

11 Vectors and the Geometry of the Space

11.1 Activity: Vectors

Prerequisites: Read Sections 11.1 - 11.4 LHE.

In this activity, you will learn how to use Mathcad to perform operations on vectors.

Instructions

After reading the comments and studying the worked example, open a blank Mathcad document and create your report there. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. Mathcad uses column notation for vectors, instead of row notation as used in the text. To define a vector in Mathcad, select **Matrices...** from the **Math** menu (or press Ctrl-M) and type the number of components in the "Rows:" box, while typing 1 in the "Columns:" box. For this example, each vector has two components. Clicking on the "Create" button will create a column vector with empty placeholders at the crosshair position. You should then type the values of the vector components in the placeholders. The "Matrices..." box will remain on the screen until you close it.
2. By default, Mathcad uses the subscript 0 for the first component. However, to represent vectors in the plane or in space, it is customary to begin numbering components with 1. To do this, we use Mathcad's built-in **ORIGIN** variable. We can either type the definition directly in our document

$$\text{ORIGIN} := 1$$

or access the **ORIGIN** value by selecting **Built-In Variables** from the **Math** menu. Remember that this must be done at the beginning of your document whenever you intend to work with vectors.

3. As stated above Mathcad uses column notation for vectors. If you were to use row-vectors, operations like dot and cross product will produce error messages.
4. Mathcad is capable of assigning units to quantities it deals with. During this activity, we will change the units of an angle from radians (default) to degrees. The following example illustrates how this can be accomplished.

$$\text{asin}\left(\frac{1}{2}\right) = 0.524 \quad \boxed{}$$

To change the units for the value displayed here, we clicked to the right of the number (0.524), so that the blue frame includes an empty placeholder positioned there. Typing "deg" over this placeholder and hitting the <Enter> key results in changing the units to degrees.

$$\text{asin}\left(\frac{1}{2}\right) = 30 \cdot \text{deg} \quad \boxed{}$$

(Instead of typing the name of the unit in, we could have selected it using the "Insert Unit" dialog box, which could be invoked by double-clicking to the right of the number, or selecting **Units** from the **Math** menu.)

Example

Use Mathcad to define the vectors: $\mathbf{u} = \langle -1, 2 \rangle$, $\mathbf{v} = \langle 3, 1 \rangle$ and $\mathbf{w} = \langle 0, -2 \rangle$.

- Have Mathcad calculate $\mathbf{u} + \mathbf{v}$, $2\mathbf{u}$, and $\mathbf{v} - \mathbf{v}$.
- Use Mathcad's numeric subscript notation (see Comment 1 on page 85) to specify the first and second components of \mathbf{u} . Calculate the length of \mathbf{u} in two ways: first by using Mathcad's "absolute value" sign, and then by having Mathcad compute the square root of the sum of squares of the components of \mathbf{u} .
- Verify the associative property of vector addition for \mathbf{u} , \mathbf{v} and \mathbf{w} .

Solution

$$\text{ORIGIN} := 1 \quad \mathbf{u} := \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{v} := \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \mathbf{w} := \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$\text{(a)} \quad \mathbf{u} + \mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad 2 \cdot \mathbf{u} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad \mathbf{v} - \mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{(b)} \quad u_1 = -1 \quad u_2 = 2 \quad |\mathbf{u}| = 2.236 \quad \sqrt{(u_1)^2 + (u_2)^2} = 2.236$$

$$\text{(c)} \quad (\mathbf{u} + \mathbf{v}) + \mathbf{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \mathbf{u} + (\mathbf{v} + \mathbf{w}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Problems

Note: You must specify ORIGIN:=1 at the beginning of your document.

