
5 Transcendental Functions

In the discussion before Problem 10 on page 53, we pointed out that $G(x) = \int_a^x f(t)dt$ is an antiderivative of $f(x)$. Subsequently, we used that fact to compare $G(x)$ with an antiderivative $F(x)$ obtained using the integration rules.

Now, suppose $f(x)$ is integrable but cannot be antidifferentiated using the integration rules available. On the other hand, $G(x)$ can be evaluated numerically for any value of x .

Many functions belong to this category - for some examples see the beginning of Section 4.6 LHE. For the moment, however, we shall focus on one particular function: $f(x) = \frac{1}{x}$. This function was the only power function not handled by the integration power rule, consequently we could not obtain an expression for $F(x)$. As you will see in Section 5.1, antiderivatives of $\frac{1}{x}$ can be successfully obtained using our "G(x)" formula - in fact, one of these antiderivatives will define the natural logarithmic function. Other functions related to this "new" function will also be investigated.

Section 5.2 introduces Mathcad's symbolic integration facility. To obtain the exact value of a definite integral, Mathcad first obtains an expression for the antiderivative (internally) and then applies the Fundamental Theorem of Calculus. In this section, you will learn how to use Mathcad to produce expressions for antiderivatives, and exact values of definite integrals; you will also see how you can use these tools to improve your own integration skills.

In Section 5.3, you will further investigate functions defined by integrals.

5.1 Activity: Logarithmic and Exponential Functions

Prerequisites: Read Sections 5.1-5.5 LHE

After illustrating the integral definition of the natural logarithmic function, this activity will use that definition to deduce some of the properties of $\ln x$. The number e and exponential functions b^x will also be discussed.

Instructions

Open a blank Mathcad document, and create your report there, answering all the problems identified by your professor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Problems

1. Plot the function $f(x) = \frac{1}{x}$ over the interval $[1, 10]$.
Double-click on the graph and make the following changes in the graph format dialog box:
 - set both X- and Y-axis "Grid Lines" on;

- set both X- and Y-axis "Auto grid" off;
- set the X-axis "No. of grids" to 9, and the Y-axis "No. of grids" to 5;
- set the X-axis "Autoscale" off, and the Y-axis "Autoscale" on.

After exiting the dialog box, change the Y-axis lower placeholder value to 0.

Highlight the graph with a dotted line box (drag the cursor onto it) and drag the right bottom corner of the box to enlarge the graph.

- (a) Without using calculus (just by looking at the graph), try to find a rough estimate of the area between the graph of $f(x)$ and the x -axis from $x = 1$ to $x = 2$. (Hint: What is the area of each little rectangle?) Repeat this for x going from 1 to 5.

Although tedious and inaccurate, the process you just followed could be repeated to get an estimate of the area from $x = 1$ to $x = t$ for any $t > 1$.

For any value of t we input, the output is a number - therefore our process can be viewed as a **function**:

$$F(t) = \boxed{\text{the area between } f(x) = 1/x \text{ and the } x\text{-axis from } x = 1 \text{ to } x = t}$$

Be sure you see why this is a function of t and *not* of x .

- (b) Use the definite integral symbol in Mathcad to define the function $F(t)$. Evaluate $F(2)$ and $F(5)$. (Your estimates from part (a) should resemble these values.)
- (c) Graph $F(t)$ and, on a separate plot, $\ln t$ for t from 0.1 to 10. The two plots should be identical. In the text, the integral you obtained in part (b) is actually used to *define* the natural logarithmic function, $\ln t$.
- (d) Evaluate $\ln 0.5$. Interpret this answer in terms of area. How about $\ln 1$? (We do not define $\ln t$ for $t = 0$ or for negative values of t .)
- (e) Evaluate: $\ln 2 + \ln 5$, $\ln 2 \cdot \ln 5$, $\ln(2 + 5)$, $\ln(2 \cdot 5)$,
 $\ln 5 - \ln 2$, $(\ln 5) / (\ln 2)$, $\ln(5 - 2)$, $\ln(5/2)$,
 $5 \ln 2$, $\ln(2^5)$, $(\ln 2)^5$.

State the logarithmic properties responsible for making some of these expressions equal.

(Note: When you type the last expression in Mathcad, it will not appear to be correct because Mathcad drops the outer parentheses.)

