# 2   Differentiation

Mathcad can perform differentiation in two, fundamentally different, ways: *numerically* and *symbolically.*

Numerical differentiation corresponds to approximating the limit of the difference quotient used in the definition of derivative [p.108 LHE]. In this mode, Mathcad is capable of obtaining *numerical estimates* of the derivative function, without actually generating its *expression*, for example

$$g(x) := \frac{d}{dx}\sin(x) \qquad g(0) = 1 \qquad g\!\left(\frac{\pi}{6}\right) = 0.866 \qquad g\!\left(-\frac{\pi}{2}\right) = 0$$

The differentiation operator ▦ can be found on the first palette strip.

Note that the following, seemingly more straightforward, way to use this operator is incorrect:

$$\frac{d}{dx}\sin(0) = 0 \qquad \frac{d}{dx}\sin\!\left(\frac{\pi}{6}\right) = 0 \qquad \frac{d}{dx}\sin\!\left(-\frac{\pi}{2}\right) = 0$$

since Mathcad is attempting to differentiate the <u>constants</u> $\sin 0, \sin\frac{\pi}{6}$ and $\sin\frac{-\pi}{2}$.

With **symbolic differentiation**, *expressions* for derivatives can be obtained, e.g.,

$$\frac{d}{dx}\sin(x) \qquad \text{yields} \qquad \cos(x)$$

In doing so, Mathcad uses differentiation rules like those introduced in Sections 2.2-2.4 LHE.

This chapter includes two laboratory assignments. The first assignment, "The Derivative", will primarily address the numerical and graphical aspects of differentiation. The second assignment, entitled "Introduction to Symbolics", addresses many of the symbolic manipulation features offered by Mathcad, including symbolic differentiation.

2.1   Activity: The Derivative

Prerequisite: Read Section 2.1 LHE.

The objective of this activity is to improve and test your understanding of what the derivative of a function means. You will investigate how the derivative is related to the secant line and to the "smoothness" of a curve. You will also have an opportunity to practice determining derivatives using the definition, and finding intervals where a function is differentiable.

Instructions

After reading the comments and studying the worked examples, open a blank Mathcad document, and create your report. Answer all the problems identified by your professor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. In the solutions to the worked examples below, you will notice a subscript notation, e.g. $y_{\tan}$. In Mathcad, you can use such a notation when naming variables and functions using "literal" subscripts.

   To type such a subscript use the "dot" (period) key. For example, to obtain $y_{\tan}$, enter:

   $$y \ .(\text{dot}) \ t \ a \ n$$

2. In the solution to part (c) of Example 1, notice how function values are tabulated. In order to accomplish this, a range variable (e.g., x) must first be defined. Then asking Mathcad to evaluate x (by entering "x=") will produce a table of all the values of that range variable, while entering f(x) will tabulate the function values corresponding to all the different values of x, e.g.

$$x := 0, \frac{\pi}{6} .. \pi$$

| x	cos(x)
0	1
0.524	0.866
1.047	0.5
1.571	0
2.094	-0.5
2.618	-0.866
3.142	-1

Example 1

(a) For the function $y(x) = x^3 - x - 1$, plot $y(x)$, the line tangent to $y(x)$ at the point $(c, y(c))$ and the secant line through the points $(c, y(c))$ and $(c+h, y(c+h))$. Take $c = -2$, $h = 1$, and x on the interval $[-3, 3]$ in steps of 0.1.

(b) On separate plots, repeat part (a) with $h = 0.5$, then $h = 0.1$.

(c) Evaluate the slope of the secant line for $h = 0.1, 0.09, \ldots, 0.01$.

(d) Evaluate the slope of the secant line for $h = -0.1, -0.09, \ldots, -0.01$.

(e) What number does the slope of the secant line approach in the limit as h approaches 0?

(f) Use Mathcad's "root" function to approximate the x-intercept(s) of $y(x)$ over the interval
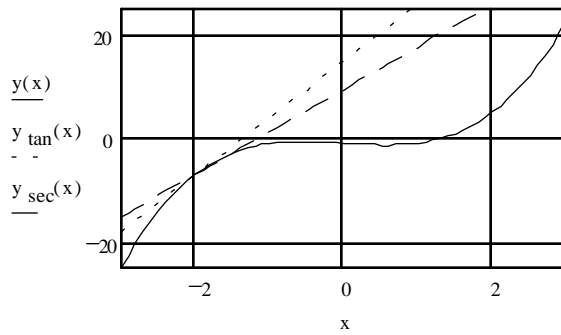
$[-3, 3]$.