1. Repeat the Example for $\mathbf{u} = \langle 1, 0, -1 \rangle$, $\mathbf{v} = \langle 0, -2, 1 \rangle$ and $\mathbf{w} = \langle \sqrt{2}, 1, \frac{1}{3} \rangle$.
2. Use Mathcad to define the vectors $\mathbf{u} = \langle 1, 4, 2 \rangle$ and $\mathbf{v} = \langle 0, -1, 3 \rangle$.
 - (a) Have Mathcad evaluate $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $3\mathbf{u}$, $\mathbf{u} \cdot \mathbf{v}$, $\frac{\mathbf{u}}{v}$, \mathbf{u}_2 and \mathbf{v}_2 .
Why does Mathcad fail to evaluate $\frac{\mathbf{u}}{v}$?
 - (b) When you entered $\mathbf{u} \cdot \mathbf{v}$ in part (a), Mathcad computed the dot product of \mathbf{u} and \mathbf{v} . Use Mathcad's subscript notation to calculate $\mathbf{u} \cdot \mathbf{v}$ from the definition of dot product (p.734 LHE) as a sum of products of components.
 - (c) Have Mathcad evaluate $\|\mathbf{u}\|$, $\|\mathbf{v}\|$ and $\|\mathbf{u} + \mathbf{v}\|$ using the "absolute value" sign. Verify that \mathbf{u} and \mathbf{v} satisfy the Triangle Inequality (p.720 LHE). (Note: Although the text only states this inequality for vectors in the plane, it is also valid for vectors in space.)
 - (d) Use Mathcad to define the vector $\mathbf{p} = \text{proj}_{\mathbf{v}} \mathbf{u}$ (vector component of \mathbf{u} along \mathbf{v}) and evaluate it. Define the vector $\mathbf{n} = \mathbf{u} - \mathbf{p}$ (vector component of \mathbf{u} orthogonal to \mathbf{v}) and evaluate it. Use Mathcad's acos function to calculate the angle between \mathbf{v} and \mathbf{u} , between \mathbf{v} and \mathbf{p} , and between \mathbf{v} and \mathbf{n} . By default, Mathcad computes all angles in radians. Have Mathcad change all three angles to degrees (see Comment 4).
 - (e) To evaluate a cross product in Mathcad, use the "button" $\overrightarrow{\mathbf{x}} \times \overrightarrow{\mathbf{y}}$ which appears on the second palette strip on the left side of the screen.
Evaluate $\mathbf{u} \times \mathbf{v}$, $\mathbf{v} \times \mathbf{u}$, $\mathbf{v} \times \mathbf{p}$ and $\mathbf{v} \times \mathbf{n}$.
Comment on how your answers illustrate some of the cross product properties listed on pp.745-746 LHE.

11.2 Activity: Lines and Planes

Prerequisites: Read Section 11.5 LHE

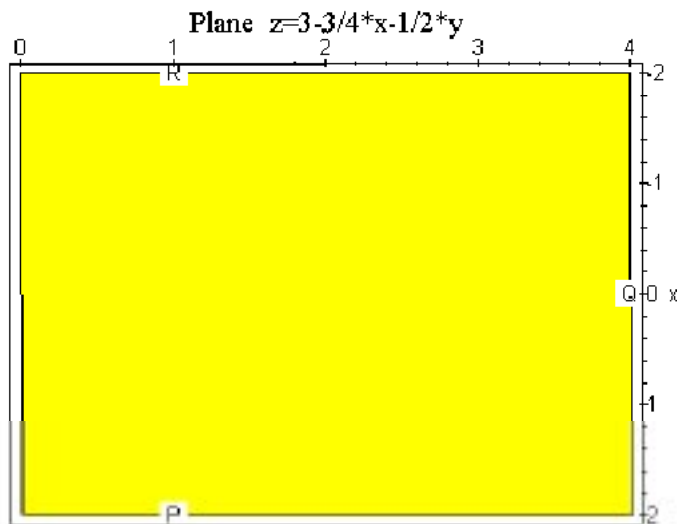
The objective of this activity is to help you visualize lines and planes in space; in particular, you will be able to visually verify solutions to various problems in this area.

