3 Applications of Differentiation

This chapter is intended to illustrate some of the applications of differentiation.

The activities of Section 3.1 illustrate the relationship between the values of first and second derivatives of a function, and the shape of its graph.

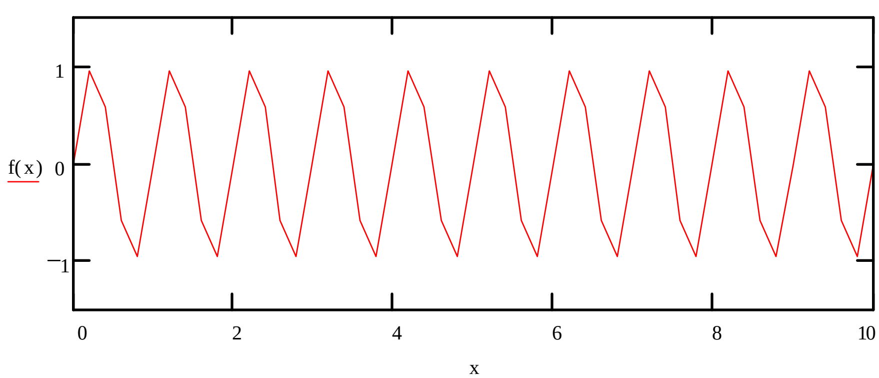
Section 3.2 demonstrates the use of Mathcad to solve optimization problems. Although computer technology cannot help you with the task of transcribing a word problem into mathematical form, it will offer some assistance with the algebra and calculus needed to solve the resulting mathematics problem.

Calculus vs. Computers (Do we really need both?)

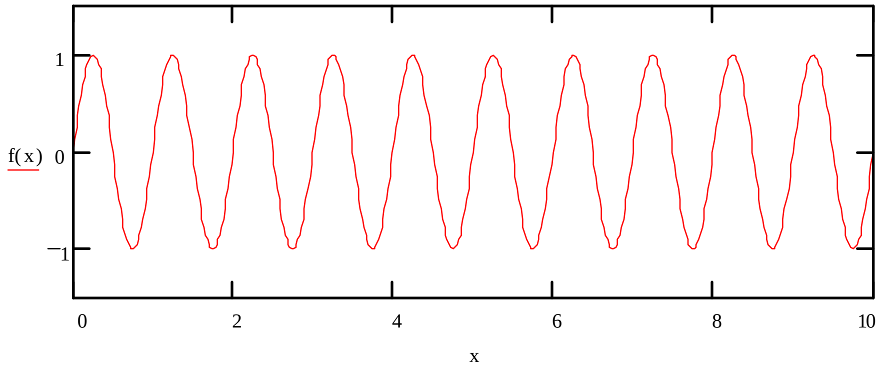
Not so long ago, when most calculations were done by hand, with a sliderule, or using a non-graphing calculator, sketching the graph of a given (even moderately complicated) function $f(x)$ was not an easy task. Instead of tediously evaluating $f(x)$ at a great many x values, qualitative curve sketching was the method of choice. Using information obtained from $f'(x)$, $f''(x)$ as well as other sources (asymptotes, intercepts, etc.), important information about the graph is extracted, including extrema and points of inflection. Then, evaluating $f(x)$ only at a *few* points would lead to a graph that would accurately depict the behavior of $f(x)$.

But now that powerful personal computers and graphing calculators are available at a relatively reasonable price, one might reasonably ask: why bother qualitatively analyzing $f(x)$, let us just sample it at sufficiently many points to generate the graph.

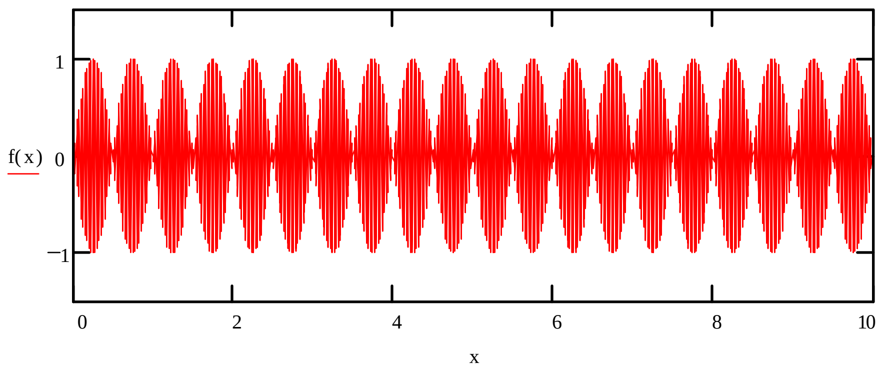
This may sound like a good idea at first - until one realizes that for many functions it is far from clear how many points is sufficiently many, as illustrated in the following example. This graph



