LOOKING FOR MATH, REDUX

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Introduction and motivation:

As mathematically minded people, it is normal for us to identify and observe the mathematics that we encounter in our daily lives. It is second nature to us to look, share, and discuss the mathematics that surrounds us. So much so that in 2007, the Mathematical Association of America (MAA) created the feature "Found Math" on their website as a space to share mathematical images. Each week, the MAA website highlights a new photo of math found and submitted by MAA members. (For more information or to submit your own photo for the "Found Math" series, please visit http://www.maa.org/community/columns/maa-found-math.) Observation and connections that we routinely make are often not made by our students, even when they are taught in a real-life environment. Whether we "see" the mathematics or not, depends, in part, on whether or not we are looking for it. Mathematics permeates our lives. Whether it's the geometry of architectural buildings or the statistics and/or functional relationships in a news article, mathematics surrounds us. This mathematics content goes unnoticed by the untrained eye. If we don't know to look for something, it's much harder to find. This paper will focus on the use of online discussion boards in motivating students to write mathematically and in fostering their tendency to see mathematics in their daily lives. Adaptation variations, assessment, insights, pitfalls, questions, and new inspirations will be included.

Using the discussion boards in Blackboard as a medium for students to post their real-life mathematical observations is an old inspiration. It all started in the early 2000s when our campus started using Blackboard as a course management system. I took to the course management aspect as an organizational tool and was intrigued by the opportunity to foster mathematical communication and dialogue in the time-delayed environment created by the discussion boards. I envisioned them as an organic space to transcend class time and continue the conversation outside of the classroom. The assignment originated in a math content course for prospective elementary teachers. As a means to provide context and as a means to help students develop mathematical connections, the course content was largely driven by real-life contexts. I routinely made observations in my daily life that were used in class as motivating activities or authentic assessment questions. It occurred to me that this is what I wanted my students to do. It also became apparent that this was not happening like I wanted for. Despite the fact that the course material was largely driven by real life phenomena, students weren't making these observations themselves. They weren't seeing the mathematics that surrounded them. It wasn't clear they knew to look for it. Maybe they just didn't know they could or why they should. I wanted my students to be able to recognize and articulate the mathematics in their lives, and to share those observations with their peers. This is the impetus for the original assignment, in which students post their real life observations in the discussion boards supported by Blackboard.

The Assignment:

Students used the discussion boards in Blackboard to post no fewer than ten incidences of mathematics found in the "real world". They were encouraged (but not required) to respond to other students' posts. A three category rubric (high, medium, and low) was used to assess student postings. Student comments were assessed on the quantity of submissions and the quality of a variety of factors including: mathematical content, mathematical language, clear and thorough explanations, posting regularity, spelling and grammar, and (when applicable) appropriate citations. It was purposely an open ended assignment. Observations could come from other courses or non-academic contexts. Students posted observations from a wide variety of real-life contexts in which they found math concepts studied in class. They responded to one another's observations and, at times, extend or generalize the original comment. Some students posed their own questions to which their classmates responded. Students learned from one another as well as the instructor. Having students share their mathematical observations created a student-centered authentic writing assessment. Students experienced mathematics in context, as suggested by the NCTM *Principles and Standards for School Mathematics* (2000).

My typical modus operandi was to align discussion board forum topics with course topics. I liked the added opportunity to assess whether students had posted in a relevant forum. In the prospective elementary education course, the mathematical content included functions (linear, quadratic, and exponential families), statistics, probability and counting, and the geometry of transformations and measurement. A course focusing on differential calculus might have forum topics that include function families (for example: power, exponential, trigonometric, logarithmic, and general functions) the derivative, and the definite integral. One variation was in a Calculus concepts course for future elementary teachers specializing in an integrated Math/Science/Technology major. In MSTI 314, discussion board forum themes were based on real-life contexts. The forums included topics like "student life math", "amusement park math", "mathematics in sports", and "math in literature". Additionally, the content based forums "conjunction, junction; what's your function" and "Rate of change ~ the Derivative" were created to see if students would find functional relationships or ways to think about "rate of change" in their lives. An example in which a student started a mathematical conversation combining "math in literature", "amusement park math", and the function concept can be found in Appendix A.