Instructions

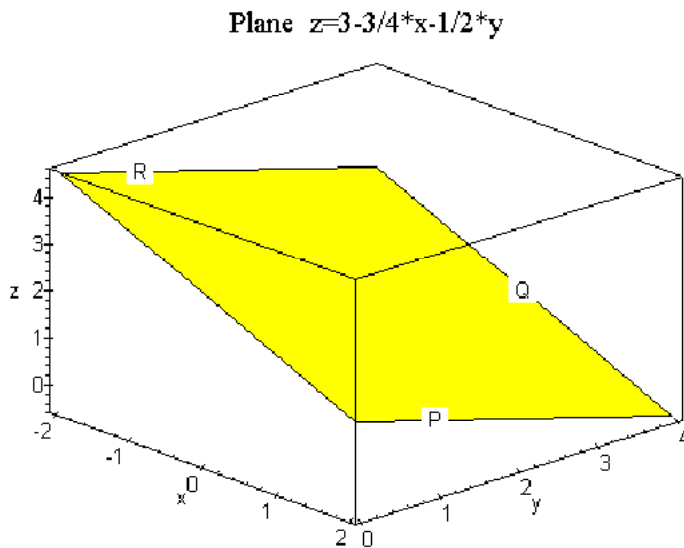
After reading the comments, open a blank Mathcad document and create your report there. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

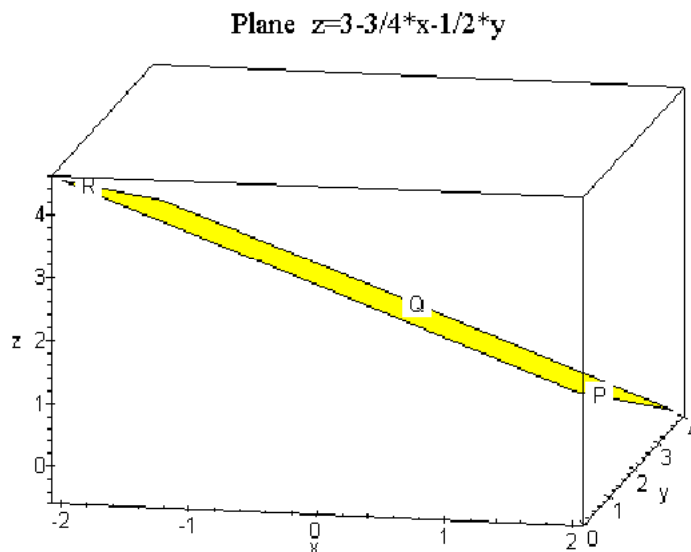
Comments

1. Throughout this activity, you will invoke several Maple template files. Upon execution of each of these files, a three-dimensional plot will be generated. You will be asked to see if the plot meets your expectations. For example, the following Maple plot was obtained to illustrate a plane passing through three given points P , Q and R .



Admittedly, it is rather hard to draw any conclusion from a plot like this. Fortunately, Maple affords us an easy way to rotate its 3-dimensional plots. When you start dragging the left mouse button within the plot window, you will begin rotating the 3-dimensional box whose edges correspond to the coordinate axes. Try dragging the mouse horizontally and vertically for rotations in different directions. When you are satisfied with the new position of the box, hit the <Enter> key to render the actual object (using the new viewpoint). Below are two such renderings, which appear to be more useful for our purpose.





Note that it usually takes several such rotations before we can get a reasonably good idea about the underlying three-dimensional objects.

Problems

1. (a) Use Mathcad to find the angle between the planes $x - 2y + z = 0$ and $2x + 3y - 2z = 0$, and to obtain a vector parallel to both planes. Then, by hand, find the parametric equations of the line of intersection of the given planes. Type the equations of the line in your document.
- (b) The two planes in part (a) are defined by RHS1 and RHS2 in Maple file PLANES1.MS. However, the line defined in this file by L1 does not correspond to the line of intersection of the two planes. Enter Maple and open this file. First, execute the code as is to see that the line "misses" the intersection.

Next, change the definition of L1 in the Maple code so that the line you derived in part (a) will be plotted. Execute the code, and then rotate the graph (using the mouse or the arrow keys) so that it is clear that this line does correspond to the intersection of the planes. Paste the plot into your Mathcad document.