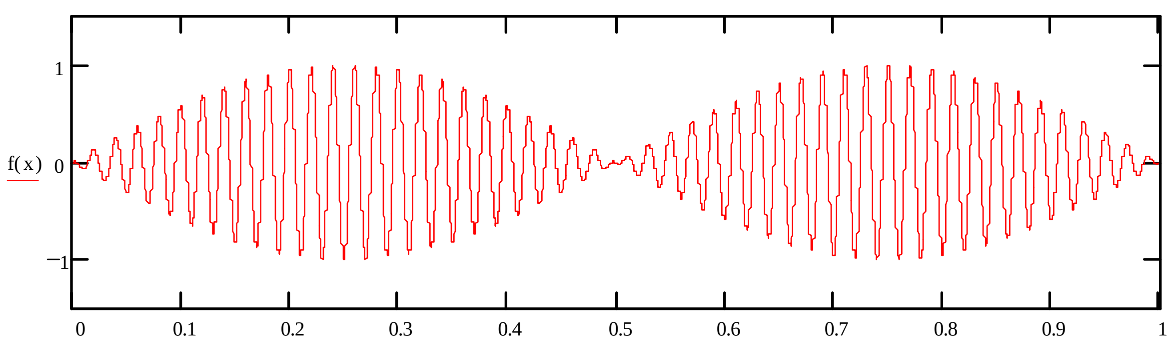
has been obtained using Mathcad with a range variable $x = 0, 0.2 .. 10$. Since the graph looks "rough", one could decide to use a "denser" range variable: $x = 0, 0.02 .. 10$, yielding



This sinusoidal shape is somewhat more satisfactory, so that one could be inclined to accept this as the ”accurate” graph of $f(x)$. Unfortunately, taking a slightly denser range variable $x = 0, 0.01$ .. 10 produces this disturbing picture:



Clearly the ”trial and error” approach in picking the samples is not working out very well here. On the other hand, analyzing the function’s expression: $f(x) = \cos(100\pi x)\sin(2\pi x)$ would enable us to identify important features of the function, which are reflected on the following graph (with the range variable $x = 0, 0.0001$ .. 1).



Being able to determine sufficiently large *number* of samples is not the only difficulty encountered when graphing a function. As you will see in some of the problems in the following section, a qualitative analysis could also be useful to determine *where* these samples need to be taken.

In spite of the above discussion, you should not be inclined to forgo using computers for curve sketching. In fact, computing technology can be extremely useful in depicting graphs of functions - as long as it is used in an *intelligent* fashion. For example, in several problems of the first section, you will use Mathcad's numerics and/or symbolics to generate not just the graph, but additional information such as derivatives and their zeros. In this fashion, your knowledge of calculus and access to computing technology can work together to enable you to accomplish a lot more.

3.1 Activity: Curve Sketching

Prerequisite: Read Sections 3.1-3.6 LHE.

In this activity you will investigate what information the graphs of the first and second derivative of $f(x)$ give us regarding the shape of the graph of $f(x)$. You will use Mathcad to find extrema and inflection points of functions. You will also determine vertical, horizontal and slant asymptotes.

Instructions

After reading the comments and studying the worked example, open a blank Mathcad document, and create your report there, answering all the problems identified by your professor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

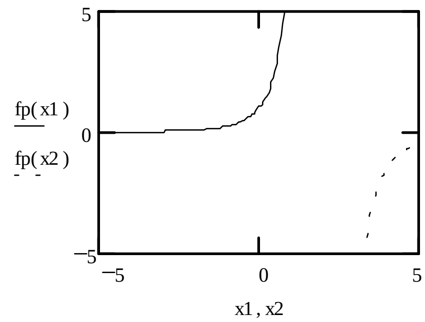
Comments

1. While working on this assignment, or performing similar tasks using Mathcad, you may experience problems with Mathcad's numerical differentiation operator. We have already mentioned (see page 30) the lack of robustness exhibited by this operator when one attempts to use it close to a point where there is no derivative. There are several ways to circumvent this problem, namely:

   (a) choose the range variable in such a way that it stays away from the points of nondifferentiability,

   (b) graph the function using two or more different range variables which, when taken together, cover the desired interval, e.g.

$$f(x) := \frac{x+3}{(x-2)^2} \qquad fp(x) := \frac{d}{dx} f(x) \qquad \begin{array}{l} x1 := -5, -4.95 .. 1.9 \\ x2 := 2.1, 2.15 .. 5 \end{array}$$



(c) follow the approach demonstrated in Problem 9 on page 30.

Example

(a) Graph $f(x) = x^5 - 5x^4 + 5x^3 + 7$ on $[-2, 4]$ in steps of 0.1. Restrict the vertical scale to obtain a useful picture.

In parts (b)-(d), use the graph and the "root" function to determine the answers in the approximate sense.

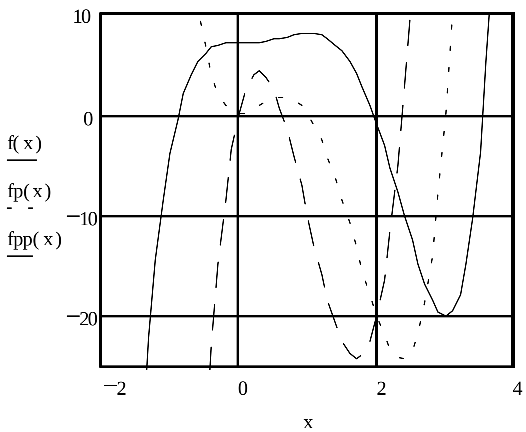
(b) On the same plot graph $fp(x) = \frac{d}{dx} f(x)$. Find the relative maxima and minima of $f(x)$ on $[-2, 4]$.