Student feedback is collected at the end of the semester in which students discuss which course components they found the most (and least) valuable to their learning. Suggestions for improvement are also solicited. Over time, student feedback for "looking for math" has pretty universally been favorable. The open-ended nature of the assignment can stifle the start of the electronic conversation, as students can be hesitant to take that initial plunge. The fact that there are many "correct answers" can be daunting, especially if students think that they don't know what is required. Once students' concern about posting a "wrong answer" is assuaged, submissions and dialogue can really get going. It's not unusual that one student's idea inspires other students to remark. Space prohibits more than just a sample of student feedback, so a small collection of student remarks follows. Student comments presented below are verbatim and generally representative of student feedback.

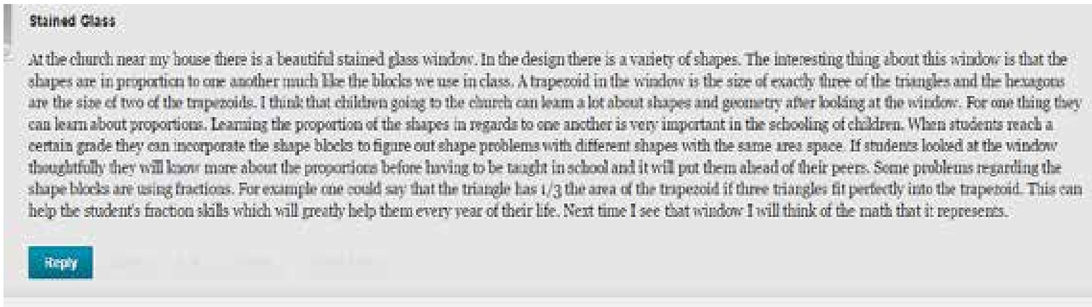
- Blackboard postings allowed me to publish thoughts while I was thinking about them and gave me an additional way to practice communicating mathematical concepts. There is a surprising amount of math I encounter through other studies and then, don't have a chance to share it, so this was a good outlet.
- This helped me to sort out my own ideas and to be able to put mathematical concepts into words. I also liked using Blackboard to post math ideas from our own lives or problems we have thought of. I think that this pushed us to look beyond math in the classroom and to see it within our world, which is very beneficial. The only hard part was the first Blackboard posting. I think that people were confused as to what we were supposed to write. (I was at first, but it became easier to see math in my life.
- The looking for math in the real world postings was something unique to the course. I really enjoyed working on those. It made me realize how much math is really used in the real world.

Requiring students to "look for mathematics" quickly became a favored activity for the professor and students alike. The assignment was a perennial staple in the calculus sequence and math content courses for future elementary teachers. Even when class content and activities were not motivated as frequently by real-life contexts, students who wrote about the mathematics they found demonstrated an appreciation for the "real-life" nature of mathematics. When we communicate our ideas or justify our reasoning, we gain insight into our own thinking. Students practice their communication skills by writing about real-world mathematics, thus promoting active learning and writing across the curriculum. In an asynchronous environment, each student has the same opportunity to contribute without the real-time pressure of a face-to-face classroom discussion. Learners are not restricted to the boundaries of class, as they are able to access and submit messages at any time and from anywhere. Using online discussion boards to discuss mathematics gives students the opportunity to practice expressing their ideas and to see when they are understood and sufficiently convincing. This can help students better understand what constitutes evidence, as endorsed by the NCTM (2000) and Common Core Standards for Mathematical Practice (CGA Center & CCSSO, 2010). The timed-delayed nature of the discussion boards means that students can organize their ideas before publishing their remarks. The additional reflection can serve to improve their writing and the quality of their mathematical reasoning and critical thinking. This is suggested by Newman, et al (1995) who indicated a relationship between critical thinking, social interaction, and deep learning. In their study, students in computer-based interactions demonstrated much deeper overall critical thinking measures than did students in the face-to-face environments.