2. Consider the two lines:

$$x = 2t + 3, y = 5t - 2, z = -t + 1 \text{ and } x = -2t + 7, y = t + 8, z = 2t - 1$$

Enter Maple and open the file LINES2.MS. In this file, the above two lines are defined by L1 and L2 and the point, P, is defined to be the origin. Execute the code to see that the lines appear to intersect, but not at the origin. By hand, find the point of intersection of the two lines. Then, in the Maple document, change the coordinates of P so that it becomes the



- point of intersection you found, and execute the code again. Rotate the graph until you are convinced that P now looks like the point of intersection. Paste the plot into your Mathcad document.
3. Consider the two intersecting lines of Exercise 20, p.760 LHE.
 - (a) Use Mathcad "Given ... Find" solve block to find the point of intersection P .
 - (b) Use the Maple template file `LINES2.MS` to illustrate the solution (follow the instructions in Problem 2).
 4. Enter Maple and open the file `PLANES2.MS`. Execute the code to see a plot of the solution to Example 3 on p.754 LHE. Solve Problem 30 p.760 LHE by hand. Modify the Maple code to illustrate that the plane you found does contain the three given points. Paste the plot into your Mathcad document.



11.3 Homework Help

- You can verify the correctness of your vector calculations in the following problems using Mathcad
 - Exercises 41-48 p.742 LHE
 - Exercises 7-12 p.750 LHE
- Following the procedure of Problem 1 and using the Maple template file referenced therein, you can illustrate the solution of
 - Exercises 41-46 and 57-58 p. 759 LHE
- Following the procedure of Problem 2 and using the Maple template file referenced therein, you can illustrate the solution of
 - Exercises 15-19 p.759 LHE
- Following the procedure of Problem 4 and using the Maple template file referenced therein, you can illustrate the solution of
 - Exercises 29-31 p.759 LHE

12 Vector-Valued Functions

12.1 Activity: Vector-Valued Functions

Prerequisites: Read Chapter 12 LHE

The objective of this activity is to help you understand that two position vector functions can describe different motions along the same path, but that curvature only depends on the location (not the motion) along a given path. You will also gain an understanding of the geometric meaning of curvature.

Instructions

After reading the comments and studying the worked example, open a blank Mathcad document and create your report there. Remember to enter your team's name at the top of the document. Upon completion of the assignment, enter the names of all team members who actively participated in the assignment. Save your work frequently.

Comments

1. Mathcad cannot differentiate (or integrate) a vector-valued function directly e.g.

$$Df(x) := \left[\begin{array}{c} \frac{d}{dx} (\sin(x) \cdot x) \\ \frac{d}{dx} \cos(x) \end{array} \right] \quad Df(1) =$$

non-scalar value

However, we can ask it to differentiate (integrate) each component:

$$Df(x) := \left[\begin{array}{c} \frac{d}{dx} (\sin(x) \cdot x) \\ \frac{d}{dx} \cos(x) \end{array} \right] \quad Df(1) = \begin{pmatrix} 1.382 \\ -0.841 \end{pmatrix}$$

2. Remember that the ORIGIN variable has to be set to one so that Mathcad will begin numbering the components of vectors (and vector-valued functions) from one (see Comment 2 in Section 11.1).
3. This activity is computationally intensive. You should be prepared to wait longer than usual for Mathcad (or Maple) to produce results. Also, there is a (greater than usual) possibility of a crash - saving your work frequently is your only safeguard against such mishaps.

Example

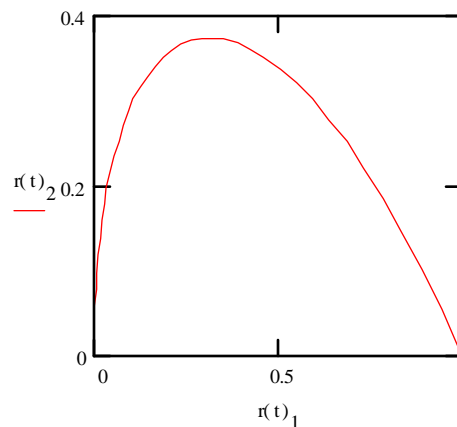
- (a) Use Mathcad to sketch the path of an object moving along the plane curve described by the position vector:

$$\mathbf{r}(t) = \left\langle \frac{-1}{65536} t^2(t+12)(t-8)(t+4)^2, \frac{-3}{2048} t(t+4)(t+8)(t-4) \right\rangle$$

for $0 \leq t \leq 4$.