2. Recall that $\ln 2 < 1$ and $\ln 5 > 1$.

Since the function $\ln x$ is continuous on $[2,5]$, by the Intermediate Value Theorem (p.91 LHE) there exists a value between $x = 2$ and $x = 5$ where $\ln x = 1$.

In part (a) we obtain an approximation to this value:

- (a) Define a range variable $u = 2, 2.5..5$. Create tables for u and $\ln u$. Between which two values of u does $\ln u = 1$?

Define v to be a range variable that goes between these values in increments of 0.1 and create tables for v and $\ln v$, as described above.

Based on these tables, what would be your rough guess of the x value that makes $\ln x = 1$?

- (b) Use Mathcad's root function to get a better estimate of this value and display the result to five decimal places.

Evaluate the constant e to five decimal places to confirm that this value has been obtained.

Indeed, the text *defines* e to be the value of x such that $\ln x = 1$.

Calculate $\ln(e^2)$, $\ln(e^{10})$, $\ln(\frac{1}{\sqrt{e}})$ without the computer, and then verify your answers using Mathcad.

Now recall the definition of an inverse function:

Function G is an inverse function of function F if

$$F(G(x)) = x \text{ for all } x \text{ in the domain of } G \text{ and}$$

$$G(F(y)) = y \text{ for all } y \text{ in the domain of } F.$$

If we let $F(y) = \ln y$ and $G(x) = e^x$ then we should see that $F(G(x)) = \ln(e^x) = x$ is true for all rational numbers x (see the discussion on top of p.344 LHE for more details).

Defining e^x to be the inverse function of $\ln x$, $F(G(x)) = x$ is now true for all real numbers x , while $G(F(y)) = e^{\ln y} = y$ is true for all positive real numbers y .

- (c) Verify $G(F(y)) = y$ for $y = 1, 10, 0.5$.
3. (a) Assign the value 2 to the base b ($b > 0$) and plot the exponential function b^x together with its derivative $\frac{d}{dx}b^x$ on $[-2, 4]$.
- (b) Repeat part (a) with
- (i) $b = 5$,
 - (ii) $b = 10$,
 - (iii) $b = 0.5$.

In parts (a) and (b) you might have noticed that the graph of $f(x) = b^x$ has a similar shape to that of $f'(x)$. We are now going to investigate that relationship.

- (c) Verify by hand that $\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{b^{\Delta x}-1}{\Delta x}b^x$. Therefore, to obtain $f'(x)$ we need only consider the limit of $\frac{b^{\Delta x}-1}{\Delta x}$ as Δx tends to zero. What property (properties) of limits (p. 71 LHE) have we used?

Denoting $A = \lim_{\Delta x \rightarrow 0} \frac{b^{\Delta x}-1}{\Delta x}$, express $f'(x)$ using A and b^x .

In parts (d) and (e), you will determine A . Of course, the value of A should depend on b .

- (d) With $b = 2$, assign Δx to take values 0.009, 0.008 ... 0.001 and display Δx and $\frac{b^{\Delta x}-1}{\Delta x}$ in table form next to each other.

What is an approximate value for A (with $b = 2$)? Your approximate value is close to a number that appeared earlier in this lab. What number is it?

- (e) Repeat part (d) with $b = 5$ this time.

What is the value of A in terms of b (for any $b > 0$)?

- (f) If you correctly answered parts (d) and (e), you should have found your approximate value for A to be less than 1 when $b = 2$, and $A > 1$ when $b = 5$.

Assign $\Delta x = 0.0001$ and define $g(b) = \frac{b^{\Delta x} - 1}{\Delta x}$. Evaluate $g(2)$ and $g(5)$.

Use the "root" function to find the value of b which makes $g(b) = 1$.

Repeat part (a) using this value of b .

With this choice for the base, how does $f'(x)$ compare to $f(x)$?