Bringing "Looking For Math" Back:

Despite its popularity, over time the Looking for math assignment fell by the wayside. Recently, and in part, inspired by MAA Found Math series, I've been champing at the bit to bring it back. Mathematics, as the study of the underlying structure of the patterns used to describe real-world phenomena, occurs all around us. I want to help my students see this. In fall 2015 it was time to take the plunge again in a mathematics content course for future elementary teachers. Typically discussion board forums were aligned with course content. This time I learned my lesson from the MSTI 314 class who readily took to the discussion board forums with real-life themes. Until I embraced this paradigm shift of using (at least some) "real-life" contexts for discussion board forums, I struggled to envision forum topics. Now I can't imagine excluding real-life forum topics. The course starts with a heavy emphasis on content involving bases other than ten and multicultural mathematics. I was concerned that students may not be able to find many examples of real-life phenomenon that exhibited non-base 10 characteristics. A natural blend of course topics and real-life contexts was the perfect direction for the inaugural return. Because of the organization of the course, I started with forum themes like "Non-base 10 and other numeration systems" and "Number and the base ten system", as well as "Geometry of Design and Art" and "Geometry of shape". Additional forums were added a few at a time, creating ongoing opportunities to revisit the assignment with the class, address student questions (as needed) and encourage them to suggest forum topics and to post their real-life math. Even without explicit reminders, these discussions serve to tacitly remind students about the assignment. Discussion boards were created for real-life themes including "Math in literature", math in sports, math in puzzles, and math in music. The students even requested and published in the forum "working with children". Over time, discussion board forums were added for most of the topics in *Mathematics for Elementary Teachers* (4^{th} ed) by Sybilla Beckmann. Mathematical topics explicitly discussed in class tended to be more popular to post on than those not discussed, with a few notable exceptions. Some topics, like Geometry (both "Design" and "shape") for example, were popular directions for students to post about even though we didn't discuss these topics formally in class. Several illustrative examples of student work follow. One student found fractions in stained glass windows at a local church, as illustrated on the next page in Figure 1.