(c) On the same plot graph $fpp(x) = \frac{d^2}{dx^2} f(x)$. Find the inflection point(s) of $f(x)$ on $[-2, 4]$.

(d) Where on $[-2, 4]$ is $f(x)$ both rising and concave up?

Solution

(a) $x := -2, -1.9 .. 4$ $\quad f(x) := x^5 - 5\cdot x^4 + 5\cdot x^3 + 7$ $\quad fp(x) := \frac{d}{dx} f(x)$ $\quad fpp(x) := \frac{d^2}{dx^2} f(x)$



(b) $x := 0$ $\quad root(fp(x), x) = -2.687 \cdot 10^{-13}$

In fact, x = 0 is the smallest root of fp(x).

(0 , 7) is not a relative extremum,
since fp does not change sign at x=0.

$x := 1$ $\quad root(fp(x), x) = 1$

(1 , 8) is a relative maximum

$x := 3$ $\quad root(fp(x), x) = 3$

(3 , -20) is a relative minimum

(c) $x := 0$ $\quad root(fpp(x), x) = 0$ $\quad f(root(fpp(x), x)) = 7$ $\quad$ (0 , 7) is an Inflection Point.

$x := .5$ $\quad root(fpp(x), x) = 0.634$ $\quad f(root(fpp(x), x)) = 7.569$ $\quad$ (0.634 , 7.569) is an I.P.

$x := 2.5$ $\quad root(fpp(x), x) = 2.366$ $\quad f(root(fpp(x), x)) = -9.319$ $\quad$ (2.366 , -9.319) is an I.P.

(d) f is rising when fp > 0 , i.e., - infinity < x < 1 and 3 < x < infinity

f is concave up when fpp > 0, i.e., 0 < x < 0.634 and 2.366 < x < infinity.

Therefore, f is rising and concave up on 0 < x < 0.634 and 3 < x < infinity.

Problems

1. Follow the procedure demonstrated in the Example to find all intercepts, relative extrema and points of inflection of $g(x) = \frac{x^3-2x^2-7x-4}{x^2+4}$ on the interval $[-5, 5]$ (use 0.03 as the increment when graphing).

2. Repeat Problem 1 for the function $f(x) = 2x\sin(2x^2) - \frac{x^3}{3}\cos(3x-2)$ on the interval $[-1.2, 1.2]$.

3. (a) Graph $f(x) = \frac{2x^3-4x^2+3}{x-2}$ on $[-5, 5]$ in steps of 0.03. Restrict the vertical scale to obtain a useful picture (try -20 to 30).

   Use Mathcad's symbolics to determine the exact expressions for the first and second derivatives of $f(x)$; define these derivatives as functions $fp(x)$ and $fpp(x)$, respectively.

   In parts (b)-(d), use the graph and the "root" function to determine answers in the approximate sense.

   (b) On a separate plot graph $fp(x)$. Find the relative maxima and minima of $f(x)$ on $[-5, 5]$.

(c) On another plot graph $fpp(x)$. Find the inflection point(s) of $f(x)$ on $[-5,5]$.

(d) Where on $[-5,5]$ is $f(x)$ both rising and concave up?

(e) By hand, find the value c such that at $x=c$, $f(x)$ is discontinuous. Based on the graph, find $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$. Is $x=c$ a vertical asymptote?

4. (a) Graph $g(x)=15x^4-x^3$ on $[-5,5]$. Now rescale the plot window and get a closer look at the graph near the origin. There is some hidden behavior of $g(x)$ to be discovered in part (b).

   (b) By hand calculation find the roots of $g(x)$ and all relative extrema and inflection points. Determine the sign diagrams for $g(x)$, $gp(x)=\frac{d}{dx}g(x)$ and $gpp(x)=\frac{d^2}{dx^2}g(x)$.
   Specify the coordinates of the relative extrema and inflection points of $g(x)$ in your document.

   (c) Based on the results of part (b), use Mathcad to regraph $g(x)$ in a plot window which allows its important features to be clearly shown.

   (d) Repeat part (c) for $gp(x)$ and $gpp(x)$.

5. Define three functions: $f1(x)=\frac{x-2}{2x^2+4}$, $f2(x)=\frac{x^2-2}{2x^2+4}$, $f3(x)=\frac{x^3-2}{2x^2+4}$ and plot them on the same graph for x in $[-4,4]$.

   (a) By hand, find horizontal or slant asymptotes for each function. Now use Mathcad to re-plot the three functions, this time over $[-10,10]$, and explain briefly how the graphs agree with the asymptotes you've just determined.

   (b) Copy and paste to create another copy of your graph of $f1$, $f2$ and $f3$. On that graph, add a plot of function $f4(x)=\frac{x^4-2}{2x^2+4}$ (define it first). Does the new function appear to have horizontal or slant asymptotes? Is that what you expected? Explain.

   (c) So far in this problem, whenever we got a horizontal or slant asymptote, it was the same asymptote for both $x\to\infty$ and $x\to-\infty$. This was so because all our functions were rational (i.e. ratios of polynomials).
   Now, for a change, define $f5(x)=\frac{(|x|)^3-2}{2x^2+4}$ and plot it for x in $[-10,10]$.
   By hand, determine both slant asymptotes (one of them you've already found in part (a)), and then use Mathcad to add their graphs to the plot of function $f5$.

