ARDUINO IN THE CLASSROOM:
CLASSROOM READY MODULES FOR UNDERGRADUATE MATHEMATICS

Michael D. Seminelli [1]
Department of Mathematical Sciences
United States Military Academy
West Point, NY 10996
Michael.Seminelli@usma.edu

INTRODUCTION

We are living in a time were technology has become an integral part of our daily lives. Our students have never known life without the internet and likely grew up with computers at school and at least one computer at home. As educators we have the opportunity to bring technology into the classroom, not as a gimmick, but in a way that can help enhance our instruction of mathematics.

Every cadet at the United States Military Academy (USMA) is required to take three courses as part of the core mathematics curriculum, totaling 12 credits hours. MA103 is the first math course every freshmen takes which emphasizes applied mathematics through mathematical modeling [1]. Students are introduced to calculus through continuous and discrete mathematics while developing effective strategies to solve complex and often illdefined problems. The course exercises a

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[1]The views expressed in this article are those of the authors and do not reflect the official policy or position of the U.S. Military Academy, the Department of the Army, the Department of Defense, or the U.S. Government.
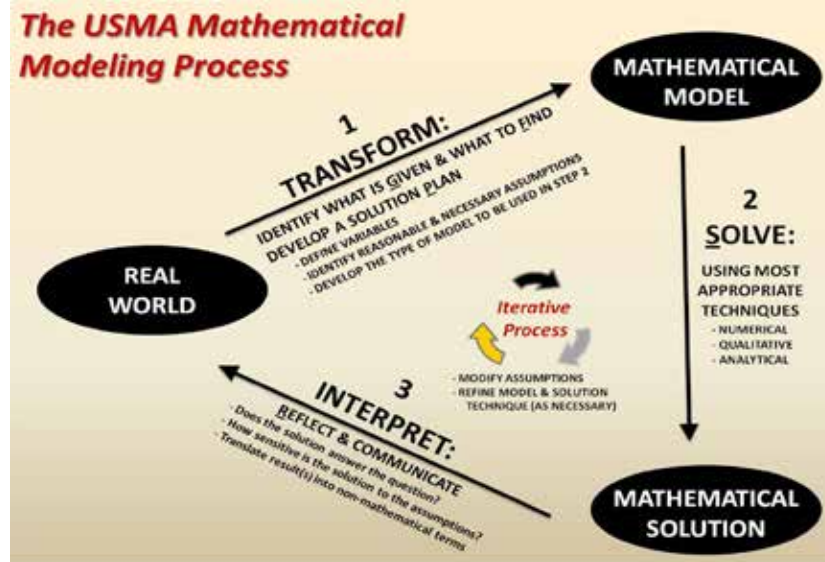
Figure 1: USMA Mathematical Modeling Triangle provides students a strategy to solve complex or incomplete problems

wide array of mathematical concepts while nurturing creativity, critical thinking, and learning through activities performed in disciplinary and interdisciplinary settings.

MA103 teaches students the problem solving process by using the "USMA Mathematical Modeling Triangle" in Figure 1. This flow chart provides students with a framework to transform the givens and unknowns of the problem statement into math, solve the problem, and finally analyze and interpret the results. Instructors also highlight the iterative approach to this problem solving process.

The second course in the core mathematics curriculum is MA104 which is single variable calculus. Students learn about differentiation, integration, and vector calculus concepts. Students apply their understanding of derivatives and integrals to solve applications based problems with the use of computational software. The students continue to use the math modeling process introduced in MA103.

MA206 Probability and Statistics is the final course in the core mathematics sequence. Students continue apply their understanding of integration as they explore probability, confidence intervals, hypothesis testing, and experimental design. Cadets continue to develop their programming skills as they use computational software packages for problem solving. Although the core mathematics program is only three courses, students majoring in STEM fields may also take multi-variable calculus, engineering mathematics, and applied statistics.

Figure 2: Arduino Starter kit is a great tool to introduce students to programming and robotics.

ARDUINO ROBOTS

Programmable robotics kits are growing in popularity and becoming increasingly more affordable. These systems range in construction and programming complexity to engage students from elementary school to college. The Lego Mindstorms system uses a control mechanism and Lego pieces to construct the robot. Students learn the essence of programming by assembling a series of command modules in the sequence the robot will execute them [2]. This approach to computer programming builds students' understanding of functions and the sets the foundation for more advanced coding.