Figure 1, Mathematics in Stained Class

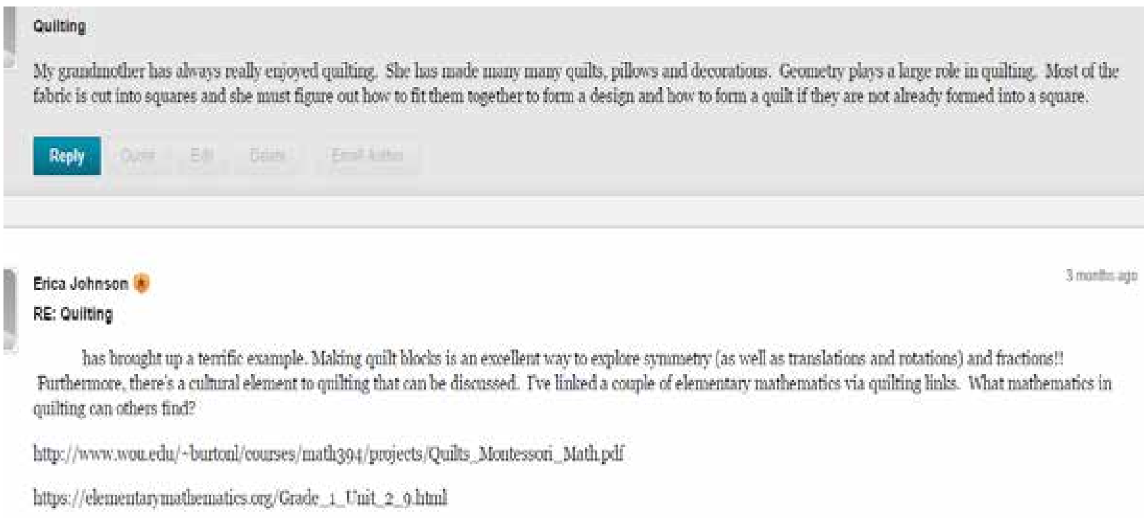

Stained Class

At the church near my house there is a beautiful stained glass window. In the design there is a variety of shapes. The interesting thing about this window is that the shapes are in proportion to one another much like the blocks we use in class. A trapezoid in the window is the size of exactly three of the triangles and the hexagons are the size of two of the trapezoids. I think that children going to the church can learn a lot about shapes and geometry after looking at the window. For one thing they can learn about proportions. Learning the proportion of the shapes in regards to one another is very important in the schooling of children. When students reach a certain grade they can incorporate the shape blocks to figure out shape problems with different shapes with the same area space. If students looked at the window thoughtfully they will know more about the proportions before having to be taught in school and it will put them ahead of their peers. Some problems regarding the shape blocks are using fractions. For example one could say that the triangle has 1/3 the area of the trapezoid if three triangles fit perfectly into the trapezoid. This can help the student's fraction skills which will greatly help them every year of their life. Next time I see that window I will think of the math that it represents.

Reply

While the mathematics in the following example is not especially deep, the observation itself is rife with potential. It invites questions about some interesting mathematics and provides an opportunity for guidance and direction. The instructor plays an important role in facilitating dialogue. Guiding discussion can be directed or it can be open-ended, or anywhere in between. In the following example, I started with very open-ended direction, in which students are exposed to a Montessori project containing deep mathematics for the young learner. This did not generate the response(s) I hoped for. Alternatively, a better approach may be a more directed guidance from the start or as a follow-up may be desired. There are many ways to foster student communication.

Figure 2, Geometry of Quilting


Quilting

My grandmother has always really enjoyed quilting. She has made many many quilts, pillows and decorations. Geometry plays a large role in quilting. Most of the fabric is cut into squares and she must figure out how to fit them together to form a design and how to form a quilt if they are not already formed into a square.