6. Consider the function of Exercise 22 p. 211 LHE.

   (a) Solve the problem completely by hand.

   (b) Use Mathcad to verify the correctness of your solution (in particular, use symbolic first and second derivatives and the function's graph). If you notice any discrepancies, then check your solution for errors.

7. Repeat problem 6 for Exercise 18 p. 211 LHE.

8. Repeat problem 6 for Exercise 26 p. 211 LHE.

3.2 Activity: Optimization

Prerequisites: Review Section 3.7 LHE, and Section 2.2 ”Introduction to Symbolics” of this manual.

In this section you will learn how symbolic and numeric features of Mathcad can be used when solving optimization problems.

Instructions

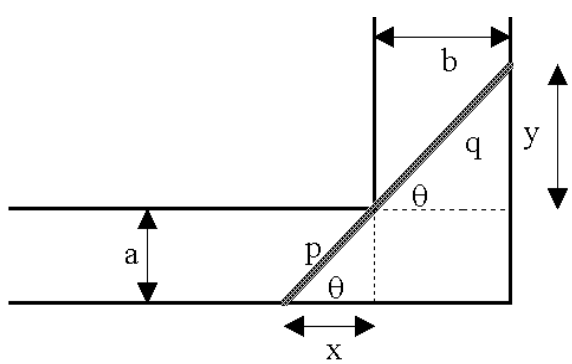
After studying the worked example, open a blank Mathcad document, and create your report there. Remember to enter your team’s name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Example

Find the length of the longest pipe that can be carried level around a right-angle corner if the two intersecting corridors are of width a feet and b feet. In particular, how long can the pipe be when $a = 5$ and $b = 8$?

Solution

A picture illustrating the problem is shown in the figure below.



For any given angle θ, the length of pipe cannot exceed the value of p+q. Note, the longest allowable length of pipe will be given by the minimum value of p+q over all possible angles θ between 0 and π/2.

The length of the pipe is $L = p + q$ where $p = \sqrt{x^2 + a^2}$ and $q = \sqrt{y^2 + b^2}$

We note that $\frac{y}{b} = \frac{a}{x}$, by similar triangles, so that

$L = \sqrt{x^2 + a^2} + \sqrt{y^2 + b^2}$ and $y = \frac{a \cdot b}{x}$

by substitution, yields

$$\sqrt{x^2 + a^2} + \sqrt{a^2 \cdot \frac{b^2}{x^2} + b^2}$$

simplifies to

$$\sqrt{x^2 + a^2} \cdot \frac{(x + b)}{x}$$

by differentiation, yields

$$\frac{dL}{dx} = \frac{1}{\sqrt{x^2 + a^2}} \cdot (x + b) + \frac{\sqrt{x^2 + a^2}}{x} - \sqrt{x^2 + a^2} \cdot \frac{(x + b)}{x^2}$$

simplifies to

$$\frac{\left(x^3 - b \cdot a^2\right)}{\left(\sqrt{x^2 + a^2} \cdot x^2\right)}$$

Therefore, $\frac{dL}{dx} = 0$ when $x^3 - b \cdot a^2 = 0$. The only real solution to this equation is

$x = \left(b \cdot a^2\right)^{\frac{1}{3}}$ Also, if $x < \left(b \cdot a^2\right)^{\frac{1}{3}}$, then $x^3 < b \cdot a^2$ i.e. $\frac{dL}{dx} < 0$.

Similarly, if $x > \left(b \cdot a^2\right)^{\frac{1}{3}}$, then $x^3 > b \cdot a^2$ i.e. $\frac{dL}{dx} > 0$.

Therefore, the First Derivative Test tells us that we have a local minimum at $x = \left(b \cdot a^2\right)^{\frac{1}{3}}$

Since, $0 < x <$ infinity , there are no end points to be concerned with. Therefore, we have found our solution, namely:

$x = \left(b \cdot a^2\right)^{\frac{1}{3}}$ and $L = \sqrt{x^2 + a^2} \cdot \frac{(x + b)}{x}$

by substitution, yields

$$\sqrt{b^{\left(\frac{2}{3}\right)} \cdot a^{\left(\frac{4}{3}\right)} + a^2} \cdot \frac{\left[b^{\left(\frac{1}{3}\right)} \cdot a^{\left(\frac{2}{3}\right)} + b\right]}{\left[b^{\left(\frac{1}{3}\right)} \cdot a^{\left(\frac{2}{3}\right)}\right]}$$

simplifies to

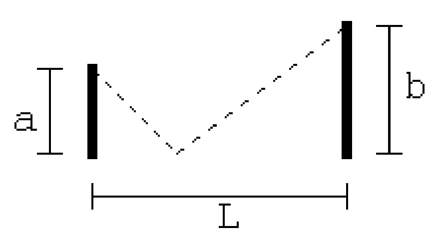
$$\left[b^{\left(\frac{2}{3}\right)} + a^{\left(\frac{2}{3}\right)}\right]^{\left(\frac{3}{2}\right)}$$

For the case $a = 5$ and $b = 8$, we have $L = \left(8^{\frac{2}{3}} + 5^{\frac{2}{3}}\right)^{\frac{3}{2}} = 18.22$ feet.

