MATH 205
Spring 2002

SAMPLE QUESTIONS FOR QUIZ 5
SOLUTION KEY

Dr. Bogacki

Differentiate each function

1. $$f(x) = -5\cdot x^3 + \frac{3}{\sqrt{x}} = -5x^3 + 3x^{\frac{-1}{2}}$$

$$\frac{d}{dx}f(x) = -5\cdot\left(3\cdot x^2\right) + 3\cdot\left(\frac{-1}{2}\cdot x^{\frac{-3}{2}}\right) = -15\cdot x^2 - \frac{3}{2x\cdot\sqrt{x}}$$

2. $$g(x) = \frac{3}{x} - 4 + 5x = 3\cdot x^{-1} - 4 + 5x$$

$$\frac{d}{dx}g(x) = 3(-1)\,x^{-2} - 0 + 5 = \frac{-3}{x^2} + 5$$

3. $$h(x) = \frac{4\sqrt{x} + 2x - 3x^2}{\sqrt[3]{x}} = \frac{4\cdot x^{\frac{1}{2}} + 2x - 3x^2}{x^{\frac{1}{3}}} = 4x^{\left(\frac{1}{2}\right)-\left(\frac{1}{3}\right)} + 2x^{1-\left(\frac{1}{3}\right)} - 3x^{2-\left(\frac{1}{3}\right)}$$

$$= 4x^{\frac{1}{6}} + 2x^{\frac{2}{3}} - 3x^{\frac{5}{3}}$$

$$\frac{d}{dx}h(x) = 4\left(\frac{1}{6}x^{\frac{-5}{6}}\right) + 2\left(\frac{2}{3}x^{\frac{-1}{3}}\right) - 3\left(\frac{5}{3}x^{\frac{2}{3}}\right) = \frac{2}{3}x^{\frac{-5}{6}} + \frac{4}{3}x^{\frac{-1}{3}} - 5x^{\frac{2}{3}}$$

4. $$f(x) = \pi + 3\cdot e^x + x^e - e^\pi$$

Note: Derivatives of the constant terms: π and e^π equal 0.

$$\frac{d}{dx}f(x) = 0 + 3e^x + e\cdot x^{e-1} - 0 = 3e^x + e\cdot x^{e-1}$$

5. $g(x) = (3x-4)\cdot\left(\sqrt{x}+x^2\right) = (3x-4)\cdot\left(x^{\frac{1}{2}}+x^2\right)$ Product Rule

$$\frac{d}{dx}g(x) = \left[\frac{d}{dx}(3x-4)\right]\cdot\left(x^{\frac{1}{2}}+x^2\right) + (3x-4)\cdot\frac{d}{dx}\left(x^{\frac{1}{2}}+x^2\right)$$

$$= 3\cdot\left(x^{\frac{1}{2}}+x^2\right) + (3x-4)\cdot\left(\frac{1}{2}\cdot x^{\frac{-1}{2}}+2x\right) = 3\cdot\left(\sqrt{x}+x^2\right) + (3x-4)\cdot\left(\frac{1}{2\sqrt{x}}+2x\right)$$

6. $h(x) = 3x^4\cdot e^x$ Product Rule

$$\frac{d}{dx}h(x) = \left[\frac{d}{dx}\left(3x^4\right)\right]\cdot\left(e^x\right) + \left(3x^4\right)\cdot\left(\frac{d}{dx}e^x\right) = 12x^3\cdot e^x + 3x^4\cdot e^x$$

7. $F(x) = \dfrac{2x^2}{x-3}$ Quotient Rule

$$\frac{d}{dx}F(x) = \frac{\left[\frac{d}{dx}\left(2x^2\right)\right]\cdot(x-3) - \left[\frac{d}{dx}(x-3)\right]\cdot\left(2x^2\right)}{(x-3)^2} = \frac{4x\cdot(x-3)-(1)\cdot 2x^2}{(x-3)^2}$$

$$= \frac{4x^2-12x-2x^2}{(x-3)^2} = \frac{2x^2-12x}{(x-3)^2}$$

8. $G(x) = \dfrac{\sqrt[4]{x^3}-1}{x^2+2} = \dfrac{x^{\frac{3}{4}}-1}{x^2+2}$ Quotient Rule

$$\frac{d}{dx}G(x) = \frac{\left[\frac{d}{dx}\left(x^{\frac{3}{4}}-1\right)\right]\cdot\left(x^2+2\right) - \left[\frac{d}{dx}\left(x^2+2\right)\right]\cdot\left(x^{\frac{3}{4}}-1\right)}{\left(x^2+2\right)^2} = \frac{\frac{3}{4}\cdot x^{\frac{-1}{4}}\cdot\left(x^2+2\right) - 2x\cdot\left(x^{\frac{3}{4}}-1\right)}{\left(x^2+2\right)^2}$$

9. $f(x) = \dfrac{\dfrac{2}{\sqrt{x}} + \dfrac{\sqrt{2}}{x}}{\sqrt{x}} = \dfrac{2x^{\frac{-1}{2}} + \sqrt{2}\cdot x^{-1}}{x^{\frac{1}{2}}} = 2x^{\left(\frac{-1}{2}\right)-\left(\frac{1}{2}\right)} + \sqrt{2}\,x^{-1-\left(\frac{1}{2}\right)} = 2x^{-1} + \sqrt{2}\,x^{\frac{-3}{2}}$

$\dfrac{d}{dx}f(x) = 2\cdot(-1)\,x^{-2} + \sqrt{2}\cdot\left(\dfrac{-3}{2}\right)\cdot x^{\frac{-5}{2}} = \dfrac{-2}{x^2} - \dfrac{3\sqrt{2}}{2x^2\cdot\sqrt{x}}$

10. $g(x) = \dfrac{x^2}{\sqrt{\pi} + \sqrt[3]{e}} = \left(\dfrac{1}{\sqrt{\pi} + \sqrt[3]{e}}\right)\cdot\left(x^2\right)$ Constant multiple rule ($\dfrac{1}{\sqrt{\pi} + \sqrt[3]{e}}$ is a constant)

$\dfrac{d}{dx}g(x) = \left(\dfrac{1}{\sqrt{\pi} + \sqrt[3]{e}}\right)\cdot(2x)$