Reply

Erica Johnson

RE: Quilting

3 months ago

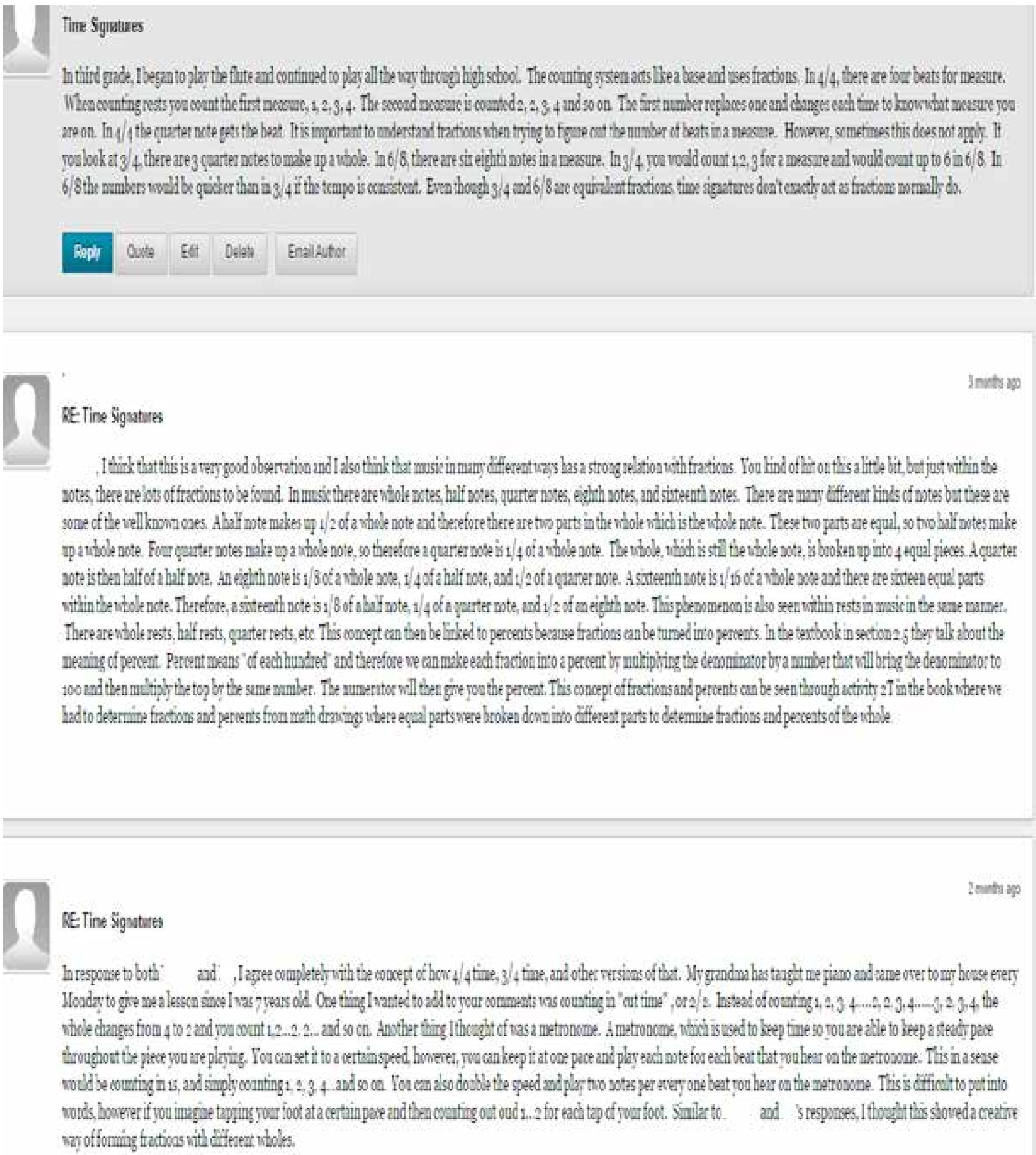
has brought up a terrific example. Making quilt blocks is an excellent way to explore symmetry (as well as translations and rotations) and fractions!! Furthermore, there's a cultural element to quilting that can be discussed. I've linked a couple of elementary mathematics via quilting links. What mathematics in quilting can others find?

http://www.wou.edu/~burtonl/courses/math394/projects/Quilts_Montessori_Math.pdf

https://elementarymathematics.org/Grade_1_Unit_2_9.html

In Figure 3 (on the next page) we see a student conversation on the mathematics of music. The first student starts the conversation by describing some of the mathematics found in music, including the counting system. This student then speaks to how it behaves like fractions and how it does not and wraps up with the topic of time signatures. The second student follows up with a thorough discussion focusing on fractions in music, paying careful attention to identifying the whole and using the meaning of fraction. The last student further adds to the conversation by noting how different "times" can be related to different wholes. While there are a few details worth pursing or clearing up, all in all, this is a lovely exchange between the participants. Had this final remark not be posted at the end of the term, the connection to the metronome might have been a promising direction to pursue and encourage further discussion. Given the preponderance of mathematics in music and how common it is for students to play an instrument, or at the very least, enjoy music, math in music can be a very desirable topic to include.

Figure 3, Mathematics in Music



Time Signatures

In third grade, I began to play the flute and continued to play all the way through high school. The counting system acts like a base and uses fractions. In 4/4, there are four beats for measure. When counting rests you count the first measure, 1, 2, 3, 4. The second measure is counted 2, 2, 3, 4 and so on. The first number replaces one and changes each time to know what measure you are on. In 4/4 the quarter note gets the beat. It is important to understand fractions when trying to figure out the number of beats in a measure. However, sometimes this does not apply. If you look at 3/4, there are 3 quarter notes to make up a whole. In 6/8, there are six eighth notes in a measure. In 3/4 you would count 1,2, 3 for a measure and would count up to 6 in 6/8. In 6/8 the numbers would be quicker than in 3/4 if the tempo is consistent. Even though 3/4 and 6/8 are equivalent fractions, time signatures don't exactly act as fractions normally do.