Solution

(a) $x := -3, -2.9 .. 3$ $y(x) := x^3 - x - 1$ $Dy(x) := \dfrac{d}{dx} y(x)$ $c := -2$ $h := 1$

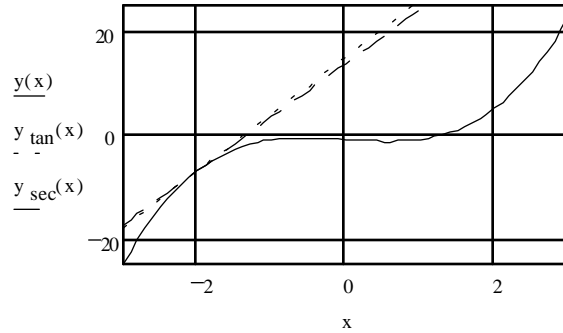$y_{tan}(x) := y(c) + Dy(c) \cdot (x - c)$ $y_{sec}(x) := y(c) + \dfrac{y(c + h) - y(c)}{h} \cdot (x - c)$
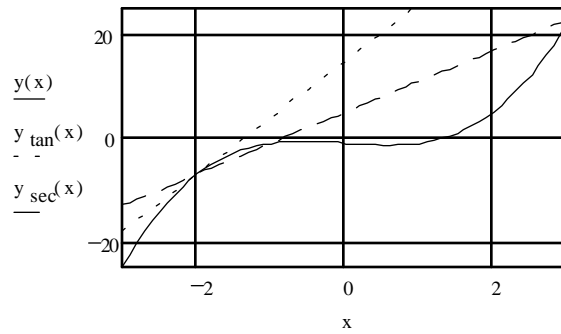
(b) $h := .5$ $y_{sec}(x) := y(c) + \dfrac{y(c + h) - y(c)}{h} \cdot (x - c)$

$$h := .1 \qquad \qquad y_{sec}(x) := y(c) + \frac{y(c+h) - y(c)}{h} \cdot (x - c)$$

(c) $h := .1, .09 .. .01$

$$\frac{y(c+h) - y(c)}{h}$$

h	
0.1	10.41
0.09	10.468
0.08	10.526
0.07	10.585
0.06	10.644
0.05	10.703
0.04	10.762
0.03	10.821
0.02	10.88
0.01	10.94

(d) $h := -0.1, -0.09 .. -0.01$

$$\frac{y(c+h) - y(c)}{h}$$

h	
-0.1	11.61
-0.09	11.548
-0.08	11.486
-0.07	11.425
-0.06	11.364
-0.05	11.302
-0.04	11.242
-0.03	11.181
-0.02	11.12
-0.01	11.06

(e) It appears that as h approaches 0, the slope of the secant line approaches 11.

In fact, this limit is equal to $Dy(-2) = 11$, which is the value of the derivative at -2, as expected.

(f) $x := 1 \qquad root(y(x), x) = 1.325 \qquad$ verify this answer by substitution: $y(root(y(x), x)) = -1.684 \, 10^{-5}$

Example 2

(a)   For the function $y = x \sin x$ and $\Delta x = 0.5$, plot $\frac{d}{dx} y(x)$, and the difference quotient function $DQy(x) = \frac{y(x + \Delta x) - y(x)}{\Delta x}$ on the interval $[-5, 5]$ in steps of 0.05.

(b)   Repeat part (a), with $\Delta x = 0.1$. From this plot, what can be said about the two graphs?

(c)   Copy the graph obtained in part (b) and zoom in to cover the interval $[1, 1.5]$. Compare the two graphs.

(d)   Copy the graph from part (c) and change the value of Δx to be 0.01 .
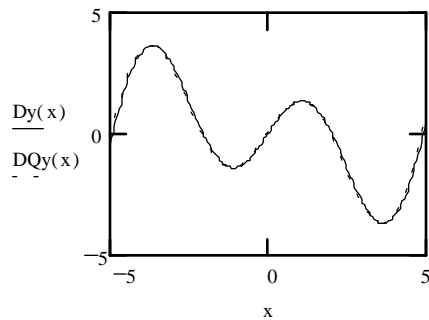
(e)   What would happen if we were to zoom in now?