Solution

$$\mathbf{r}(t) := \begin{bmatrix} \frac{-1}{65536} \cdot t^2 \cdot (t+12) \cdot (t-8) \cdot (t+4)^2 \\ \frac{-3}{2048} \cdot t \cdot (t+4) \cdot (t+8) \cdot (t-4) \end{bmatrix} \quad t := 0, 0.1..4$$



- (b) For $t = 0, 1, 2, 3, 4$, tabulate the position vector, velocity, speed, and acceleration. For efficiency, you will need to perform symbolic differentiation.

Solution

For efficiency reasons, we define the velocity and acceleration vectors using symbolic differentiation:

Velocity:

$$\mathbf{v}(t) := \begin{bmatrix} \frac{d}{dt} \left[\frac{-1}{65536} \cdot t^2 \cdot (t+12) \cdot (t-8) \cdot (t+4)^2 \right] \\ \frac{d}{dt} \left[\frac{-3}{2048} \cdot t \cdot (t+4) \cdot (t+8) \cdot (t-4) \right] \end{bmatrix} \quad \text{yields} \quad \mathbf{v}(t) := \begin{bmatrix} \frac{-3}{32768} \cdot t \cdot (t+4) \cdot (t+2) \cdot (t^2 + 4t - 64) \\ \frac{-3}{512} \cdot (t+2) \cdot (t^2 + 4t - 16) \end{bmatrix}$$

$$\begin{array}{l} \text{Acceleration:} \\ \text{yields} \end{array} \quad \mathbf{a}(t) := \begin{bmatrix} \frac{d}{dt} \left[\frac{-3}{32768} \cdot t \cdot (t+4) \cdot (t+2) \cdot (t^2 + 4t - 64) \right] \\ \frac{d}{dt} \left[\frac{-3}{512} \cdot (t+2) \cdot (t^2 + 4t - 16) \right] \\ \left[\begin{array}{l} \frac{-15}{32768} \cdot t^4 - \frac{15}{4096} \cdot t^3 + \frac{9}{1024} \cdot t^2 + \frac{33}{512} \cdot t + \frac{3}{64} \\ \frac{-9}{512} \cdot t^2 - \frac{9}{128} \cdot t + \frac{3}{64} \end{array} \right] \end{bmatrix}$$

In the calculation of $\mathbf{v}(t)$ and $\mathbf{a}(t)$, the following steps were performed:

- (i) Derivation format was chosen as "horizontally", with "show comments" checked on
- (ii) **Evaluate Symbolically** was chosen to perform differentiation
- (iii) **Derive in Place** was checked, and result simplified using the **Simplify** and **Factor** expression commands
- (iv) **Derive in Place** was turned off.

s := 0..4	Position		Velocity		Speed	Acceleration	
s	r(s) ₁	r(s) ₂	v(s) ₁	v(s) ₂	v(s)	a(s) ₁	a(s) ₂
0	0	0	0	0.188	0.188	0.047	0.047
1	0.035	0.198	0.081	0.193	0.21	0.116	-0.041
2	0.185	0.352	0.229	0.094	0.247	0.174	-0.164
3	0.505	0.338	0.413	-0.146	0.439	0.183	-0.322
4	1	0	0.563	-0.563	0.795	0.094	-0.516

Note: we changed the global setting of Exponential Threshold (in **Numerical Format** on the **Math** menu) to 15.

- (c) For $t = 0, 1, 2, 3, 4$, tabulate the unit tangent vector, principal unit normal vector, tangential and normal components of acceleration, and curvature.