Reply Quote Edit Delete Email Author

3 months ago

RE: Time Signatures

, I think that this is a very good observation and I also think that music in many different ways has a strong relation with fractions. You kind of hit on this a little bit, but just within the notes, there are lots of fractions to be found. In music there are whole notes, half notes, quarter notes, eighth notes, and sixteenth notes. There are many different kinds of notes but these are some of the well known ones. A half note makes up 1/2 of a whole note and therefore there are two parts in the whole which is the whole note. These two parts are equal, so two half notes make up a whole note. Four quarter notes make up a whole note, so therefore a quarter note is 1/4 of a whole note. The whole, which is still the whole note, is broken up into 4 equal pieces. A quarter note is then half of a half note. An eighth note is 1/8 of a whole note, 1/4 of a half note, and 1/2 of a quarter note. A sixteenth note is 1/16 of a whole note and there are sixteen equal parts within the whole note. Therefore, a sixteenth note is 1/8 of a half note, 1/4 of a quarter note, and 1/2 of an eighth note. This phenomenon is also seen within rests in music in the same manner. There are whole rests, half rests, quarter rests, etc. This concept can then be linked to percents because fractions can be turned into percents. In the textbook in section 2.5 they talk about the meaning of percent. Percent means "of each hundred" and therefore we can make each fraction into a percent by multiplying the denominator by a number that will bring the denominator to 100 and then multiply the top by the same number. The numerator will then give you the percent. This concept of fractions and percents can be seen through activity 2T in the book where we had to determine fractions and percents from math drawings where equal parts were broken down into different parts to determine fractions and percents of the whole.

2 months ago

RE: Time Signatures

In response to both and , I agree completely with the concept of how 4/4 time, 3/4 time, and other versions of that. My grandma has taught me piano and came over to my house every Monday to give me a lesson since I was 7 years old. One thing I wanted to add to your comments was counting in "cut time", or 2/2. Instead of counting 1, 2, 3, 4....2, 2, 3, 4.....3, 2, 3, 4, the whole changes from 4 to 2 and you count 1,2...2, 2... and so on. Another thing I thought of was a metronome. A metronome, which is used to keep time so you are able to keep a steady pace throughout the piece you are playing. You can set it to a certain speed, however, you can keep it at one pace and play each note for each beat that you hear on the metronome. This in a sense would be counting in 1s, and simply counting 1, 2, 3, 4...and so on. You can also double the speed and play two notes per every one beat you hear on the metronome. This is difficult to put into words, however if you imagine tapping your foot at a certain pace and then counting out oud 1...2 for each tap of your foot. Similar to and 's responses, I thought this showed a creative way of forming fractions with different wholes.

Pleasant Surprises and Future Plans:

The mathematical content of student submissions were assessed (low, medium, or high) using a posting rubric that can be found in Appendix B. The rubric is an updated adaptation of rubric(s) used in previous iterations of this assignment. Through the term, students were given formative feedback, including possible suggestions to improve the student submission. Students were encouraged to submit more than the minimum of four if they wanted to compensate observations assessed as “low”. This resulted in students posting far more than the minimum requirement of four and may possibly be why students were not as hesitant to post as with groups who were required to post ten instances of real-life math. Students were encouraged to bounce their ideas off of me prior to posting and several did just that. Furthermore, requiring students to respond to a minimal number of their peers’ submissions ensured that students read other’s posts and created an online culture of responding to one another.