Solution

(a) $y(x) := x \sin(x)$ $Dy(x) := \dfrac{d}{dx} y(x)$ $\Delta x := 0.5$ $DQy(x) := \dfrac{y(x + \Delta x) - y(x)}{\Delta x}$ $x := -5, -4.95 .. 5$
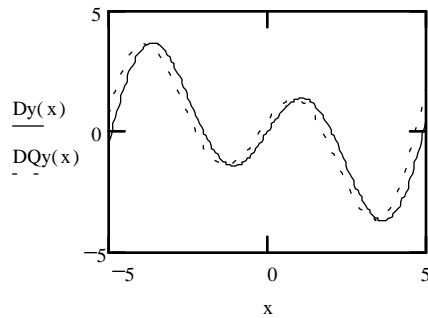
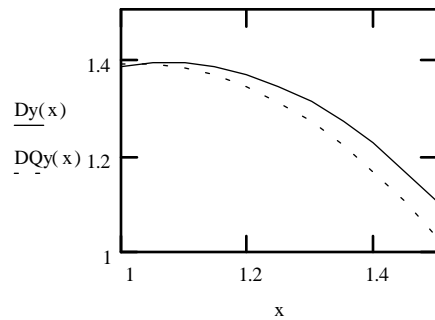**(b)** $\Delta x := 0.1$ $DQy(x) := \dfrac{y(x + \Delta x) - y(x)}{\Delta x}$
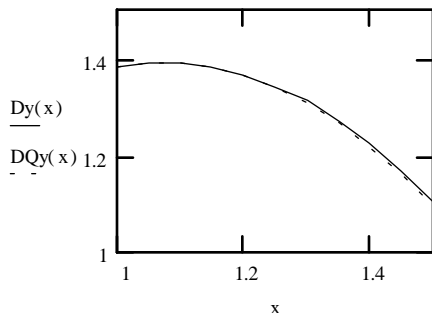

In this plot the graphs
are indistinguishable

(c) $x := 1, 1.05 .. 1.5$

After zooming in on the plot, differences
in the graphs can be seen.

(d) $\Delta x := 0.01$ $DQy(x) := \dfrac{y(x + \Delta x) - y(x)}{\Delta x}$

$\underline{Dy(x)}$

$\underset{-\,-}{\underline{DQy(x)}}$

Again the two graphs are indistinguishable.

(e) If we were to zoom in further we would be able to detect differences.

Problems

1.   Repeat Example 1 for $y(x) = x + 1 + \sin x$.

2.   Repeat parts (a)-(e) of Example 1 for $y = x \cos(x^2 - 2)$.

3.   Repeat parts (a)-(e) of Example 1 for $y = \sqrt{\left| \sin\left(3\,(x + 1.99999)^2\right)\right|}$. (Be alert!)

4.   (a)   Graph $f(x) = \sqrt{x^2 + k}$ and $\frac{d}{dx} f(x)$ on $[-2, 2]$ in steps of 0.01 for $k = 1$. On separate plots, regraph for $k = 0.1$, 0.01 and 0. In each case, find the slope of $f(x)$ at $x = 0$.

   (b)   What happens to the slope of $f(x)$ at $x = 0$ in the limit as k approaches 0 from the right? What happens in the case $k = 0$?

   (c)   What value does the slope of $f(x)$ approach as x tends to negative infinity?

5.   (a)   Graph $f(x) = \left| \dfrac{x^2(2-x)(1+x)}{x^3 - 4x^2 + 3x + 3} \right|$ on $[-3, 4]$ in steps of 0.001. Restrict the range of the vertical axis to $[-1, 3]$ in order to obtain a useful graph.

   (b) $f(x)$ has a vertical asymptote at $x = c$. Use the "root" function to estimate the value of c.

   (c)   Copy the graph obtained in part (a), and zoom in on the curve around the point $(1, f(1))$. Notice that the graph is almost a straight line.

   (d)   Copy the graph from part (a) again, and this time zoom in on the curve around the point $(2, f(2))$.

   (e)   If a function is differentiable (or smooth) at a point, then when you zoom in, you see a non-vertical straight line. Find all points where f is not smooth (use the information obtained in parts (b)-(d); you may also want to zoom in on other points).