4. (a) Consider the exponential function a^x and the power function x^a for $a = 2, 3$ and 4 . For each value of a , graph these functions (in appropriate plot windows) to illustrate that in each case the power and exponential curves intersect at two points, and that eventually the exponential function always increases much faster than the power function.
- (b) Find the exact value of a for which the graphs of a^x and x^a are tangent. At the point of tangency, evaluate a^x and x^a (choose displayed precision 15). Using this special value of a , graph a^x and x^a in a plot window which shows that the curves are tangent.
- (c) As the value of a approaches the special value found in part (b), the points of intersection of the graphs of a^x and x^a come closer together. To see this, graph $f(x) = a^x - x^a$ for several values of a near the special value found in part (b), and observe how the zeros of $f(x)$ change their position.
- (d) Find all points of intersection of 4^x and x^7 . (Hint: First locate the points of intersection by plotting $f(x) = 4^x - x^7$; then use the "root" function.) Also, graph 4^x and x^7 in plot windows which show their points of intersection.

5.2 Activity: Symbolic Integration

Prerequisites: Review the activity "Introduction to Symbolics" (Section 2.2). Read Sections 4.5, 5.2, 5.4, 5.5 and 5.8 LHE.

In this activity, you will use Mathcad's symbolic processor to find expressions for antiderivatives and evaluate definite integrals exactly. You will learn about some of the symbolic processor's limitations; you will also learn how to use its power to help you improve your integration skills.

Instructions

A "template" document SYMBINT.MCD is available for this assignment - your professor will inform you whether you are to use it. Answer all the problems identified by your professor. Remember to



enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. The symbols \int (from the third palette) and \int_a^b (from the first palette) can be used to symbolically evaluate indefinite and definite integrals, respectively. To symbolically evaluate an integral:
 - (a) select the appropriate integration symbol from the strip,
 - (b) fill in the integrand, variable of integration and the limits of integration (if applicable),
 - (c) hit the SPACE BAR repeatedly until the entire integral is included in the blue frame,
 - (d) choose **Evaluate Symbolically** from the Symbolic menu.

2. In the activity of Section 2.2, we mentioned a mistake frequently made by students when using Mathcad to symbolically obtain the first derivative of a given expression: differentiating $\frac{d}{dx}(\text{expression})$ leads to the **second** derivative. Likewise, if you want to symbolically obtain an antiderivative, make sure you either **Evaluate Symbolically** $\int(\text{expression})dx$ or **Integrate on Variable** (expression), but do not integrate $\int(\text{expression})dx$.

Problems

From the Symbolic menu, choose Load Symbolic Processor. Once the loading is complete, from the same menu, choose Derivation Format. In the Derivation Format dialog

- *switch "Show derivation comments" to ON (make sure a check appears in the box);*
- *select "Show derivation steps" horizontally.*

1. Consider the integral $\int x \sin(x^2 + \frac{\pi}{4})dx$.
 - (a) Evaluate it by hand, with a pencil and paper. (Record your answer in your document.)
 - (b) Use Mathcad to evaluate the integral symbolically.
What is missing in Mathcad's answer?
Differentiate this answer symbolically.

2. (a) By hand, evaluate $\int_0^{\sqrt{\pi}} x \sin(x^2 + \frac{\pi}{4}) dx$. (Record your answer in your document.)

- (b) Numerically evaluate the integral in Mathcad.
- (c) Evaluate it symbolically using Mathcad.
3. (a) Have Mathcad evaluate $\int_{-2}^{-1} \frac{1}{x} dx$ numerically.
- (b) Evaluate the integral symbolically. Anything surprising?
- (c) Have Mathcad evaluate the indefinite integral $\int \frac{1}{x} dx$ symbolically. What is wrong with this answer (other than what you already detected as missing in Problem 1(b)).
- (d) By hand, determine the exact value of the definite integral of part (a).
4. In the "Introduction to Symbolics" assignment of Section 2.2, you were asked to symbolically differentiate functions from Exercises 23, 25 p.134 LHE and 5 p.161 LHE.
- (a) Open up your solution for that assignment and copy to the clipboard each of the three final answers you obtained there, and paste them into your document.
- (b) Have Mathcad symbolically integrate each expression.
- (c) Compare the results obtained with the original functions. Are they the same? If not, how do they differ? Explain. Hint: Have Mathcad symbolically simplify the difference in the two functions.

Chances are that by now, in your study of calculus, you've encountered functions whose antiderivatives were difficult (or seemingly impossible) to find. One possible reason for this might have been your lack of skill in using techniques of integration. You are going to learn more techniques of integration, which will enable you to tackle more problems successfully. On the other hand, there exist elementary functions (i.e. functions involving algebraic, trigonometric, exponential or logarithmic functions) whose antiderivatives cannot be expressed in terms of elementary functions. Consider the following problem.