This group of students, more than previous groups, jumped right in to the assignment. One of my previous challenges was often getting the ball rolling. For the first time in an introductory course that was not an issue. It wasn’t that everyone jumped in immediately, but a critical mass engaged in the conversation early and often. Their observations were not always as mathematically detailed (or clearly communicated) as desired, but these students hesitated less than all other previous students, and that’s a huge win. Based on the end of the term feedback, even when students struggled with the class, they still had high praise for their experiences “looking for mathematics”. This is supported by the following student comments taken from the end of term feedback.

- I think that the “Looking for Mathematics” blackboard assignment contributed positively to our learning experience because it was a way for us to all realize how math is all around us and how we will be able to incorporate it into our math lessons when we are teachers in the future. … By learning to see how our math concepts can relate to our everyday lives, we are able to better comprehend them and better remember them.
- I really enjoyed the blackboard postings as a whole, I thought that relating math to the “real world”; was a great way to gain a better understanding of the course material. Seeing a ton of different examples especially ones that other people in the class came up with had a positive impact on me because they were usually things that wouldn't think of.
- This really opened my eyes on how math is really involved a lot in every day life and when you are looking for it, you see it a lot more.
- Honestly when first getting this assignment I thought it was going to be very challenging and dumb. But I found it to be very helpful. It was interesting to see how many different people saw things and then how we could reflect on it.

In fall 2016, I plan to incorporate the Looking For Math assignment in my Math for Elementary Teachers course. My emphasis will be on improved communication about the assignment. In particular, I will emphasize the role that formative feedback plays into their summative assessment, so they are less anxious about their grade. Regular communication will also serve as friendly reminders about the assignment and to facilitate questions. The natural blend of course topics and real-life contexts worked well, so it's worth trying again. So as to not overwhelm the students, I'll start with a handful of forums in the beginning, adding additional forum topics over time. This should create multiple opportunities to discuss the assignment, address student questions, and solicit their suggestions for discussion board topics. All details about the assignment will also be posted online so that students have an electronic record. I'm also thinking of incorporating more photography images as suggested by Furner and Marinas (2015). They suggest ways to use photography and GeoGebra to facilitate students identifying mathematics in historical buildings. Historical buildings are a great context but we need not limit ourselves. I anticipate using a photograph to start a class discussion on the mathematics found within the picture will. Suggests a good opportunity to share a picture with the class to start a discussion of the mathematics they observe. Through this class brainstorm and dialogue, I can provide general information and specific examples about what high level postings should and could include. Interested students will be welcome to incorporate the use of GeoGebra and students will be encouraged include their own (relevant) photographs related to their posting.

I encourage you to experiment with using online discussion boards to have students write about the mathematics they find in their lives. The goal is that students communicate mathematical ideas in contexts that interest them. Thus promoting a community of learners reflecting the different interests and abilities of those who form the community. Furthermore, when students use online discussion boards to share their "real-life" observations, they are able to answer for themselves the familiar question: "Where are we ever going to use this?"

References:

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Furner, J. and Marinas, C. (2015) "Teaching Math Concepts Through Historical locations using GeoGebra and Photography", *Proceedings for the ICTCM 2015 International Conference,* Electronic proceedings of the Twenty-Seventh Annual International Conference on Technology in Collegiate Mathematics, Volume 27, Las Vegas, NV March 12 - 15, 2015. Available online for access or retrieval on May 17th, 2016 at http://archives.math.utk.edu/ICTCM/VOL27/A017/paper.pdf.

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Appendix A

Amusement park mathematics from (Calculus Concepts) MSTI 314

Current Forum: Amusement Park Math Read 6 times
Date: Fri Oct 28 2005 12:02 am
Author: .@sjfc.edu>
Attachment: ferris_wheel.xls (17920 bytes)
Subject: The ferris wheel