Arduino is an open-source electronics platform based on easy-to-use hardware and software [3]. The boards read inputs from sensors to trigger an output action or response - such as illuminate an LED, power a servo motor, or display data to a screen. Arduino has created an online community where users have posted tutorials and project descriptions to inspire the creativity in others. The Arduino Starter Kit in Figure 2 contains all of the tools needed to build and program a robot, all for around $100. The Arduino processing boards use the Arduino programming language which is based on the open-source language *Wiring*, used as a cross platform framework for microcontrollers [4]. The Arduino programming interface enables users to program content and then upload the code via USB cable to the Arduino board, allowing it to operate without the computer.

We selected the Boe-Bot Robot Kit from Parallax which utilizes the Arduino Uno processing board for use in our classrooms, Figure 3. These robots are more advanced to construct and program as well as easily interface with a robust array of sensors. Each robot kit used is available for less than $200 and are sold from many online vendors [5]. Although the programming the robots requires an understanding of computer coding, the kits have a well developed manual and online tutorials that walk students through the construction of simple robot processes like making an
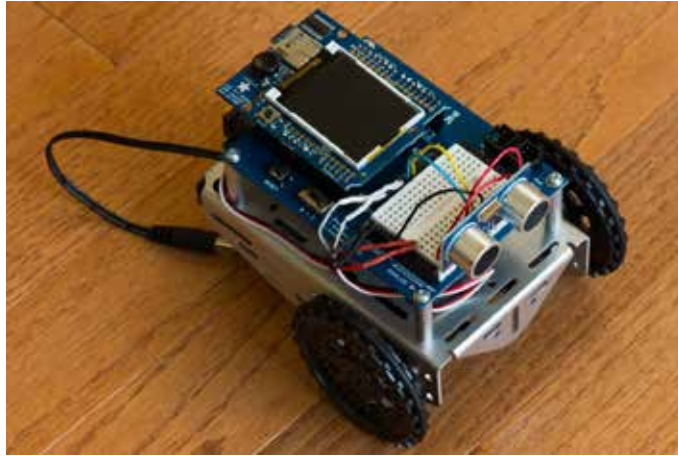
Figure 3: Boe-Bot Robot Kit with Arduino Uno board, ultrasonic range finder, and LCD screen.

LED blink, up to obstacle avoidance using an ultrasonic range finder [6].

ROBOTS IN THE CLASSROOM

The Arduino robots were used to help teach students pre-calculus and calculus topics by generating data for hands on experiments. Students were involved in robot assembly by attaching and wiring the sensor to their robots. However, the robots used were programmed by the instructor in advance based on the students' limited experience with coding and the short 55 minute class period. If time allows we recommend dedicating a lab hour or multiple lessons to robotics which allows students the opportunity to program their own robots. This engages student creativity and problem solving skills as well as enhances the interdisciplinary aspect of the exercise.

The following sections serve as a collection of lesson plans designed to demonstrate two calculus topics using robotics. The lessons are designed to be completed in 55 minutes with a class of twenty students.

EXERCISE I: Average and Instantaneous Rates of Change (Calculus I)

Lesson Objectives:

1. Explain the geometric interpretation of average and instantaneous rates of change.

2. Explain why the slope of the tangent line is the limit of the slopes of the secant lines.

3. Compute average rates of change (slopes of secant lines) and approximate instantaneous rates of change.

4. Understand the definition of velocity.

Assignment:

- **READ:** Section 2.1 in *Stewart Early Transcendentals 8E*

- **DO:** *Stewart Early Transcendentals 8E*: Section 2.1, Exercises 1, 2, 5, 7

Think About:

1. What is the relationship between secant lines and average rate of change?

2. What is the relationship between tangent lines and instantaneous rate of change?

Materials Needed:

- Arduino Robots - Programs loaded and acoustic sensor removed (Enough for groups of 2-3 students)

Activity 1: Introduction/ History of Robotics

- Discuss the early development of robots.

- Discuss the "Three Laws of Robotics" from Issac Asimov's *I, Robot* novels.

- Discuss modern applications of robotics (self-driving cars, drones for filming, manufacturing).

Activity 2: Robot Construction

- Students start by attaching the acoustic sensor to the bread board.

- Students then follow instructions (printed/ slide) to connect the four wires from the bread board (sensor) to the designated ports. Finally, Attach the power supply cable.