   (f)   Now zoom out on the curve by graphing $f(x)$ on $[-1000, 1000]$ in steps of 1.

   (g)   What shape does the graph take now?

    (h)   What is the behavior (limit) of $f(x)$ as x tends to $+$ or $-$ infinity?

6.   Consider the function $g(x) = \frac{1}{3-x^2}$ and the point $(2, -1)$.

    (a)   By hand, use the definition of the derivative to find $g'(x)$.

    (b)   By hand, find the equation of the tangent line to the graph of g at the given point.

    (c)   Use Mathcad to plot $g(x)$ and the tangent line you obtained in part (b). (Make sure it **looks** tangent!)

7.   Repeat Problem 6 for the function $f(x) = \sqrt{x+1}$ and the point $(3, 2)$.

8.   Repeat Problem 6 for the function and the point given in Exercise 22 on page 113 LHE.

The following group of problems will deal with the issue of differentiability of a function. There are several ways in which a function can fail to be differentiable at a point (see the discussion on pp. 110-112 LHE). Unfortunately, Mathcad's numerical differentiation procedure tends to be very sensitive to nondifferentiable functions. While one can hardly blame it for failing to produce a derivative at a point where it doesn't exist, the algorithm will also fail to produce derivatives near that point.

Because of that, in the following problems, we shall bypass the internal differentiation procedure altogether. As we saw in Example 2, $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ tends to be very close to $f'(x)$ for small enough Δx; this gives rise to the approach known as a divided difference approximation of a derivative. We shall use this approach below.

9.   Given the function $y(x) = |x|$,

    (a)   by hand, find every point where the function is differentiable,

    (b)   in Mathcad, define the function $DQ(x) = \frac{y(x+0.001)-y(x)}{0.001}$ and plot it together with $y(x)$. Switch the "trace type" to "points" for both graphs. Is the plot consistent with your findings of part (a)?

10.   Repeat Problem 9 for the function given in Exercise 44 on page 114 LHE.

11.   Repeat Problem 9 for the function given in Exercise 46 on page 114 LHE.

Caution: Differentiability of functions like $y = x^{1/3}$ or $y = x^{2/5}$ is quite an interesting issue. However, we can't use Mathcad to graph these, or similar functions reliably. This will become obvious if you ask Mathcad to evaluate $(-1)^{1/3}$ - instead of the expected value of -1, you will see some complex number. This answer is correct, but is useless to us here, since we do not deal with complex arithmetic in this course.

2.2   Activity: Introduction to Symbolics

Prerequisite: Read Sections 2.1-2.4 LHE.

This activity introduces Mathcad's symbolic processor, and its differentiation tools in particular. You will then use these tools to help you master the mechanics of performing differentiation by hand (using the differentiation rules).

Instructions

A "template" document SYMBOLIC.MCD is available for this assignment - your professor will inform you whether you are to use it. Answer all the problems identified by your professor. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. As explained in the course of the assignment, the symbolic processor has to be loaded manually. While the symbolic processor is loading, the cursor turns into a Maple leaf, which indicates the Maple V kernel (which is responsible for handling symbolics in Mathcad) is being loaded. You must remember that the symbolic engine is not entirely integrated into Mathcad. In particular, it processes only one expression at a time, completely unaware of any other definitions made in your document, e.g. while

$$\frac{d}{dx}\tan(x) \qquad \text{simplifies to} \qquad \frac{1}{\cos(x)^2}$$

   works fine, the following code accomplishes nothing:

$$f(x) := \tan(x) \qquad\qquad \frac{d}{dx}f(x) \qquad \text{simplifies to} \qquad \frac{d}{dx}f(x)$$

2. To evaluate an expression at a specific value of the variable symbolically, the **Substitute for Variable** command should be used, as demonstrated in the following example:

   In order to symbolically evaluate $\frac{x^3+x+1}{x^2-x-2}$ at $x = c + \Delta x$ we need to

   (a) select the $c + \Delta x$ expression using the blue frame (click on it, and use the "up" arrow if necessary),

   (b) copy the expression to the clipboard,

   (c) select with the blue frame any x in the target expression ($\frac{x^3+x+1}{x^2-x-2}$),

   (d) choose **Substitute for Variable** from the **Symbolic** menu.