Remove

While I was reading Go, Dog, Go!, I came to the part of the story when the dogs were riding on the ferris wheel. This part of the story made me think about the function of a ferris wheel and what it would look like to graph. I wanted to find the function of a ferris wheel as height as a function of time. Before I could graph this function I first had to find some variables that would fit what I was looking for. I found variables at this web site : http://jwilson.coe.uga.edu/emt669/Student.Folders/Jeon.Kyungsoon/IU/trig/F.wheel.html. The variables that were found at this web were height in feet and time in seconds. The variables also only went from 0 seconds to 10 seconds, which is one revolution. I expanded this in the excel document to find what 4 revolutions would be (see link). The variables in the domain start at 0 seconds and end at 40 seconds. The range values go up and down from 4 feet to 44 feet. The reason that the wheel does not start at 0 feet is because if were zero, the ride would not work, you would hit your feet. This ferris wheel has a platform where you get on that is 4 feet above the ground.

The graph that I found (see link) looks cyclic, or repeats itself, and reminds me of a sin curve. I noticed on my calculator that there is a sin regression that I might be able to use to find out the function for this wild graph. I plugged the same data in excel into my calculator and then found the sin regression for the variables. This is what I found:

y= a*sin(bx+c) + d (the function for the ferris wheel!!)

a= 17.2456 (The distance on the Y axis from the midline of the sin curve)

b= .6274 (2pie/11.25) 11.25 is what the x axis is at the midline.

c= -1.5540 (The shift in the sin curve)

d= 23.8992 (The value of the midline on the y axis)

Plugging in the numbers for the function will give us the sin regression for our ferris wheel with these variables. With the help of Mrs. Ti I was able to explain what a,b,c, and d are. To tell you the truth, I have never learned this, but sounds very interesting to me. Can anyone please help me out more with this sin function!

Reply

Current Forum: Amusement Park Math Read 5 times

Appendix B:

Student Posting Rubric

Low-level Postings (L)	Moderate-level postings (M)	High-level postings (H)
• Postings contain frequent grammatical and/or spelling errors. • Postings rarely contain precise and correct mathematical language. • Waiting until midterm and/or end of term to post and not reading classmates postings. • Most postings just report an observation without making connections to material discussed in class or bringing up relevant questions. • Posting the minimum • Postings are often "obvious" and/or lack depth ~ for ex: multiplication is used in situations where one might need to multiply. Addition and subtraction are necessary to balance one's checkbook. • Postings focus on one (or a few) topics discussed in class or are essentially a "repeat" of another post. • Postings stay at the "low" level for most/all of the term. • Postings are rarely cited (as appropriate) so the reader can locate the reference.	• Postings are usually free from grammatical and/or spelling errors. • Postings usually contain precise and correct mathematical language. • Posting enough to "get by" ~ Posting just a few times a term and reading a few peers' postings. • Postings are related to topics covered in class, but connections are sometimes not carefully explained. • Submitting at most the minimum number of relevant ("low-level"-included) postings, but not taking it to "the next level" by not thoroughly describing or connecting it to relevant mathematical concepts. • Postings reflect many topics discussed in class. • Postings stay at the "low" or "medium" level for most of the term. • Some postings include appropriate citations.	• Postings are mostly free from grammatical and/or spelling errors. • Postings mostly contain precise and correct mathematical language. • Regular postings and reading of most of the other posts. • Postings generally include clear and thorough explanations and connections between mathematics concepts are clearly explained. • Submitting more than the minimum # of (medium – high level) postings • Building on other's postings ~ asking questions and/or carefully describing how the mathematical ideas could be built into an activity for elementary schoolers. • Postings reflect most (or at least a wide gamut of) topics discussed in class. • While postings may start at the "low" or "medium" level, they are primarily "medium" or "high" level for most of the term. • Postings mostly contain appropriate citations.

Disclaimers:

1) Characteristics of Good Explanations in Mathematics* will provide the guidelines for assessment.
2) This is a general guideline ~ a starting point, if you will… I do not purport that this covers every conceivable situation, but instead, a basic idea of my expectations with this activity.

*"Characteristics of Good Explanations in Mathematics" as outlined in *Mathematics for Elementary Teachers* (4th ed.) by Sybilla Beckmann.