Problems

1. An open-topped box is to be made from a sheet of paper a inches wide and b inches long by cutting a square from each corner of the paper and folding up the remaining sides. Determine the length of the cutout (in terms of a and b) that will maximize the volume of the box. (Hint: Use the Second Derivative Test to verify your solution.) In particular, determine the volume of the largest box made this way from an 8.5 by 11 inch sheet of paper.

2. Two posts, one a feet high and the other b feet high, stand L feet apart. They are to be attached to a single stake by wires running from the top of each post to the stake (see the diagram below). Determine where the stake should be located to minimize the amount of wire used.



3.3 Homework Help

- For Exercises 3-20 and 25-34 in Section 3.2 LHE, you may find Mathcad helpful when verifying your answers. If your answer involves a c value, you can use Mathcad to create plots containing three items:
  - the graph of each function $f(x)$ over the designated interval $[a, b]$,
  - the *secant* line passing through the points $(a, f(a))$ and $(b, f(b))$ and
  - the line *tangent* to the graph of f at the value c you identified.

  By visually inspecting each graph, you should be able to verify your solution.

  (If your answer to some of the Exercises 3-20 is negative, i.e., you found that Rolle's Theorem cannot be applied, you may plot the first two items above. Note that even if Rolle's Theorem cannot be applied, it does **not** automatically follow that no c exists within the interval such that $f'(c) = 0$.)

- You could follow the approach of Problem 6 on page 44 when working on most of the exercises in Sections 3.3, 3.4 and 3.6 as well as the Review Exercises (p.244 LHE) in the text. Caution: remember Mathcad's limitations, already mentioned in the previous chapter on page 30, when dealing with derivative graphs or functions like $x^{1/3}$.

3.4 Questions

1. Based on the following graph of f(x) and g(x) which one of the following statements is true?

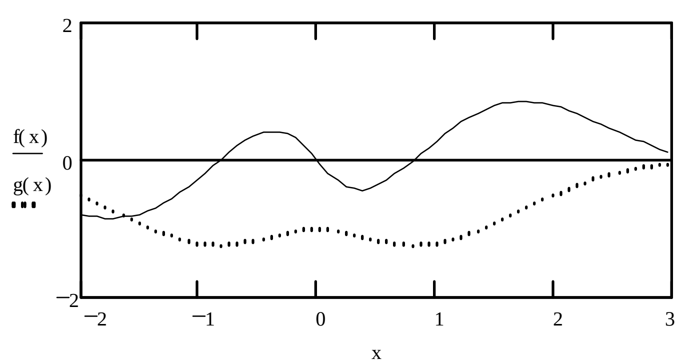
A. f(x) = g' (x)

B. g(x) = f ' (x)

C. f(x) = g''(x)

D. g(x) = f''(x)

E. None of the above.



2. Based on the graphs of g(x) and h(x) choose the one statement which is true.

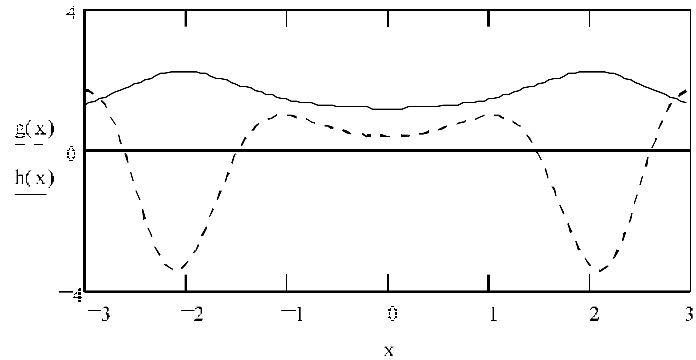
A. g(x)=h ' (x)

B. h(x)=g' (x)

C. g(x)=h " (x)

D. h(x)=g'' (x)

E. None of the above



3. Based on the following graph, choose the equation that best represents f(x).

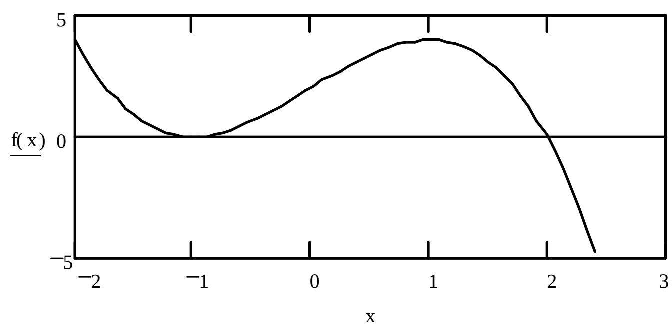
A. $f(x)=(x+1)\cdot(x-2)^2$

B. $f(x)=(x-1)^2\cdot(x-2)$

C. $f(x)=(x+1)^2\cdot(2-x)$

D. $f(x)=(x+1)^2\cdot(x+2)$

E. $f(x)=\left(x^2-1\right)\cdot(2-x)$



4. Based on the graph of y ' (x), which one of the following statements is true?

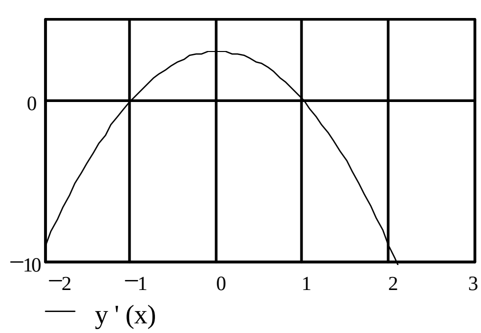
A. y(x) is concave down on [-1,1].