3. Two common mistakes worth avoiding:

   (a) *differentiating* a derivative

$$\frac{d}{dx}x^5 \quad \text{by differentiation, yields} \quad 20x^3$$

... instead of evaluating it

$$\frac{d}{dx}x^5 \quad \text{yields} \quad 5{\cdot}x^4$$

... or just differentiating the expression:

$$x^5 \quad \text{by differentiation, yields} \quad 5{\cdot}x^4$$

(b) differentiating only a portion of the expression

$$\frac{d}{dx}x^4 + x^2 \quad \text{yields} \quad 4{\cdot}x^3 + x^2$$

instead of differentiating the entire expression:

$$\frac{d}{dx}\left(x^4 + x^2\right) \quad \text{yields} \quad 4{\cdot}x^3 + 2{\cdot}x$$

Problems

1. (a) In Mathcad, evaluate numerically $\frac{1}{2} - \frac{1}{3}$ and $\sin(\frac{\pi}{3})$.
        Mathcad carries out numerical calculations keeping approximately 15 decimal places. We can have Mathcad show any result with up to 15 digits displayed.

    (b) Double-click on the first expression of part (a), and change the "Displayed Precision" value in the "Numerical Format" dialog, to 15. Repeat this for the second expression.

    Although more accurate, these values still remain only numerical approximations of the exact values. The first expression is a <u>rational</u> number (i.e., a fraction) and has the repeating decimal expansion $0.1\underline{6}\ldots$; the second one is an <u>irrational</u> number, and therefore cannot be expressed as a fraction or repeating decimal.

    In other words, even if Mathcad could process and display more than 15 decimal places, it still couldn't numerically calculate the exact values of the two expressions above.

    To evaluate expressions exactly, Mathcad contains a symbolic processor.

2. From the **Symbolic** menu, choose **Load Symbolic Processor**. Once the loading is complete, choose **Derivation Format** from the same menu. In the Derivation Format dialog box

- Switch "Show derivation comments" to ON (make sure a check appears in the box);

- Select "Show derivation steps" horizontally.

(a) Enter the expression 1/2-1/3 (in math mode), and use the "up" arrow if necessary to have the entire expression within the blue box. Choose **Evaluate Symbolically** from the **Symbolic** menu.

(b) Repeat this for the expression $\sin(\frac{\pi}{3})$.

The Symbolic Processor can also carry out algebraic operations involving <u>unassigned</u> variables. First, recall that trying to have Mathcad numerically evaluate something like $x - x^2 + \frac{x}{2}$ without having previously assigned a value to x, leads to an undefined variable error.

3. (a) Try to evaluate $x - x^2 + \frac{x}{2}$ *numerically* (type the expression followed by =).

   (b) Type (or copy) the same expression, enclose it in the blue frame, and evaluate it *symbolically.*

The Symbolic menu is quite extensive. Some of its features are beyond the scope of calculus, while others will be mentioned only briefly. In any case, you can always use Mathcad's on-line Help to obtain additional information.

4. Enter the expression $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$. Copy it onto the system clipboard, and paste it <u>underneath</u> the original expression three times. You should now have four identical copies of the expression.

   Enclose the entire first expression in the blue frame, then choose **Evaluate Symbolically** from the **Symbolic** menu. In a similar manner, apply **Simplify** to the second copy, **Expand Expression** to the third and **Factor Expression** to the fourth copy. (The last result will extend beyond the right margin. Drag it underneath.)

   In this example "Evaluate Symbolically" did not change the expression at all, whereas the other three commands resulted in a transformed (but equivalent) version of the expression.

5. (a) In Mathcad, define $G(x) = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$. Evaluate $G(\frac{\pi}{12})$ numerically.

   (b) Type $G(\frac{\pi}{12})$, and evaluate it symbolically. Are you satisfied with the result? (If necessary, see Comment 1 for an explanation.)

   (c) Use the **Substitute for Variable** procedure, described in Comment 2, to symbolically substitute the expression $\frac{\pi}{12}$ for x into the expression $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$. Evaluate the resulting expression numerically.

<u>Recap:</u> *In symbolic mode, use **Substitute for Variable** to evaluate a function for a specific*

value of a variable. Do not use function notation in symbolic mode; it is ignored by Mathcad in this mode.