5. Try to have Mathcad symbolically evaluate $\int \sqrt{1-x^3} dx$. Is the answer entirely satisfactory?

In fact, the antiderivative of $\sqrt{1-x^3}$ cannot be expressed in terms of elementary functions. Since Mathcad always looks for an antiderivative which can be expressed in terms of elementary functions, its response to the problem above was appropriate. However, its response is not always so appropriate. Consider the following two problems.

6. (a) Have Mathcad symbolically evaluate $\int \sqrt{\sin x} \cos^3 x dx$.
- (b) Repeat for $\int \sqrt{\sin x} (1 - \sin^2 x) \cos x dx$.
- (c) Mathcad responded in two dramatically different ways, even though these integrals are mathematically equivalent. Which trigonometric identity is responsible for their equivalence?

7. (a) Have Mathcad symbolically evaluate: $\int \cot x (1 + \ln |\sin x|)^2 dx$.
- (b) By hand, make the substitution $u = \ln(|\sin x|)$, and have Mathcad symbolically integrate the resulting function of u .
- (c) Copy the answer from part (b) and use the **Substitute for Variable** command (see Comment 2 on page 31) to express the answer in terms of x .
- (d) Copy the answer (in terms of x) from part (c) and use Mathcad to verify that the derivative of the answer is $\cot x(1 + \ln |\sin x|)^2$.

By now, you should be convinced that one cannot blindly rely on Mathcad's symbolic integration features without carefully thinking about the validity of its answers; we've just seen examples where an integral expression could obviously be obtained, yet, Mathcad "gave up" on it.

Here is another way in which symbolic integration can go wrong.

8. (a) Have Mathcad evaluate the definite integral $\int_{-2}^2 \sqrt{x^2} dx$ twice: numerically and symbolically.
- (b) Plot the function $f(x) = \sqrt{x^2}$ over the interval $[-5, 5]$. Then have Mathcad *simplify* the expression $\sqrt{x^2}$ symbolically and, on a separate plot, graph the result over the same interval.
- (c) Explain what appears to have gone wrong in Mathcad's symbolic evaluation of the integral.

Fortunately, for most integrals that you will encounter (in calculus and elsewhere) Mathcad does not suffer from the kinds of problems encountered above, and thus, can be a useful tool.

In the next problem, we will demonstrate one of the possible ways in which Mathcad can help you practice some techniques of integration.

9. Consider the definite integral $\int_0^{\pi/4} \sin^6 x \cos x dx$.
- (a) Use Mathcad to evaluate the integral symbolically.
- (b) By hand, determine the u -substitution to be used, as well as the new limits of integration. In Mathcad, set up the integral in terms of u , and have Mathcad evaluate it symbolically.

Note that both results obtained above should be identical (if they are not, look for a mistake, and ask for help if you need it). Although it's possible that the values are equal by accident, typically their equality results from your having correctly carried out the substitution, including change of the limits. Therefore, when you are solving similar problems, you might use Mathcad to check your solution by symbolically evaluating your u -integral.

10. Repeat Problem 9 for the integral of Exercise 54 p.301 LHE.

11. Repeat Problem 9 for the integral of Exercise 55 p.301 LHE.
12. Repeat Problem 9 for the integral of Exercise 37 p.333 LHE.
13. Repeat Problem 9 for the integral of Exercise 74 p.351 LHE.

5.3 Activity: Functions Defined by Integrals

In this activity, you will investigate functions defined by integrals of the form $G(x) = \int_a^x f(t)dt$, where $G(x)$ cannot be expressed in "closed form" (that is, an antiderivative of $f(t)$ cannot be found in terms of elementary functions).