B. y(x) has a relative maximum at x=0.

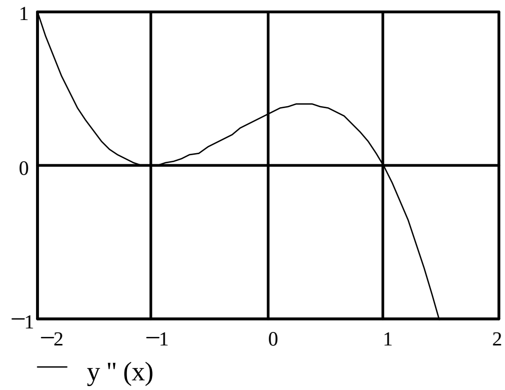
C. y(x) has a relative maximum at x = -1.

D. y(x) is decreasing on [0,2].

E. y(x) has a relative maximum at x=1.



5. Based on the graph of y '' (x), which one of the following statements is true?



A. y(x) is concave down on [-2,0].

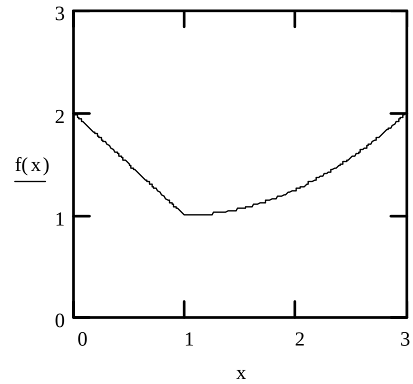
B. y(x) is concave down on [0,1].

C. y(x) has a relative maximum at x = -1.

D. y(x) has a point of inflection at x=1.

E. y(x) has a point of inflection at x= -1.

6. Which one of the following statements about f(x) on [0,3] is **not** true.



A. f(x) has a maximum value of 2.

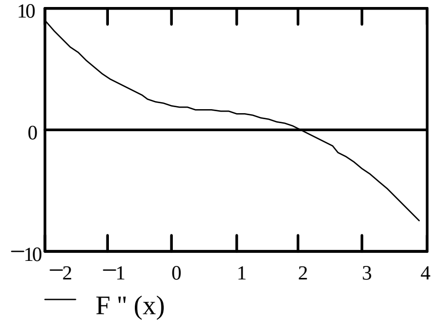
B. f(x) has no critical points.

C. f(x) is not differentiable at x=1.

D. The minimum value occurs at x=1.

E. f(x) does not satisfy the conditions of the Mean Value Theorem.

7. Based on the following graph of F ''(x), and the fact that F ' (- 1) = 0 , decide which one of the following cases appears to be true



A. F(x) has a point of inflection at x= - 1

B. F(x) is decreasing on [- 1, 3]

C. F(x) has a relative minimum at x= - 1

D. F(x) has a critical number x= -1, but the Second Derivative Test fails to identify it as a relative maximum/minimum.

E. F(x) has a relative maximum at x= - 1

4 Integration

The chapter on integration in LHE begins with an introduction to the *indefinite integral*,

$$\int f(x)dx = F(x) + C$$

where the function F is an *antiderivative* of f (i.e., $F' = f$) and C is an arbitrary constant. Later in the chapter, a number called the *definite integral*, $\int_a^b f(x)dx$ is discussed. It is initially introduced as a limit of a Riemann sum; then, as the theory is further developed, the Fundamental Theorem of Calculus formula relating the definite and indefinite integrals is presented.

Rather than following the text's order of presentation, we shall begin by focusing on the definite integral. One of the reasons for doing this is that definite integrals can be evaluated *numerically*. By its definition, the definite integral is a number, and very efficient ways exist to approximate this number on a digital computer. For example, selecting the definite integral symbol from the first palette, filling the placeholders with appropriate items, and then evaluating:

$$\int_0^2 \frac{3}{4}\cdot\sqrt{x}\,dx = 1.414$$

we obtain an approximation of the true value (in this case, $\int_0^2 \frac{3}{4}\sqrt{x}dx = \sqrt{2}$).

In the following section, the limit process that gives rise to the definition of definite integrals is explored. Further exercises will involve numerical methods to calculate definite integrals, and interpretation of such integrals as areas or as averages.

In Section 5.2, we will investigate Mathcad's *symbolic* integration tools. This will enable us to obtain expressions for indefinite integrals, and evaluate definite integrals exactly (rather than approximately).

4.1 Activity: Riemann Sums and the Fundamental Theorem of Calculus

Prerequisites: Read Sections 4.1 through 4.4 and 4.6 LHE.

One of the main objectives of this activity is to emphasize that a definite integral is a number which is defined to be the limit of a Riemann sum; in conjunction with this you will be asked to verify the Fundamental Theorem of Calculus. Different ways of evaluating definite integrals will be introduced and compared.