6. *To solve an equation for an unknown, we must put the equation in the form $F(x) = 0$. If we type in the expression for $F(x)$, then select x and choose the Solve for Variable command, Mathcad will attempt to find an exact solution(s) to the problem.*

   (a) Enter the expression $\frac{6}{x} + \frac{x+10}{4} - 6$, enclose any x in the blue frame, then choose **Solve for Variable** from the **Symbolics** menu.

   (b) Copy the expression from part (a), and use **Substitute for Variable** to verify that your solution(s) are correct.

7. Use the **Solve for Variable** to solve the equation $x^2 + 1 = 0$.

   The expression obtained should involve the letter i . This of course denotes the imaginary number whose square is -1. Mathcad will attempt to find solutions using <u>complex</u> number arithmetic. In this course we are interested in real solutions only.

8. Find all <u>real</u> solutions to the equation $2x^3 - x^2 - 4x = 15$.

9. *While Mathcad can solve many equations, it must be remembered that not all equations can be solved.*

   Have Mathcad try to solve for x in the expression $x^2 + e^x \cos x$.

   Watch the cursor: when the "Maple leaf" turns into an arrow, Mathcad will be done with this calculation. Actually, this time Mathcad will "discreetly" give up. To have it acknowledge its defeat, ask it to solve the same equation AGAIN. What error message did Mathcad produce?

   To aid us in our investigation of calculus, the symbolic processor contains a differentiation command. There are two ways to obtain derivatives symbolically in Mathcad:

   – Click on the $\boxed{\frac{d}{dx}}$ symbol from the strip on the left, fill in the placeholders with the expression and the variable, then hit the SPACE BAR and choose **Evaluate Symbolically** from the **Symbolic** menu. (Note: To make sure the <u>entire</u> expression is differentiated, enclose it in <u>parentheses</u>.)

   – Enter the expression, then have the blue frame contain the independent variable, and choose **Differentiate on Variable** from the **Symbolic** menu.

10. Use *both* of the above methods to symbolically differentiate $x^2 + \csc x$ with respect to x.

11. Perform the symbolic differentiation to solve Exercises 23, 25 p.134 LHE and 5 p.161 LHE. In each case, perform additional operations (by choosing items from the Symbolic menu) until your final answer resembles that at the end of the book (on pages A38 and A41 LHE) - note: for Exercises 23 and 25, try to match the final form given in the book.

   Higher-order derivatives can be obtained by repeating the symbolic differentiation process as many times as necessary. Equivalently, we can use the $\boxed{\frac{d^n}{dx^n}}$ symbol from the third palette

strip, e.g.

$$\frac{d^2}{d\,x^2}\,\sin(x) \qquad \textsf{yields} \qquad -\sin(x)$$

12. Symbolically evaluate the third and the fifth derivatives of $3x^4 - 2x^3 + 7x - 1$ with respect to x.

13. Consider the functions $f(u) = u^2 + \csc u$ and $g(x) = \sin x$.

    (a) Use the **Substitute for Variable** operation to obtain the composite function $F(x) = f(g(x))$. Evaluate $F'(x)$ symbolically. (Throughout this problem, do <u>not</u> assign function names to your expressions.)

    (b) Evaluate $f'(u)$ symbolically, and then substitute $\sin x$ for u into the resulting expression, thereby obtaining $f'(g(x))$.

    (c) Evaluate $g'(x)$ symbolically, and then use copy and paste to multiply it by the result obtained in (b). Expand the expression.

    (d) Compare the final results of parts (a) and (c). What important theorem have you just verified?

14. Draw the graph of $f(x) = 2x^3 - x^2 + 6x - 3$ on the interval $[-5, 5]$ with step size 0.04.

    (a) Based on the graph, how many roots does the function appear to have?

    (b) Use the root function to find approximate values to all possible roots.

    (c) Use the **Factor** command from the **Symbolic** menu to factor the expression of $f(x)$.

    (d) How many real roots does $f(x)$ have?

    (e) What are the exact values of the root(s)?

15. Repeat Problem 14 for the function $f(x) = 20x^3 - 14x^2 - 10x + 7$ on the interval $[-2, 2]$.

16. Given $g(x) = \sin^2(2x^3 + x)$,

    (a) find $g'(x)$ *by hand*,

    (b) use Mathcad to *symbolically* find $g'(x)$,

    (c) compare your answers in (a) and (b). If you cannot see that they are *algebraically equivalent*, use Mathcad to verify that (one possible way: **subtract** the answer (b) from the answer (a) and symbolically **simplify** the difference - is it 0?)

