EFFECTS OF TECHNOLOGY-AIDED MULTIPLE-REPRESENTATIONS APPROACH ON STUDENTS' UNDERSTANDING OF DERIVATIVES

Hana Shatila Samer Habre Iman Osta

Lebanese American University

P.O. Box 13-5053

Beirut-Lebanon

hana.shatila@lau.edu.lb shabre@lau.edu.lb iman.osta@lau.edu.lb

Introduction

For decades the teaching of calculus has mostly been based on rote memorization of formulas and procedures, algebraic manipulation, and solving drill problems. Calculus however is used in many scientific disciplines, and therefore this teaching approach may not always be beneficial because conceptual understanding and transfer of knowledge are needed rather than mere memorization. Calls for change in calculus instruction have been paramount since the late 1980's (e.g. Douglas, 1986; Steen, 1987; Vinner, 1989). Recognizing the shortcomings of procedure based curricula, the 1989 Standards issued by the National Council of Teachers of Mathematics (NCTM, 1989) recommended that the teaching of mathematics "emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving" (p. 125). According to Kaput (1987), each mode of representation of the multiple representations approach, i.e. the numeric, graphical, and symbolic representation, is special because it offers a unique perspective to the concept being investigated. On the graphical approach and its importance for Calculus, Zimmermann (1991) argued that "the role of visual thinking is so fundamental to the understanding of calculus that it is difficult to imagine a successful calculus course which does not emphasize the visual elements of the subject" (p. 136). Many steps have been carried out since then in many countries to ensure changes in the curriculum, be it by emphasizing conceptual understanding and visualization, or through a change in the delivery style (e.g. group work discussion and student-teacher interactions). The development of graphic calculators in the previous century, and in parallel of dynamic mathematical software (*GeoGebra, Maple, Cabri, Geometer's Sketchpad,* and *Autograph*, to name a few) played an important role in implementing the new reformed curricula. Research (Fey 1989; Habre & Abboud, 2006; Kaput, 1992; Porzio, 1999) has shown that technology in general provides students with an easier and more effective access to multiple representations of mathematical concepts. More particularly, the dragging and animation features of dynamic software programs provide students with an environment of discovery, experimentation, identifying patterns, generating and testing conjectures, and visualizing mathematical objects in ways that were not possible using paper and pencil (Gonzalez & Rodriguez, 2011; Herceg, 2010; and Sacristán et al., 2010).

The derivative concept is one of the key ideas in calculus. Among other things, the derivative measures the steepness of a function, the slope of a tangent line to a curve at a given point, the rate of change of the output relative to the input, and helps in finding the critical points of a graph. The derivative can also be used as a tool to model the behavior of changing quantities such as population dynamics, decaying radioactive materials, finding velocity and acceleration of moving objects and others. Therefore, having a solid understanding of the derivative is important. According to Ellison (1993), a good conceptual understanding of the derivative includes the following: The formal definition, the idea of a differentiable function at a point, the derivative as a function (all 3 falling within the algebraic mode of representation), the instantaneous rate of change (numeric mode), and the slope of a tangent line (graphical mode). Making connections and translations among and within these representations is key for understanding this important calculus concept (Ferrini- Mundy & Graham, 1994; Orton, 1983; Zandieh, 1998). Research on understanding derivatives has shown however that students may face difficulties in any one of the 3 modes, be it the numeric mode of representation (Bezuidenhout, 1998), the graphical mode (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Ferrini- Mundy & Graham, 1994; Orton, 1983; Vinner 1982), or the algebraic mode particularly in the formal definition of derivatives (Zandieh, 1998). This has been attributed partly to traditional instructional methods in schools and colleges that place a strong emphasis on memorizing rules and manipulating symbols, while less emphasis is placed on the derivative concept as a rate of change and even lesser emphasis on its graphical representation.

This research report examines the impact of two different instructional methods on students' conceptual understanding of the derivative concept. The research is conducted in two Calculus 1 classes where the derivative is taught using two textbooks that adopt two distinct pedagogical approaches: a formal symbolic approach (Book 1), and a multiple-representation approach with a focus on visualization (Book 2). The study addresses the following research questions:

1. What are the differential effects of the two approaches (formal symbolic approach and multiple-representation approach) on students' conceptual understanding of the derivative?
2. Does the use of a multiple-representation visual approach improve students' motivation and attitude towards math?

Methodology and Collection of Data

The study was conducted during the fall of 2013 and the spring of 2014 at a Lebanese university located in Beirut, and that adopts an American type of education. It offers a sequence of four calculus courses: Calculus I, Calculus II, Calculus III, and Calculus IV. Several factors determine which calculus course is required for different students, such as students' SAT scores, their school background, and their future field of study. In Calculus I students cover the following topics: functions, limits, continuity of functions, the derivatives of functions and their applications. It is to be noted that some students taking Calculus I might have a prior knowledge of derivatives carried from their school education.