Instructions

A "template" document has been created for this assignment. Before proceeding, you should first read the comments below. Unless your professor instructs you otherwise, you should load the document RIEMANN.MCD. Follow the instructions on the screen to begin creating your lab report. After the "fill-in the blanks" portion, your report should include answers to the problems identified by your instructor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

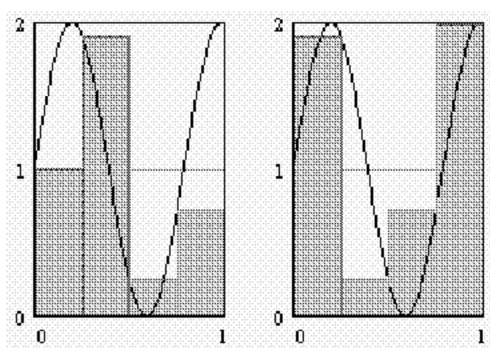
Comments

1. Mathcad has several different summation operators: $\boxed{\sum_{n=1}^{m}}$ (accessed from the first palette strip) which corresponds to the standard mathematical summation, and $\boxed{\sum_{i}}$ (accessed from the third palette strip) which we will use during this activity. The latter operator requires that the summation index be already defined as a range variable.

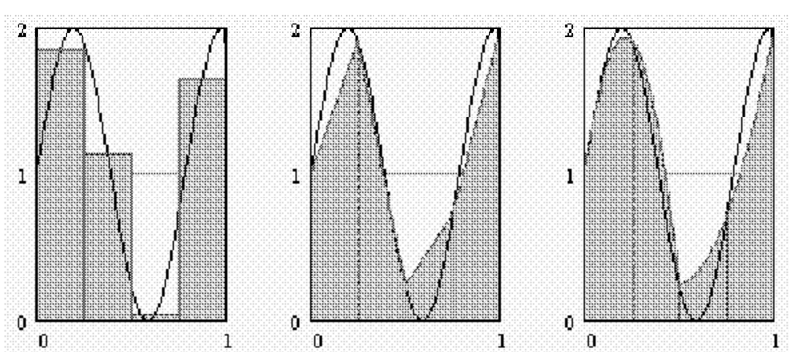
   Here are examples of usage of both operators:

   $$\sum_{i=1}^{5} i^2 = 55 \qquad\qquad i := 1..5 \quad \sum_{i} i^2 = 55$$

2. While working on the template document, you will encounter depictions of left and right Riemann sums:



   as well as Midpoint, Trapezoidal and Simpson's rules



Make sure you carefully watch these pictures as they change in response to your changing the number of partitions (n).

Problems

1. Consider the definite integral of Exercise 6, p. 289 LHE

   (a) Evaluate the integral by hand. Include the complete solution.

   (b) Have Mathcad numerically evaluate the integral. Does the answer agree with part (a)?

   (c) Does the integral correspond to an area of any region? Justify your answer. (You may want to use Mathcad to plot the integrand over the interval of integration).

2. Repeat Problem 1 for the integral of Exercise 8, p.289 LHE.

3. Repeat Problem 1 for the integral of Exercise 18, p.289 LHE.

4. Repeat Problem 1 for the integral of Exercise 19, p.289 LHE.

5. Repeat Problem 1 for the integral of Exercise 20, p.289 LHE.

6. Repeat Problem 1 for the integral of Exercise 24, p.289 LHE.

7. Consider the function $f(x) = (2 - x)\sqrt{x}$ and the interval $[0, 2]$.

   (a) Use Mathcad to evaluate the average value of the function over the given interval; assign it to a variable A.

   (b) In Mathcad, plot $f(x)$ and the constant function $y = A$ on the same graph. Use the combination of crosshair, zoom and Mathcad's "root" function to approximate all values of x where the function equals A.

8. Repeat Problem 7 for $f(x) = (x^2 + x + 1)\sec x$ on $[-1.2, 1]$.

9. Repeat Problem 7 for $f(x) = \frac{|x|}{x}\cos x$ on $[-0.6, 1.2]$. Be very careful. (Make sure you plot the function with "trace type" set to "points".)

   Let f be a function integrable over the interval $[c, d]$, and let us define

$$G(x) = \int_a^x f(t)dt$$

with x and a in $[c, d]$. Then, from the Second Fundamental Theorem of Calculus, we have $G'(x) = f(x)$, i.e., G is an antiderivative of f. In the following problems, you will compare $G(x)$ (for a chosen a value) and the antiderivative $F(x)$ obtained when evaluating the definite integral: $\int f(x)dx = F(x) + C$.

10. Consider the function $f(x) = x^3 - x + 3$.

   (a) By hand, evaluate the indefinite integral $\int f(x)dx = F(x) + C$.

   (b) In Mathcad, define the function $G(x)$ as discussed above, using the value $a = 2$. Plot both $F(x)$ and $G(x)$ on the same graph. Determine the value C for which $F(x) + C = G(x)$.

11. Repeat Problem 10 for $f(x) = 4\sin x - 3\cos x$ and $a = 0$.

12. Repeat Problem 10 for $f(x) = \sec^2 x - x^2$ and $a = \frac{\pi}{4}$. (Be careful when selecting the range variable for plotting.)

