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The Use of Maple in Graphing Solution Curves to Initial Value Problems

Abstract: Definitely it is worthwhile spending time to use paper and pencil to graph solution curves to initial value problems. Here we illustrate with examples the use of Maple software to graph solution curves in seconds. Some examples are chosen so that analytical solutions are not employable.

Discussion: There are problems occurring in science and engineering that can be modeled by initial value problems containing ordinary differential equations with some initial conditions. There are some analytical methods available to solve differential equations. It is indeed worthwhile the time spent on studying these analytical methods. However in case analytical methods are not available we still ask ourselves how the solution curves behave or even look like. Software engineers who understand these analytical methods have succeeded in creating software that will allow mathematicians to use them as tools in solving these initial value problems as well as graphing the solution curves. Mathematica and Maple are two packages widely used nationally and internationally to find computer aided solution that can be obtained in seconds. We will discuss in this paper by use of examples of initial value problems how to graph the solutions of initial value problems using “phaseportrait” command in Maple.

When we have the initial value problem that contains a single differential equation or a system of differential equations we will assign a name to the equation using the assignment operator, colon followed by equal sign. If it is a system of equations that contain two or more equations the equations are entered into a computer that is separated by commas. Entering the equation(s) against the assigned name of the equation becomes the first line of Maple code that ends with a semicolon that shows the end of the statement. For instance “ode1” can be assigned as a name to denote the differential equation. The second line of Maple code will contain the initial conditions separated by commas entered against a pre-assigned name for the initial conditions. For instance “IC” can be assigned as a name to denote the initial conditions. The final Maple code will contain the “phaseportrait” command that will contain both the differential equation and the initial conditions. We will now illustrate seven examples and their solution curves by the use of Maple.

Example1: Use Maple to graph the computer generated pictures for the solutions to the four initial value problems given below. Draw all four graphs in the same window.

$$\frac{dy}{dx} = x^2 - y^2$$

(a) $y(-2) = 1$,

(b) $y(3) = 0$,

(c) $y(0) = 2$

(d) $y(0) = 0$

Example2:Use Maple to graph the computer generated pictures for the following three initial value problems given as a system of two differential equations with initial conditions specified at $t = 0$. Draw all three graphs in the same window.

$$\frac{dx}{dt} = y, \frac{dy}{dt} = -x$$

(a) $x(0) = 0, y(0) = 2$

(b) $x(0) = 0, y(0) = 1$

(c) $x(0) = 0, y(0) = 1.5$

Conclusion:

The graph of the solution of an initial value problem takes quite amount of time when analytical methods are employed while the graphing by using Maple will take only a few seconds. We can really appreciate the software that is used as a tool in graphing solutions that involves a differential equation with some prior known initial conditions and we cannot employ analytical methods to find or graph the solution.

References:

(1) A First Course in Differential Equations with Modeling Applications, Dennis G.Zill, Ninth Edition, Brroks/Cole, Cengage Learning.

(2) Soma Velummylum, *"The Use of Maple and Laplace Transforms in Solving Initial Value Problems,"* ICTCM, Electronic Proceedings (2009), C008-ISBN: 0-321-68983-6

(3) Soma Velummylum, *"Exploring Solutions of a Dynamical System with Maple",* ICTCM Electronic Proceedings (2007) -Vol-19-C018, pp227-231