MATH 205
Spring 2002

QUIZ 9
SOLUTION KEY

Dr. Bogacki

1. In the following problems, use $f(x) = x^4 + 4x^3 - 2$,

   $f\,'(x) = 4x^3 + 12x^2 = 4x^2(x+3)$

   $f\,''(x) = 12x^2 + 24x = 12x(x+2)$

2. Find all critical numbers of f(x)

   f ' is never undefined. Therefore, the only way to get critical numbers is by setting f ' = 0.

   Critical numbers are: $x = 0, x = -3$

3. Find the absolute maximum and the absolute minimum values of f on [- 1, 1]

   Using the Closed Interval Method (p.282)

   Since -3 is not in (-1,1), the only critical number to consider is 0. $f(0) = -2$

   Evaluate f at each endpoint: $f(-1) = 1 - 4 - 2 = -5$ $f(1) = 1 + 4 - 2 = 3$

   The largest of the three values, f (1) = 3 is the absolute maximum value.

   The smallest of the three values, f (- 1) = - 5 is the absolute minimum value.

4. (a) Find the intervals on which f is increasing or decreasing.

   Test the signs of f ' between the critical points.

   On the interval (-infinity,-3), we can test f ' (- 4) = $4(-4)^2 \cdot (-4+3) < 0$

   On the interval (-3,0), we can test f ' (- 1) = $4(-1)^2 \cdot (-1+3) > 0$

   On the interval (0, infinity), we can test f ' (1) = $4(1)^2 \cdot (1+3) > 0$

x		-3		0	
f'	-	0	+	0	+
f	decr.		incr.		incr.

Answer: f is increaing on the intervals $(-3, 0)$ and $(0, \infty)$

f is decreasing on the interval $(-\infty, -3)$

(b) Find the local maximum and minimum values of f.

Using the first derivative test, we conclude:

$f(-3) = (-3)^4 + 4\cdot(-3)^3 - 2 = 81 - 108 - 2 = -29$ is a local minimum since f ' changes from negative to positive

$f(0) = -2$ is not a local extremum since f ' does not change its sign there

f does not have a local maximum

5. (a) Find the intervals of concavity of f.

Since f ' ' (x) = 12x(x+2) is continuous for all x, the only way for it to change sign is when f ' ' (x) = 0

The possible inflection points are at x = 0 and x = - 2.

Test the signs of f ' ' between these points.

On the interval (-infinity,-2), we can test $f''(-3) = 12(-3)\cdot(-3+2) > 0$

On the interval (-2,0), we can test $f''(-1) = 12(-1)\cdot(-1+2) < 0$

On the interval (0, infinity), we can test $f''(1) = 4(1)\cdot(1+2) > 0$

x		-2		0	
f''	+	0	-	0	+
f	conc. up		conc. down		conc. up

The graph of f is concave upward on the intervals $(-\infty, -2)$ and $(0, \infty)$.

The graph of f is concave downward on the interval $(-2, 0)$.

(b) Find the inflection points of f.

f has two inflection points: at x = - 2 and at x = 0.