Activity 3: Data Collection

- Students place the robot perpendicular to a fixed vertical surface (ie. wall or door)with acoustic sensor facing the surface. Students then turn on the robot (using switch) and robot will drive way from the wall, reporting it's distance from the wall every 2 seconds.

- The experiment ends when the robot starts reporting unrealistic distance values (sensor no longer parallel to wall or effective range of sensor is exceeded). Students record the output from the LCD screen and repeat the experiment at least two more times, Table 3.

Activity 4: Analysis

- Students plot the data from the three trials using a spreadsheet or analysis program and build a continuous model to represent the position function for

the robot's movement, Figure 7. Students can differentiate the model to predict the robot's speed at a teacher specified time.

Pseudo Code for Exercise

```
 1: procedure RATEOFCHANGEEXPERIMENT
 2:   Initialize:
 3:       eventTime ← clock for the experiment
 4:       distance ← distance from the start point
 5:       velocity ← robot speed
 6:       incriment ← used to increase robot speed
 7:       delay ← time between distance readings
 8:   Loop:
 9:       loop conduct experiment repeat
10:           eventTime = getTime()
11:           print ('Time: ', eventTime, ' ', distance, 'cm')
12:           velocity = velocity + incriment
13:           vehicleForward (velocity)
14:           distance = getDistance()
15:           pause delay
16:       end loop
```

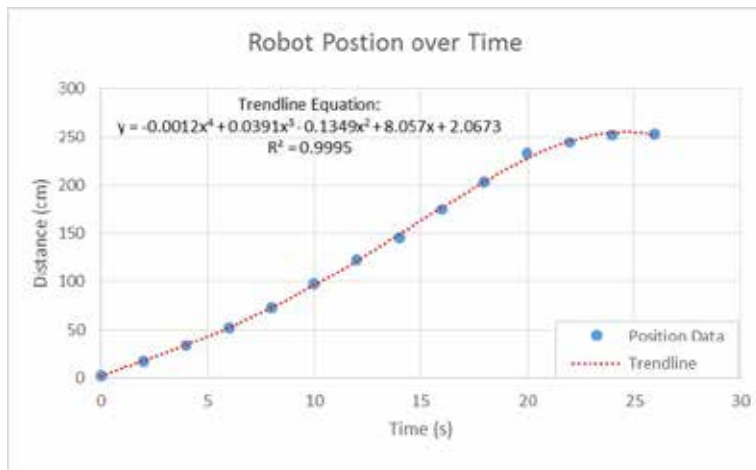| Time(s)	Data(cm)
0	2.76
2	16.84
4	33.86
6	52.61
8	73.32
10	97.87
12	122.69
14	145.95
16	174.98
18	203.2
20	232.61
22	244.92
24	252.05
26	253.21

Table 1: Experimental Data

Figure 4: Plot of Robot Position from Experimental Data

EXERCISE II: Related Rates (Calculus I)

Lesson Objectives:

1. Model and solve problems involving related quantities that change over time.

2. Implement the USMA Modeling Process for Related Rates Problems, Table 2.

Assignment:

Phase	Step	Descriptive Title
Transform	1	Read and Understand the Problem
	2	Draw a Picture/Diagram
	3	Define all Variables
	4	Define all Derivatives
	5	Define the Relationship Equation
Solve	6	Differentiate with Respect to Time
Interpret	7	Substitute and Answer the Question

Table 2: USMA Modeling Process for Related Rates Problems

- **READ:** Section 3.9 in *Stewart Early Transcendentals 8E*

- **DO:** *Stewart Early Transcendentals 8E*: Section 3.9, Exercises 5, 13, 14, 15, 22, 29, 33, 37

Think About:

1. How can a good diagram or picture assist in setting up a problem that relates rates of change?

2. How can dimensional analysis assist in setting up a related rates problem?

Materials Needed:

- Two Arduino Robots - Programs loaded and acoustic sensor removed. (Enough for groups of 4-5)

Activity 1: Robot Construction

- Students start by attaching the acoustic sensor to the bread board.

- Students then follow instructions (printed/ slide) to connect the four wires from the bread board (sensor) to the designated ports. Finally, Attach the power supply cable.

Activity 2: Modeling Process

- Introduce students to the seven step modeling process in Table 2.