    For Problems 17-22 repeat Problem 16 for the given function.

17.   The function of Exercise 32 on page 123 LHE.

18.   The function of Exercise 42 on page 134 LHE.

19.   The function of Exercise 2 on page 143 LHE.

20.   The function of Exercise 4 on page 143 LHE.

21.   The function of Exercise 20 on page 143 LHE.

22.   The function of Exercise 44 on page 143 LHE.

2.3   Homework Help

- Problem 6 on page 30 may be repeated for Exercises 17-21 p.113 LHE. Using the differentiation rules (instead of the definition), you can also solve Exercises 43-46 p. 124, 51-56 p.134 and 61-64 p.144 LHE.

  Generally, you can apply this procedure to *any* problem in Sections 2.1 - 2.4 LHE where you are asked to differentiate a function - you will just have to come up with coordinates of a point of tangency.

- Each of the exercises listed here involves determination of point(s) at which the given function has a horizontal tangent line. You can use Mathcad to draw each of the graphs, switch the **Show Markers** ON for both axes, and use the markers to create a cross at each point you've calculated.

  - Exercises 47-52 p.124 LHE

  - Exercises 57-58 p.134 LHE

- Problem 16 on page 35 can be repeated for Exercises

  - Exercises 3-16, 31-37, 41-42 p.123 LHE

  - Exercises 13-26, 29-46 p.134 LHE

  - Exercises 1-10, 13-14, 16-17, 19-30, 41-50 p.143 LHE

  - Exercises 7-14, 16-26, 28-32 p.161 LHE

- The following exercises in the text require finding the second or higher-order derivative - you can use an approach similar to that of Problem 16 on page 35 to verify your answers:

  - Exercises 72-80 p.135 LHE

  - Exercises 69-72 p.145 LHE

    – Exercises 41-44 p.162 LHE

- To help you in verification of tangency for implicit functions (Section 2.5 LHE) a *Maple V* template file `IMPLICTN.MS` is provided. To access that file, load Maple (double-click on its icon), select **Open** from the **File** menu, then select the `IMPLICTN.MS`. Select **Execute Worksheet** from the **Format** menu. The file will generate an illustration of Example 4 p.148 LHE. In order to illustrate one of the Exercises 1-28 p.151 LHE, you will need to enter different equations into the template. Note that you will need to calculate the equation of the tangent line (rather than just its slope). Remember: You can always reopen the original document if the modified document fails to execute properly. Also, do not hesitate to ask for help if you need it.

2.4   Questions

1.   The function g(x) produced the following output.
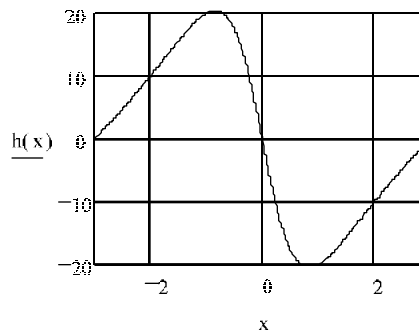   Which of the following statements is most likely to be true?

   A.   The function g is not defined at x=2.

   B.   g' (2) = -16

   C.   The limit of g(x) as x approaches 2 is -16

   D.   g is not continuous at x= 2

   E.   g(2) = -16.

Δx	$\dfrac{g(2+\Delta x) - g(2)}{\Delta x}$
0.1	-15.157
0.08	-15.32
0.06	-15.486
0.04	-15.654
0.02	-15.826

Δx	$\dfrac{g(2+\Delta x) - g(2)}{\Delta x}$
-0.1	-16.92
-0.08	-16.729
-0.06	-16.542
-0.04	-16.358
-0.02	-16.178

2.   Based on the graph of h(x) provided, estimate the value of $\dfrac{\Delta h}{\Delta x}$ at c=2 when $\Delta x = -2$

   A.   2

   B.   -5

   C.   13.666

   D.   -10

   E.   10

3. The function p(x) is plotted below on the interval [-3 , 3]. Which one of the following statements
is **FALSE**.

    A.   p(x) is not a polynomial

    B.   p(x) is continuous on [-3 , 3].

    C   As tends to 0, p(x) tends to 1

    D.   p(x) is not differentiable at 0

    E.   p(x) is not differentiable at 1.