LEARNER-CENTERED MATH TECHNOLOGY

Monika Kiss

Saint Leo University

33701 State Road 52, St Leo, FL 33574

Monika.kiss@saintleo.edu

Today, more than ever, student-centered technology for learning is the driving force behind how we are teaching mathematics. As mathematics educators, we are bombarded by the number of publishers who are trying to sell us their new eBooks which come with, of course, a type of learning management system such as MyMathlab, or WebAssign (to name only two). These technology based math resources can help teachers develop innovative and effective tools for teaching math, as well as "provide students with ample opportunity to explore mathematical concepts through graphical illustrations and hands-on exercises" [4].

When used correctly, this technology can decrease withdrawal rates and increase the number of students who pass the course, both of which are very positive outcomes [5]. We, the educators, are able to delegate some of the work we do, such as grading or other types of assessment, to technology. Students benefit by having access to a large number of online educational resources, such as "work out this example" or "show me how" applications that some of the programs offer. In addition, nobody can deny the fact that having instantaneous feedback is wonderful for students. It allows students to correct their misconceptions right away, using a non-judgmental interface with instantaneous feedback; the teacher 's time can then be spent helping students who need additional guidance and academic support. We can promote meaningful learning by assisting students to use the technology as a support, rather than a replacement, for real understanding. However, when students circumvent the learning process by using technology to achieve the grade wanted rather than the knowledge needed, then the design of the learning process and its software has impeded student understanding.

That is the dark side to this technology. We lose some student understanding in this way. There are some students who, for one reason or another, take advantage of the system. This may be related to their self-efficacy and confidence concerning math, or it may be simply a means to an end. It is my position that if we as educators provided additional opportunities for practice of the concepts, and had a better understanding of why some students choose to try to circumvent the system, we could develop meaningful ways of addressing the problem, one of which is discussed in this paper. If, for example, it was determined that the students who most frequently try to obtain the correct answer by cycling through the problems did so because their confidence in their own abilities was low, we could work to increase their confidence and develop a stronger sense of self-efficacy by providing additional opportunities for practice and understanding.

According to Bandura [1], self-efficacy is one's belief in his or her ability to execute a particular task or behavior; many students believe they "are not good at math." As you already know, math problems consist of a set of tasks, which one must complete by following a set of rules.

The problem as I see it is that students quickly realize that if they want to retry a problem within the program, there is, in most cases, a limited number of different questions of the same type. So, if they are just patient enough, sooner or later, they will have the problem they started with and so they will have access to the correct answer. (Unfortunately, in most cases, there are only a limited number of computer generated problems of the same type.) Is that clever? Yes, of course it is. The students are learning a skill, but that is not the point of the lesson. We want them to learn the math concept at hand and not how to use the system to increase their grade.

So, how can we use technology to achieve our goal of supporting student understanding and perhaps even inspiring students to go beyond rote memorization to learn how to see beyond the basic concepts and synthesize the information? Can we hope for such understanding? We can! We can use the technology available to provide additional practice opportunities. We can also teach not just mathematics but also some basic programming to students for deeper understanding, for critical thinking and in general for more creative thinking and problem solving by using a Computational Algebra System (CAS) setup and supported by the teacher. A CAS is "capable of working symbolically as well as numerically. It does on a computer the manipulation that has traditionally been done with pencil and paper. For instance, it immediately processes computational tasks such as obtaining derivatives, integrals, or graphs of one and several variable functions" [2].

Our claim is simple. Students must do the math to learn the math, rather than rely on locating the correct answer in a learning system or online. Without practice there is no real understanding; and while the short-term goal may be to pass the class, the long-term goal should be to develop a real understanding of math. However, in order to get students to want to understand the content in a deeper manner, we need to help students believe that using a particular system will enhance their performance in the class, and increase their math confidence. How? Practice! We propose that by introducing interactive worksheets we can bring students to not only learn mathematical concepts but also increase their confidence. The technology tool which I am proposing is called *Mathematica*. As many of you are aware this tool is expensive and thus many schools may not have access to this tool. Fortunately, there is at least one other alternative tool that you can use to adapt this idea. The alternative software is the open source tool called Sage [2]. By teaching students how to program in *Mathematica* as well as similar CAS and possibly create generalizations, "students taught in a CAS environment achieve at least as well as those taught in traditional classes on measures of pen and paper skills at the first year level, and in many cases achieve a higher level of conceptual understanding" [3].

How does it work? An interactive worksheet is simply a worksheet the instructor creates in *Mathematica*. In the classroom we will traditionally introduce concepts, define terms and create opportunities for students to apply formulas to different problems by using worksheets. An interactive worksheet is simply one where you have all the definitions, and examples in a Mathematica worksheet. The students will go through examples and definitions, just as in any other worksheet, but in this worksheet they will also be introduced to small programs created in Mathematica. These small programs, created by the faculty, will be where mathematical concepts will be demonstrated.

For example, one of our favorite problems in College Algebra is the study of transformations. It is an incredibly visual concept. Students can easily discover for themselves the three types of transformations using *Mathematica*. In *Mathematica*, one can create a simple program that will demonstrate vertical shift for the student. They can discover what happens using the Manipulate function. By creating a simple program to discover the vertical shift, the students can then be guided to adopt and modify the program that they already were given to the other types of transformations and ultimately even to discover the general rule on what should happen in the event that there are multiple transformations.

First, the instructor would type up some preliminary information to set the tone of the lesson.

Transformation Worksheet.nb * - Wolfram Mathematica 10.3

Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Transformations - An Introduction

Now that you have a library of elementary functions, we want ot learn how to construct new functions from these using geometry, called transformations.