- Highlight to students that the seven problem solving steps are nested within the USMA Math Modeling Process: Transform, Solve, Interpret. In steps 1-5, students transform the word problem into math notation. Then, in step 6 they solve by differentiating the relationship equation. Finally, students answer the question in step 7 and interpret the results to ensure it makes sense in the context of the problem.

Problem Scenario:

Robot A is initially located 300 cm west of Robot B. At the same time, Robot A travels east at a constant speed for 15 seconds while Robot B travels north at a constant speed for 15 seconds. How fast is the distance between the robots changing at the instant before they stop?

Activity 3: Robot Demonstration

- Start with the robots 300 cm apart with Robot A's direction of travel perpendicular to Robot B's direction of travel, as in Figure 5. Turn on the robots and run the program. Both robots will drive straight for 15 seconds and stop. The robots should be configured in a right triangle with the start point of Robot B. Students will

- Students take position measurements of each robot relative to the start point and then the distance between them. Students will use the data to compare to their theoretical answers.
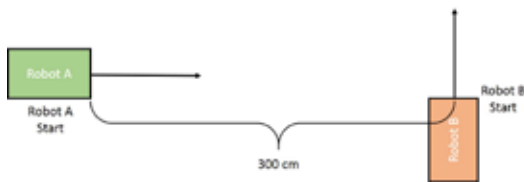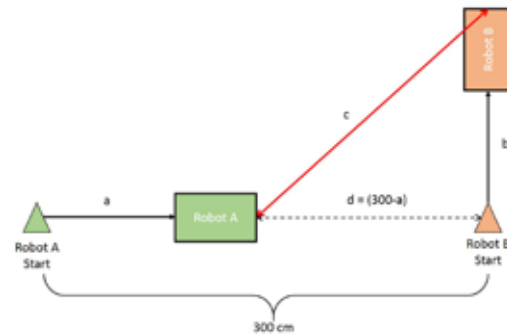

Figure 5: Initial Positioning of Robots

Figure 6: Step 2 Diagram

Activity 4: Problem Solving

1. Read Problem.

2. Draw a picture of the scenario, Figure 6.

3. Define variables.

   - $a=$ distance traveled by Robot A

   - $b=$ distance traveled by Robot B

   - $c=$ distance between Robot A and Robot B

   - $d=$ distance from Robot A to Robot B's start point (300 cm - a)

   - $t=$ time robots are traveling

4. Define derivatives.

   - $\frac{da}{dt}=$ speed of Robot A; $\frac{db}{dt}=$ speed of Robot B

- $\frac{dc}{dt}$ = rate at which distance between Robot A and Robot B changes

5. Define the Relationship Equation. Pythagorean Theorem.

   - $c^2 = (300 - a)^2 + b^2$

6. Differentiate the Relationship Equation.

   - $2c\frac{dc}{dt} = -2(300 - a)\frac{da}{dt} + 2b\frac{db}{dt}$

7. Substitute and Solve.

   - $Rate = \frac{distance}{time}; \frac{da}{dt} = \frac{a}{t}; \frac{db}{dt} = \frac{b}{t}$

   - $c = \sqrt{(300 - a)^2 + b^2}$

   - $\frac{dc}{dt} = \frac{(a-300)\frac{a}{t} + b\frac{b}{t}}{\sqrt{(300-a)^2 + b^2}}$

Additional Activities: Speed Estimation/ Model Development

- In this lesson students assume each robot has a constant acceleration and use the relationship that $Rate = \frac{distance}{time}$. However, each robot is programmed to use the acoustic sensor to determine its position and display the distance traveled each second. Students can use this data and develop a continuous model that represents each robot's position as a function of time, see Table 3. Instructors can tie in concepts discussed in during earlier lessons on instantaneous rates of change and differentiation.

- Students can also use the robot data to fit a continuous linear or power model using a *MS Excel* or similar software package. A power model in the form $f(x) = a(x - c)^b + d$, with $b = 1$ was used to represent the movement of the robots, see Table 4.

- The initial model is created by estimating or solving for the parameters using data points and systems of equations. Students analyze the model's accuracy using r^2 or Sum of Squared Error then adjust the parameters to achieve a better fit to the data. Table 4 includes the Sum of Squared Error (SSE), Sum of Squared Totals (SST), and r^2 values for each model after optimizing with *MS Excel Solver*. Each model achieved an r^2 value of 0.99 which indicates a well fitting model.