Solution

$$\begin{array}{ll} \text{Unit tangent vector:} & \mathbf{T}(t) := \frac{1}{|\mathbf{v}(t)|} \cdot \mathbf{v}(t) \\ \text{Principal unit normal vector:} & \mathbf{N}(t) := \frac{1}{|\mathbf{T}_p(t)|} \cdot \mathbf{T}_p(t) \\ \text{Tangential component of acceleration:} & a_T(t) := \mathbf{a}(t) \cdot \mathbf{T}(t) \\ \text{Normal component of acceleration:} & a_N(t) := \mathbf{a}(t) \cdot \mathbf{N}(t) \end{array} \quad \begin{array}{ll} d\mathbf{T}/dt : & \mathbf{T}_p(t) := \begin{bmatrix} \frac{d}{dt} \mathbf{T}(t)_1 \\ \frac{d}{dt} \mathbf{T}(t)_2 \end{bmatrix} \\ \text{Curvature:} & \mathbf{K}(t) := \frac{|\mathbf{T}_p(t)|}{|\mathbf{v}(t)|} \end{array}$$

s	Position function		Unit tangent vector		Principal unit normal vector		Components of acceleration		Curvature
	$r(s)_1$	$r(s)_2$	$T(s)_1$	$T(s)_2$	$N(s)_1$	$N(s)_2$	$a_T(s)$	$a_N(s)$	
0	0	0	0	1	1	0	0.047	0.047	1.333
1	0.035	0.198	0.386	0.922	0.922	-0.386	0.007	0.123	2.795
2	0.185	0.352	0.925	0.38	0.38	-0.925	0.099	0.218	3.572
3	0.505	0.338	0.943	-0.334	-0.334	-0.943	0.28	0.243	1.261
4	1	0	0.707	-0.707	-0.707	-0.707	0.431	0.298	0.471

Problems

Note: Throughout this activity you should change the global setting of Exponential Threshold (in Numerical Format on the Math menu) to 15.

- (a) Repeat all three parts of the Example for the path described by the position vector:

$$\mathbf{p}(t) = \left\langle \frac{-1}{128}t^2(t-12), \frac{-3}{32}t(t-4) \right\rangle \text{ for } 0 \leq t \leq 4.$$

To save time, you should insert the file (select **Insert** from the **File** menu) **VVFUN1.MCD** into your solution document. Remember to change the definition of $\mathbf{p}(t)$. Also, you will need to perform symbolic differentiation (as described above) to obtain velocity and acceleration vectors for $\mathbf{p}(t)$.



- (b) Compare the path and tabulated values from part (a) to the values obtained in the Example. Observe that the paths are alike, but the tabulated values are different. In fact, $\mathbf{p}(t)$ and $\mathbf{r}(t)$ describe the same path. However, $\mathbf{p}(t)$ may not equal $\mathbf{r}(t)$ for the same value of t . In other words, the point described by $\mathbf{p}(t)$ traverses the same path as $\mathbf{r}(t)$, but moves either faster or slower at each value of t .

Use <Alt>-<Tab> to go to Program Manager and run Maple. Open the file **VVFUN2.MS**. While playing the animation of the two moving points, notice that the value of t is displayed above the graph.



Use the "advance" key $\boxed{\rightarrow}$ and the direction key ($\boxed{\rightarrow}$ and $\boxed{\leftarrow}$) to reach the frame with $t = 1$. Click on the point labeled $\mathbf{r}(t)$ and look at the x and y coordinates displayed by Maple in the bottom of the animation window. Check that these coordinates are close to the corresponding values given in the solution to part (b) of the Example.

Notice that the point $\mathbf{p}(1)$ is not very close to $\mathbf{r}(1)$; therefore, we should not expect the velocity and acceleration at $\mathbf{p}(1)$ to be the same as at $\mathbf{r}(1)$.

For what value of t will the point $\mathbf{p}(t)$ be at the same location as $\mathbf{r}(1)$? To obtain an approximate answer to this question, use the "advance" and the direction key to find $t = T1$ where the point $\mathbf{p}(T1)$ is approximately equal to $\mathbf{r}(1)$. (You should click on the point $\mathbf{p}(t)$, and compare the coordinates with those of $\mathbf{r}(1)$.) Write down the $T1$ value

for use in part (c) of this problem. Based on the animation, which point ($\mathbf{p}(T1)$ or $\mathbf{r}(1)$) appears to be moving faster? Again, write down your answer for use in part (d) of this problem.

Repeat the above procedure to identify $T3$ where $\mathbf{p}(T3)$ is close to $\mathbf{r}(3)$. Write down the $T3$ value for use in part (c) of this problem. Now which point appears to be moving faster? Again, write down your answer for use in part (d) of this problem.