Instructions

After reading the comments below, open a blank Mathcad document, and create your report there, answering all the problems identified by your professor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. (a) If your limits of integration are variable names or numbers without decimal points, Mathcad's symbolic processor will try to find an exact symbolic or numerical value for the integral.
 - (b) If you use decimals in the integrand or limits of integration, Mathcad's symbolic processor will return a 20-place numerical value if it can evaluate the integral.
 - (c) If the symbolic processor can't find an indefinite integral or an exact value for a definite integral, you will see the message "No closed form found for integral" (or Mathcad will simply return the same integral). If all you need is a numerical approximation to a definite integral, try using Mathcad's numerical integration routine: select the filled-in integral operator and press =.
2. There are many functions $f(t)$ whose antiderivatives exist in the form $G(x)$, but cannot be expressed in terms of elementary functions; for example,

$$\sqrt{1-t^3}, \sqrt{t} \cos t, \sin t^2, e^{t^2}, \frac{\sin t}{t}, \sqrt{1+\sin^2 t}$$

3. Functions defined by integrals of the form $G(x)$ are important in physics and engineering, and may appear in the solution to certain integration problems when using Mathcad's symbolic processor. Several of these functions are defined below (others can be found in Mathcad Help).

The *Fresnel Cosine Integral* is a function of x defined by $\text{FresnelC}(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt$ with

domain $x \geq 0$.

The *Fresnel Sine Integral* is a function of x defined by $\text{FresnelS}(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt$ with domain $x \geq 0$.

The *Sine Integral* is a function of x defined by $\text{Si}(x) = \int_0^x \frac{\sin t}{t} dt$ with domain $x \geq 0$.

The *Error Function* is a function of x defined by the integral $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ with domain $x \geq 0$.

Like $\ln x$, the function $\text{erf}(x)$ is a Mathcad built-in numerical function so you can numerically evaluate it by pressing =. For example, we obtain

$$\text{erf}(2) = 0.995322265018953$$

You can also obtain a numerical value for $\text{erf}(2)$ by including a decimal point after the 2 and evaluating it using the symbolic processor:

$$\text{erf}(2.) \quad \text{yields} \quad .99532226501895273416$$

or by using the symbolic processor to evaluate the integral definition of $\text{erf}(2)$:

$$\frac{2}{\sqrt{\pi}} \int_0^2 e^{-t^2} dt \quad \text{yields} \quad \frac{1.7641627815248433599}{\sqrt{\pi}} \quad \text{simplifies to} \quad .99532226501895273415$$

Note that if a decimal point is not included, then the symbolic processor cannot evaluate the Error Function:

$$\text{erf}(2) \quad \text{yields} \quad \text{erf}(2)$$

and

$$\frac{2}{\sqrt{\pi}} \int_0^2 e^{-t^2} dt \quad \text{yields} \quad \text{erf}(2)$$

The functions $\text{FresnelS}(x)$, $\text{FresnelC}(x)$ and $\text{Si}(x)$ listed above are not built-in Mathcad numerical functions. One way to find a numerical value for one of these functions is to numerically evaluate the integral which defines the function. For example, $\text{Si}(3)$ can be evaluated:

$$\int_0^3 \frac{\sin(t)}{t} dt = 1.847760218568742$$

We could also evaluate $\text{Si}(3)$ by including a decimal point and using the symbolic processor:

$$\text{Si}(3.) \quad \text{yields} \quad 1.8486525279994682564$$

or

$$\int_0^3 \frac{\sin(t)}{t} dt \quad \text{yields} \quad 1.8486525279994682564$$

The symbolic processor can differentiate and integrate these functions too. For example,

$$\frac{d}{dx} \text{FresnelS}(x) \quad \text{yields} \quad \sin\left(\frac{1}{2} \cdot \pi \cdot x^2\right)$$

$$\int \text{FresnelS}(x) dx \quad \text{yields} \quad x \text{FresnelS}(x) + \frac{\cos\left(\frac{1}{2} \cdot \pi \cdot x^2\right)}{\pi}$$

Problems

From the **Symbolic** menu, choose **Load Symbolic Processor**. Once the loading is complete, from the same menu, choose **Derivation Format**. In the Derivation Format dialog