- The plot in Figure 7 depicts the experimental data and velocity model for each robot. Students can utilize this plot to assess and confirm the accuracy of each robot's power model through visual inspection, "eyeball test." Students can also interpret the plot and identify robot velocity as a linear function and therefore conclude that each robot had a constant velocity.
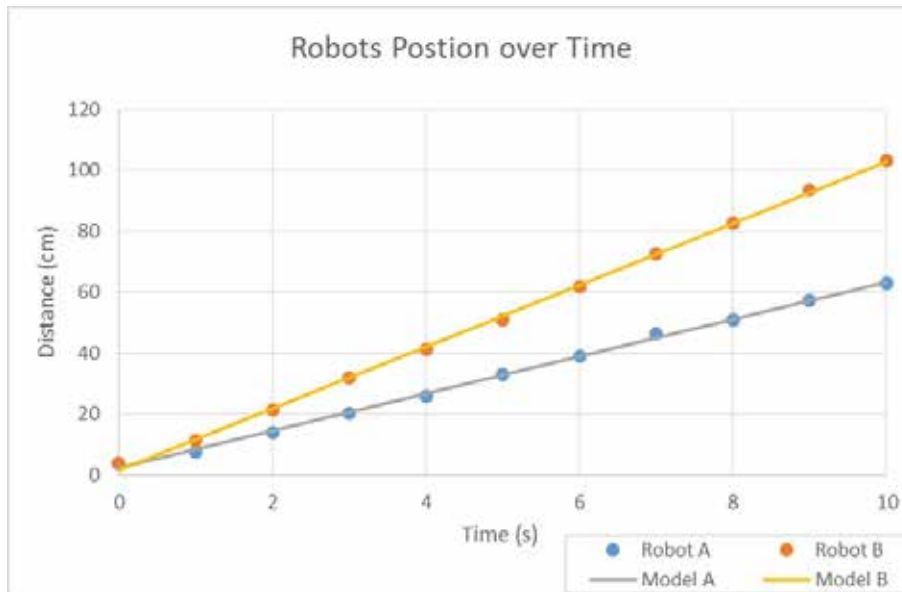
Figure 7: Robots A and B Position Data and Corresponding Velocity Models

Time (s)	Robot A (cm)	Model A (cm)	Robot B (cm)	Model B (cm)
0	3.7	2.4	3.8	1.6
1	7.8	8.5	11.2	11.8
2	13.9	14.6	21.5	21.9
3	20.5	20.7	31.8	32.0
4	26.1	26.8	41.2	42.1
5	33.2	32.9	51.0	52.2
6	38.8	38.9	61.7	62.3
7	46.4	45.0	72.6	72.4
8	50.8	51.1	82.7	82.5
9	57.3	57.2	93.4	92.7
10	63.0	63.3	103.2	102.8

	Model A	Model B
a	6.1	10.1
b	1	1
c	0	2.9
d	2.4	31.3
SSE	5.4	8.3
SST	4082.7	11250.1
r^2	0.99	0.99

Table 3: Experimental Data and Fitted Power Models

Table 4: Velocity Model Parameters

Pseudo Code for Exercise

```
1:  procedure RELATEDRATEEXPERIMENT
2:  Initialize:
3:      eventTime ← clock for the experiment
4:      distance ← distance from the start point
5:      velocity ← robot speed
6:      runTime ← time that robot drives from start point
7:  Loop:
8:      while eventTime <= runTime do
9:          eventTime = getTime()
10:         print ('Time: ', eventTime, ' ', distance, 'cm')
11:         vehicleForward (velocity)
12:         distance = getDistance()
13:     end while
14: Stop Exercise:
15:     if eventTime > runTime then
16:         vehicleForward(0)
17:     end if
```

CONCLUSION

Using robotics in the classroom helps to enhance student understanding of keystone calculus topics. These exercises take to seemingly intangible concepts from the students homework and allow them to fully experience the math. When students came to class they were excited to build the robots and then use them to collect data. Hands on experiments also allow students to engage their creativity and critical thinking skills, especially when they obtain unexpected data points. Educators can leverage these "teachable" moments to expand student learning beyond the calculus lesson. The affordability of programmable robotics kits drastically increases the feasibility of conducting class wide instruction for both students and educators. Incorporating innovative technology into the classroom will help inspire students and bring your calculus lessons to life.

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