Copy the last frame you found (where $\mathbf{p}(T3)$ is close to $\mathbf{r}(3)$) onto the clipboard. Use $\langle \text{Alt} \rangle - \langle \text{Tab} \rangle$ to switch to your Mathcad document, and paste the frame there. Use Alt-Tab to go back to Maple and EXIT FROM MAPLE . In Program Manager, use $\langle \text{Alt} \rangle - \langle \text{Tab} \rangle$ to go to Mathcad.

- (c) Use Mathcad's root function to improve upon your estimate of $T1$ found in part (b), as follows:

Solve the equation $\mathbf{p}(t)_1 = \mathbf{r}(1)_1$ using the $T1$ value as your initial guess. Call this improved estimate $t1$, and verify that both $\mathbf{p}(t1)_1 = \mathbf{r}(1)_1$ and $\mathbf{p}(t1)_2 = \mathbf{r}(1)_2$ are satisfied. *In a similar manner, find $t3$ such that $\mathbf{p}(t3) = \mathbf{r}(3)$.*

- (d) Define the range variable $s = t1, t3..t3$ (Therefore, s will assume just two range values: $t1$ and $t3$.) For these two range values, create tables for the same quantities that you tabulated in part (a). To save time, just copy-and-paste all the tables from your solution to part (a) of this problem.

Now compare these tables with the second and fourth rows of the tables created in the Example. Observe that some quantities have the same value in both tables, while others don't.

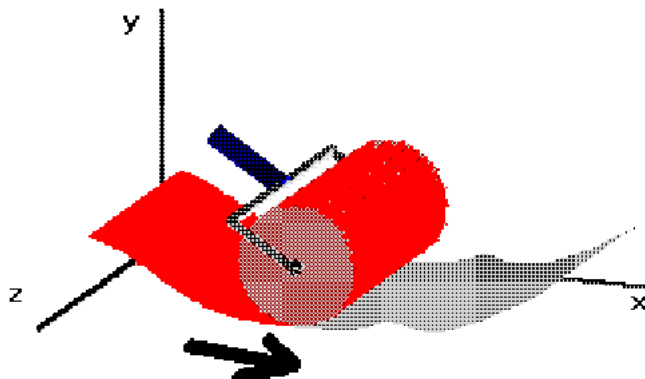
How does the speed of \mathbf{p} at $t1$ compare with that of \mathbf{r} at 1? How about the speed for $\mathbf{p}(t3)$ vs. $\mathbf{r}(3)$? Are these findings consistent with your observation of the animation in part (b)? Which quantities have the same values in both tables? Comment on why this should be expected.

- (e) Use Mathcad to calculate the arc length of the curve described by $\mathbf{p}(t)$ from $t = 0$ to $t = 4$. Then define $\mathbf{vr}(t)$ to be the velocity corresponding to $\mathbf{r}(t)$, and use it to verify that the arc length of $\mathbf{r}(t)$ from $t = 0$ to $t = 4$ is the same as that of $\mathbf{p}(t)$.

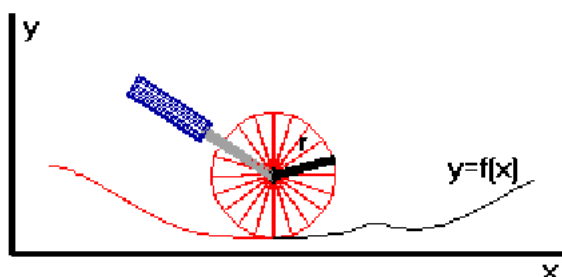
Define L to be the value of the arc length obtained above. Find the value of t for which the point $\mathbf{p}(t)$ travels a distance of $L/2$. (Hint: Use Mathcad's root function.) Repeat this for the point $\mathbf{r}(t)$.

2. In problem 1(d) you observed the curvature had the same value in both tables. Therefore, curvature does not appear to depend on the motion along a given path. Rather, it only depends on the location on that path. The solution to the following problem should provide you with an understanding of the geometric meaning of curvature.

Consider a cylindrical surface whose generators are parallel to the z -axis. Suppose the trace of the surface in the xy -plane is described by a given function $y = f(x)$.