- switch "Show derivation comments" to ON (make sure a check appears in the box);
 - select "Show derivation steps" horizontally.
1. (a) Use the symbolic processor to find the derivative of the Fresnel Cosine Integral. Which calculus theorem tells you how to obtain this derivative without using the symbolic processor?
 - (b) On separate plots, graph $\text{FresnelC}(x)$ and $\cos(\frac{\pi}{2}x^2)$ for $0 \leq x \leq 5$. Let p be the smallest positive root of $\cos(\frac{\pi}{2}x^2)$. Determine the value of p and evaluate $\text{FresnelC}(p)$ to eight decimal places. What is the maximum value of $\text{FresnelC}(x)$ on $[0, \infty)$?
 - (c) Find $\lim_{x \rightarrow \infty} \text{FresnelC}(x)$. Confirm your answer numerically.
 - (d) Evaluate $\text{FresnelC}(x)$ at $x = 2$ and describe the region whose area you've computed.
 - (e) Find the area under $\text{FresnelC}(x)$ and above the x -axis on the interval $[0, 2]$.
 - (f) Use the symbolic processor to evaluate $\int \sqrt{x} \sin x dx$. Use pencil and paper to confirm Mathcad's answer.
 2. (a) Use the symbolic processor to find the derivative of the Error Function. Which calculus theorem tells you how to obtain the derivative without using the symbolic processor?
 - (b) On separate plots, graph $\text{erf}(x)$ and e^{-x^2} for $0 \leq x \leq 3$. Find $\lim_{x \rightarrow \infty} \text{erf}(x)$. Confirm your

answer numerically.

- (c) Evaluate $\operatorname{erf}(x)$ at $x = 2$ and describe the region whose area you've computed.
- (d) Find the area under $\operatorname{erf}(x)$ and above the x -axis on the interval $[0, 2]$.
- (e) Solve the equation $\operatorname{erf}(x) = 0.7$ in two ways:
- use the symbolic processor's **Solve for Variable** command, and
 - use Mathcad's "root" function.
- (f) Use the symbolic processor to evaluate $\int x^2 e^{-x^2} dx$. Use pencil and paper to confirm Mathcad's answer.
3. (a) Use the symbolic processor to find the derivative of the Sine Integral. Which calculus theorem tells you how to obtain the derivative without using the symbolic processor?
- (b) On separate plots, graph $\frac{\sin x}{x}$ and $\operatorname{Si}(x)$ for $0 < x \leq 10$ in steps of 0.2. Find $\lim_{x \rightarrow \infty} \operatorname{Si}(x)$.
- (c) Find the maximum value of $\operatorname{Si}(x)$ on the interval $(0, \infty)$.
- (d) Find the area between $\frac{\sin x}{x}$ and the x -axis on the interval $[0, 5]$.
- (e) Repeat part (d) for the function $\operatorname{Si}(x)$.
- (f) Use Mathcad to antidifferentiate $x \sin\left(\frac{1}{x}\right)$. Find the limit of the antiderivative as $x \rightarrow 0$.
- (g) Use the symbolic processor to evaluate $\int \frac{\sin \sqrt{x}}{x} dx$. Use pencil and paper to confirm Mathcad's answer.
4. Consider the function of x defined by $f(x) = \int_0^x \frac{1}{2+\cos t} dt$ where $x > 0$.
- (a) On separate plots, graph $\frac{1}{2+\cos x}$ and $f(x)$ on the interval $[0, 10]$. What is the smallest and largest slope of $f(x)$?
- (b) Evaluate $f(x)$ at $x = 2\pi$ and describe the region whose area you've computed.
- (c) Find the area between $f(x)$ and the x -axis over the interval $[0, 2\pi]$.
- (d) Use the symbolic processor to evaluate $\int_0^x \frac{1}{2+\cos t} dt$. Mathcad will return the function $g(x) = \frac{2}{3}\sqrt{3} \arctan\left(\frac{\sqrt{3}\tan(x/2)}{3}\right)$. Which calculus theorem tells you that the derivative of $g(x)$ must be $\frac{1}{2+\cos x}$? Verify this by using the symbolic processor.
- (e) Graph $g(x)$ on the interval $[0, 10]$ and observe that it differs from the graph of $f(x)$. Therefore, $g(x)$ is not equivalent to $f(x)$ and so Mathcad's symbolic evaluation of $f(x)$ cannot be correct for all $x > 0$. For which values of x is $g(x) = f(x)$? How should $g(x)$ be amended in order to obtain an expression for $\int_0^x \frac{1}{2+\cos t} dt$ valid for all $x > 0$? (Hint: Remember that Mathcad omits an arbitrary constant of integration.)