There are three types of transformations we will discuss;
- Shifts,
- Reflections and
- Stretching or shrinking.

Let's investigate these tranformations and let's understand what happens when we apply these transformations.

Figure 1. Statement of the problem

Then the instructor would provide an example to represent a vertical shift that the students can interact with, such as the following example:

Example 1: We will take the graph of f(x) = sin x and we will want to study what happens to this graph when we add a number to the function on the outside of the function. That is, we would like to know how the algebra of adding a constant on the outside of the function affects the graph of the function. So, let's consider what the graphs of f(x) = sin x + C look like, where we will let C range over the numbers -2 to 2. To do this, we need to learn some basic programming commands as well!

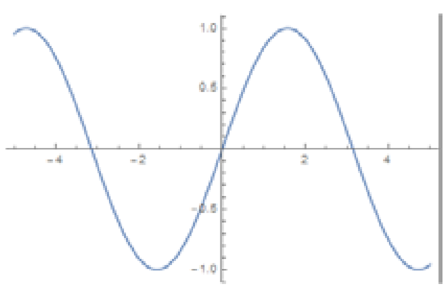
So, what do we need to learn to do this? Let's take it one step at a time!

First, we need to learn how to ask *Mathematica* to graph a function. the syntax of graphing is simple:

Plot[Sin[x], {x, -5, 5}]

Once you type this command into the *Mathematica* worksheet, you will have to press shift enter.
This command will graph f(x) = sin x on the interval [-5, 5].
Please note and take good care on using the [ ] as well as the upper case letters
that are used in *Mathematica* for pre-defined functions, such as Plot and Sin. Let's look at it together!

```
Plot[Sin[x], {x, -5, 5}]
```



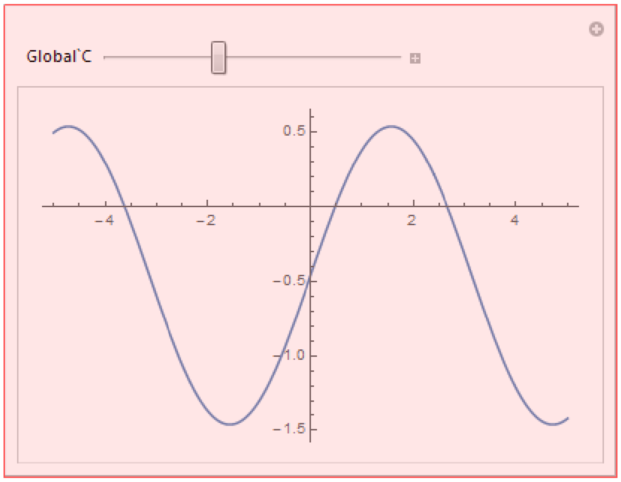
Next we want to learn how to add a constant C to the functions f(x) = sin x and allow that constant to change.
To do this we learn about a function that allows you to interact with your graph. This function is called Manipulate.
let's see how this works.

The syntax for this function to add C to the function has the following form

Manipulate[Plot[Sin[x] + C, {x, -5, 5}] , {C, -2, 2}]

Once you type this command into the *Mathematica* worksheet, you will have to press shift enter.
This command will create an interactive box with a sliding bar to change the value of C for the graph of
f(x) = sin x + C on the interval [-5, 5]. Let's see what this looks like.

```
Manipulate[Plot[Sin[x] + C, {x, -5, 5}], {C, -2, 2}]
```



Now that we completed this step, talk to your neighbors and discuss what you observe!

Figure 2. The introduction to programming.

As you can see, the students can learn about the process of using Mathematica to graph, which is a wonderful skill for students to have as they venture into College Algebra and beyond. In addition, they are encouraged to have discussions with their classmates to come up with their own conclusions and discover for themselves about this type of transformation.

At this point, you can encourage students different ways. One option would be to have students investigate the plot and manipulate functions of Mathematica to graph other types of functions and their vertical transformations. Or you can encourage them to determine what would happen if they were to graph f(x) = sin (x + C), that is to learn about horizontal transformations. Some other exercises you can incorporate in your lesson would include the students to explore the other types of transformations, namely reflections and stretching or shrinking. The opportunities for ways to have the students learn more about the functionality of Mathematica as well as the beauty of transformations is only as limited as the time you may wish to spend on this activity in your courses. Students will gain greater understanding of mathematics as well as programming.

I hope that after reading this paper you will be inspired to introduce these interactive worksheets into your classes. We need more students who can be great problem solvers of mathematics as well as creative programmers of the future.

References

[1] Bandura, A. (1986). *Social functions of thought and action: A social cognitive theory.* Upper Saddle River, NJ: Prentice-Hall.

[2] Botana, F., Abánades, M. A., & Escribano, J. (2014). Using a free open source software to teach mathematics. *Computer Applications In Engineering Education*, *22*(4), 728-735. doi:10.1002/cae.21565

[3] Langtry, T., Coupland, M., & Moore, B. (2003). Mathematica™ in context. *International Journal Of Mathematical Education In Science & Technology*, *34*(5), 699. doi:10.1080/0020739031000148967

[4] Smith, C. J., Tefera, A., & Zeleke, A. (2012). Computerized proof techniques for undergraduates. *International Journal Of Mathematical Education In Science & Technology*, *43*(8), 1067-1077. doi:10.1080/0020739X.2011.644335

[5] STEWART, P. (2012). Closing the Math Achievement Gap. *Diverse: Issues In Higher Education*, *29*(3), 13.