Your problem is to determine the *largest radius* of the roller which can be used to *completely* paint *both* sides of the surface. (We assume the axle of the roller is parallel to the z -axis.) Equivalently, your problem can be rephrased as follows: Find a circle with as big a radius r as possible which can be rolled over the upper and lower sides of $y = f(x)$ while touching every point of the curve.



- (a) The particular curve to be painted in this problem is given by

$$f(x) = \sin\left(\pi \cos \frac{x}{2}\right) \quad \text{for } 0 \leq x \leq 5.$$

Use Mathcad to plot this curve.

- (b) Use <Alt>-<Tab> to access Program Manager, invoke Maple, and then open the file **ROLLER1.MS**. Follow the instructions included in the file to produce two animations of circles of different radii rolling along the curve. (Note that the red layer of paint is artificially elevated to make the painting easier to follow.) Observe that the circle of radius 1 (first animation) is too big to paint the entire curve, whereas the circle of radius 0.4 (second animation) successfully paints it. Copy the final frame of each animation onto the clipboard, and paste it into your Mathcad document. **EXIT FROM MAPLE**.
- (c) Invoke Maple again, this time opening the file **ROLLER2.MS**. In that file, you are asked to choose a value of the radius r (type it over "???") between 0.4 and 1 and then observe if the circle successfully paints the curve. Copy the final frame of the animation onto the clipboard and paste the plot into your document. Then, type the value of r next to the plot. **EXIT FROM MAPLE**.



- (d) Plot the curvature $K(x)$ of the given function $f(x)$ for x on $[0, 5]$ in steps of 0.02. (Use the formula of Theorem 12.10 on page 822 LHE.) Next, plot the derivative of the curvature and change the trace type to "points." Finally, use Mathcad's "root" function to find the two x values where the curvature attains its maximum value for x on $[0, 5]$. Calculate this maximum curvature at each x value and define it to be $Kmax$.

Read the discussion on page 822 LHE, directly underneath the proof of Theorem 12.10. In that discussion, the definition of radius of curvature is given:

$$\text{radius of curvature} = \frac{1}{\text{curvature}}$$

Pay attention to Figure 12.35 on page 822 LHE. Before proceeding with this problem, you should understand the geometric meaning of radius of curvature.

- (e) Since curvature is never negative, maximizing it is equivalent to minimizing the radius of curvature. Use the value of $Kmax$ you obtained in part (d) to calculate the minimum radius of curvature, denoted by $Rmin$, of the given curve $f(x)$ for x on $[0, 5]$.
Use <Alt>-<Tab> to switch to Program Manager, invoke Maple, and return to the document ROLLER2.MS. Follow the instructions included in the file to see the animation of a rolling circle of radius equal to $Rmin$ obtained above. (You will need to type the value of $Rmin$ over "???.") Use copy and paste to paste the final frame into your Mathcad document. Then in the main Maple window, type in (at the last ">" prompt) `Roller(vvv)`; and hit ENTER (where vvv stands for the value of $Rmin + 0.02$). Copy and paste the final frame of the resulting animation into your Mathcad document. EXIT FROM MAPLE.
- (f) What is the radius of the largest circle that paints the given curve? In your own words, explain the geometric meaning of "curvature" and how it differs from the "concavity" of a curve.
3. Calculate the curvature of the function $f(x) = \frac{x(2-x)(x-5)}{8}$ and evaluate for $x = 1$. Determine the circle of curvature for $f(x)$ at $x = 1$. Using parametric equations to describe the circle of curvature, graph both $f(x)$ and the circle of curvature on the same plot. (HINT: You can use Mathcad's symbolic features to help you find the center of the the circle.)
4. Consider Problem 16 p. 814 LHE.
- Solve the problem by hand.
 - Use the symbolic features of Mathcad to verify your answer.
5. Consider Problem 30 p. 815 LHE
- Solve the problem by hand.
 - Use Mathcad's numerical or symbolic features to verify your answer.

12.2 Homework Help

- Use the template of Problems 4 and 5 of Section 12.1 to solve these similar exercises
 - Exercises 13-18, 27-29 p.814 LHE
 - Exercises 33-40 p.828 LHE