$

The point is not on the curve
- it is not possible to find a tangent line

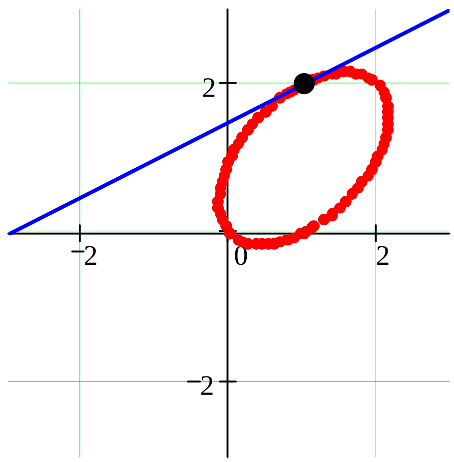


(b) $(1,2)$

Check that the point is on the curve:

$$\text{LHS} = 1^2 + 2^2 = 5$$

$$\text{RHS} = 1 \cdot 2 + 1 + 2 = 5 \qquad \text{OK}$$



Slope:

$$m = \frac{-2(1) + 2 + 1}{2(2) - 1 - 1} = \frac{1}{2}$$

Tangent line equation:

$$y - 2 = \frac{1}{2} \cdot (x - 1)$$