4.2 Homework Help

- Solutions to most of the exercises on evaluating definite integrals, can be verified as in Problem 1 on page 53. These include
  - Exercises 1-11, 14-16, 19-26 p.289 LHE
  - Exercises 51-55, 57, 58, 61-64 p.301 LHE
  - Exercises 33-44 p.314 LHE

- Exercises 39-46 p.289 LHE, dealing with Mean Value Theorem for Integrals and average values, could be accompanied by activities like those in Problem 7 on page 53.

- Most of the exercises in which an indefinite integral is evaluated, could be supplemented with a computer activity along the lines of Problem 10 on page 53. This could serve as illustration as well as verification of correctness of the solution obtained by hand.
  - Exercises 11-14, 16-34 p.256 LHE
  - Exercises 7-12, 14-49 p.300 LHE
  - Exercises 3-20 p.313 LHE

4.3 Questions

1. Based on the graph of y(x), determine the answer that best approximates $\int_{-2}^{0} y(x)\,dx$

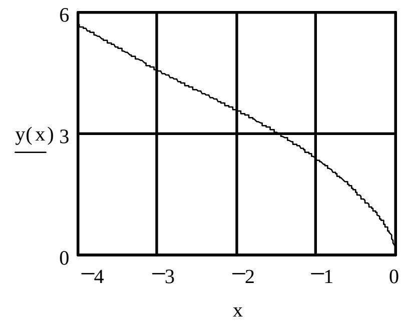
A. 24.566

B. - 4.754

C. 4.525

D. 11.125

E. - 1.541



2. Given that $F(x)=\int_{0}^{x} f(t)\,dt$ use the graph of f(t) to approximate F'(3).

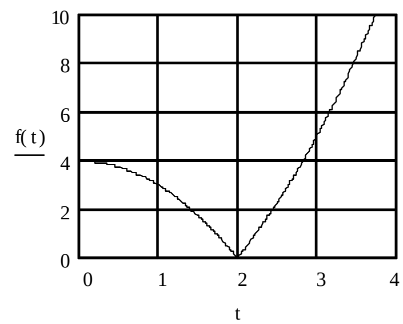
A. 12.566

B. 7.667

C. 2.108

D. 5

E. 0



3. Given that $G(x)=\int_{2}^{x} g(t)\,dt$ use the graph of g(t) to approximate G(1).

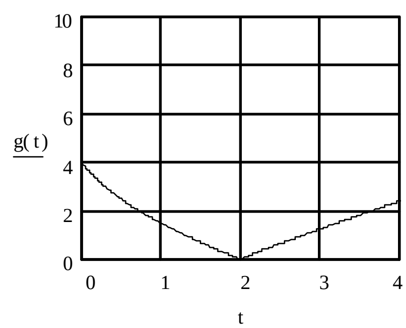
A. 2.625

B. - 0.716

C. 1.508

D. 0.891

E. - 1.641

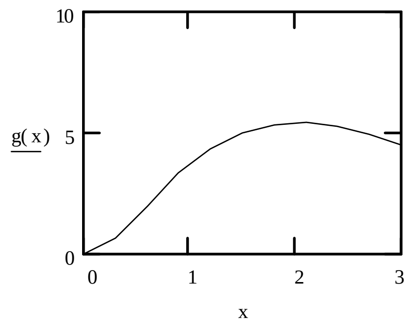


4. Based on the graph of g(x), which one of the following statements is true.

A. $\int_0^2 g(x)\,dx = 10.187$ B. $g(2) = 2.357$

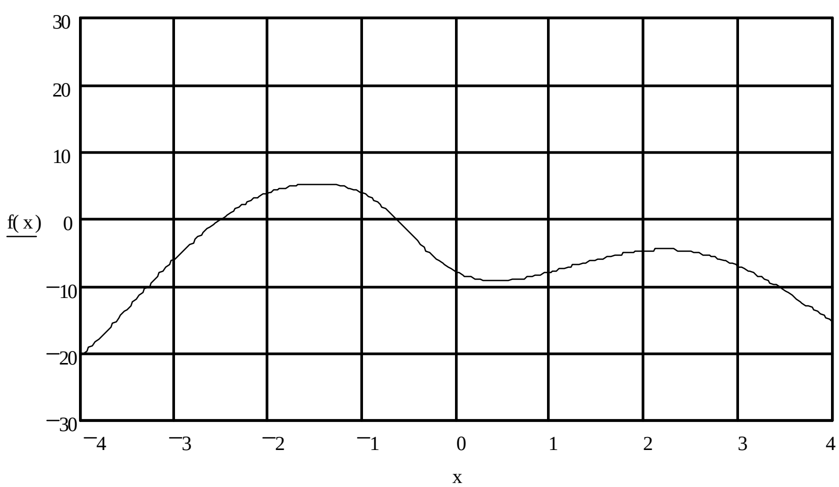
C. $\frac{d}{dx}\int_0^x g(t)\,dt = 5.413$ at x =2.

D. $\frac{d}{dx}g(2) = 2.357$ E. $\frac{d}{dx}g(1) = 1$



5. Based on the graph of f(x) shown below, choose the best answers to questions (i) and (ii) from the following list:

A. 70
B. 49
C. 20
D. 11
E. undefined
F. -11
G. -20
H. -49
I. -70



(i) The area between the graph of y = f(x) and the x-axis over the interval [0,3] is ____

(ii) The value of $\int_3^1 f(x)\,dx$ is ____