5. (a) Use Mathcad to define the function $F(x) = \int_0^x \frac{1}{\sqrt{1-t^2}} dt$. Graph $F(x)$ on the interval $[-0.99, 0.99]$. On a separate plot, graph the function $\arcsin x$ over the same interval to see that the graph is the same as that of $F(x)$.
- (b) Define the function $G(x) = \int_0^x \frac{-1}{\sqrt{1-t^2}} dt$. On the same plot, graph $G(x)$ and $\arccos x$ over the interval $[-0.99, 0.99]$. From the graph, you should see that there is a constant difference between the two functions; graph the difference $\arccos x - G(x)$ to make this clear. What is the exact value of the constant $K = \arccos x - G(x)$? (Hint: by hand, evaluate each function at $x = 0$.)
- (c) By hand, verify that $F'(x) = \frac{d}{dx} \arcsin x$ and $G'(x) = \frac{d}{dx} \arccos x$ (Which calculus theorem did you use in the process?). Explain how the findings of parts (a) and (b) are consistent with these equalities.
- (d) Using a definite integral, define a function $H(x)$ which is related to $\arctan x$ in the same way that $F(x)$ and $G(x)$ are related to $\arcsin x$ and $\arccos x$, respectively. Graph $H(x)$, $\arctan x$ and $H(x) - \arctan x$.

5.4 Homework Help

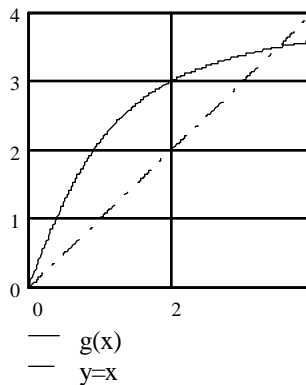
- You may want to use some of the techniques introduced in Section 2.3 for exercises involving differentiation:
 - Exercises 35-60 p.324 LHE
 - Exercises 27-46 p.350 LHE
 - Exercises 25-36 p.359 LHE
 - Exercises 47-64 p.377 LHE
 - Exercises 15-28 p.396 LHE
- Solutions of exercises involving curve sketching could be verified using Mathcad (as done in Problem 6 on page 44):
 - Exercises 67-72 p.325 LHE
 - Exercises 53-58 p.351 LHE
- When working on integration exercises, you may want to consider using Mathcad in a way similar to Problem 1 on page 53, Problem 1 on page 61, or Problem 9 on page 63.
 - Exercises 1-38 p.333 LHE

- Exercises 67-84 p.351 LHE
- Exercises 60-65 p.360 LHE
- Exercises 1-44 p.386 LHE

5.5 Questions

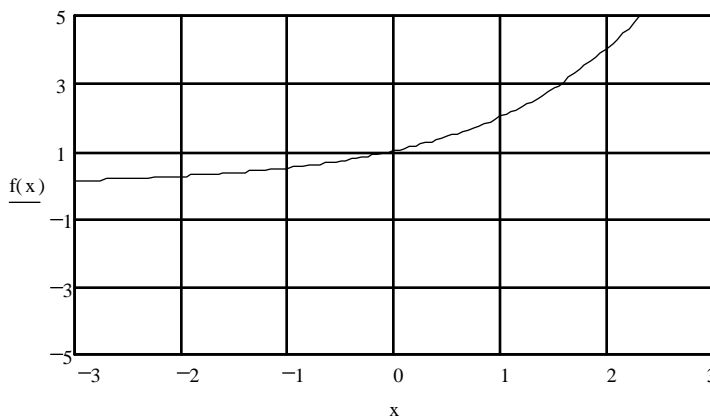
1. Based on the graph of $g(x)$, which one of the following statements is **False**.

- A. $g^{-1}(x)$ is increasing on $(0,4)$.
- B. $g^{-1}(0)=0$
- C. $g^{-1}(x)$ is concave up on $(0,4)$.
- D. $g^{-1}(2)=3$
- E. $g^{-1}(3)=2$



2. Choose the most appropriate function, based on its graph below.

- A. $f(x) = \ln(x)$
- B. $f(x) = -e^{-x}$
- C. $f(x) = 2^x$
- D. $f(x) = -\ln(x)$
- E. $f(x) = 10^x$



3. Choose the most appropriate function, based on its graph below.

- A. $f(x) = \ln(x)$
- B. $f(x) = -e^{-x}$
- C. $f(x) = 2^x$
- D. $f(x) = -\ln(x)$
- E. $f(x) = 10^x$