In the fall of 2013 two sections of Calculus I were offered by the same instructor and using Book 1(control group). In spring of 2014, only one section of Calculus 1 was offered using Book 2 (experimental group) and taught by a different instructor. The collection of data included classroom observations, assessment of students learning (tests), and interviews. In all, 52 students whose ages range between 17 and 20 years old participated the study, and they were equally distributed between the control and experimental group. The selection of students took into consideration those who completed all the requirements for the study. The twenty-six students of the control group were evenly split between males and females while there were 14 males and 12 females in the experimental group. Students' SAT scores on the math section were almost normally distributed, forming a normal bell curve. To assess their prior mathematical knowledge, a 60-minute diagnostic test was administered to all students at the beginning of each semester. The test consisted of nine questions covering topics such as equations of lines, x-and y-intercepts, the equation of a tangent line to a curve at a point, and one question that tested the procedural knowledge of the derivative. After the implementation of the derivative unit, a common written test (Post Test) was administered to all students of the two groups; it included five conceptual understanding-based problems. Finally, interviews were conducted with 12 students (5 males and 7 females) from the control and experimental group. The purpose of the interviews was to obtain a clearer, more explicit and better picture of students' conceptual understanding of derivatives. All interviews were audio-recorded and transcribed for analysis.

In this paper, we shall report on the classroom observations, the results of the Post Test, and triangulate the results with the interviews.

Preliminary Results and Discussion

The classroom observations

During the implementation of the derivative unit, one of the researchers was present in both groups (control and experimental) to observe and document the instructional method/s and strategies (e.g. lecture, group work, use of technology, individual work), as well as students' participation. The implementation of the derivative unit in the control group was carried out over five class sessions (50 minutes each). No group work or technology was part of the learning process. Homework exercises and problems were all selected from the book. In the experimental group, the implementation of the unit was carried out over 6.5 class sessions because group work and technology (*Autograph*) were used. In contrast to the control group, the classroom environment of the experimental group was cooperative and interactive; in addition, students were asked to complete a set of activities designed to encourage the exploration of the derivative in different representations (See Appendix A for a sample of such activities).

Autograph, published by Eastmond Publishing Ltd., is a dynamic software conceived initially to teach mathematics at the secondary and university levels. According to Butler, D. (2013), the creator of the software, "In the design of Autograph, there was an overriding determination to make the creation of dynamic objects as straightforward as possible" (p. 114). For instance, to understand the concept of derivatives visually, one can use the software to plot a function, pick a point on the function, plot the tangent at that point and then drag the point along the graph. While this is being done, the screen shows at its bottom the varying coordinates of the point and the equation of the tangent line (see the two snapshots in Figure 1). Another feature of Autograph is that it illustrates graphically the tangent line to a given curve as it moves along the curve while, at the same time, showing the tangent line and plotting the derivative function. Figure 2 shows two additional snapshots of the software in action: The red curve is the plot of the initial function, while the dotted is for the derivative function.

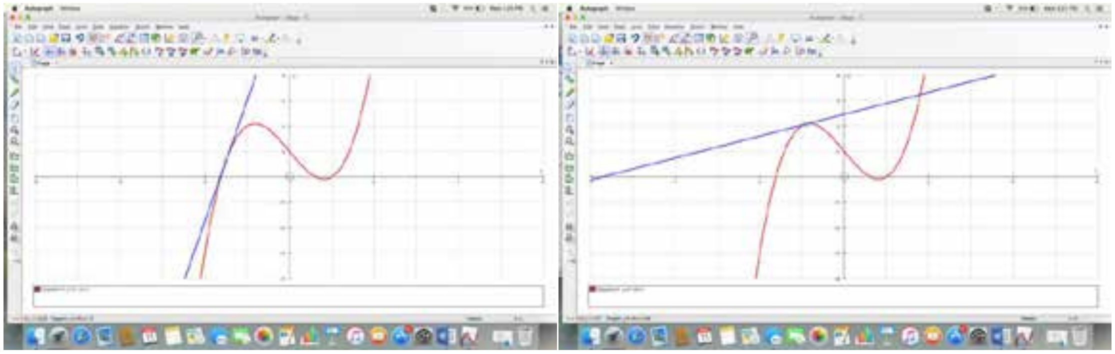


Figure 1. Two snapshots showing the manual dragging of the tangent line. The snapshots also show the coordinates of the point of tangency and the slope of the tangents.

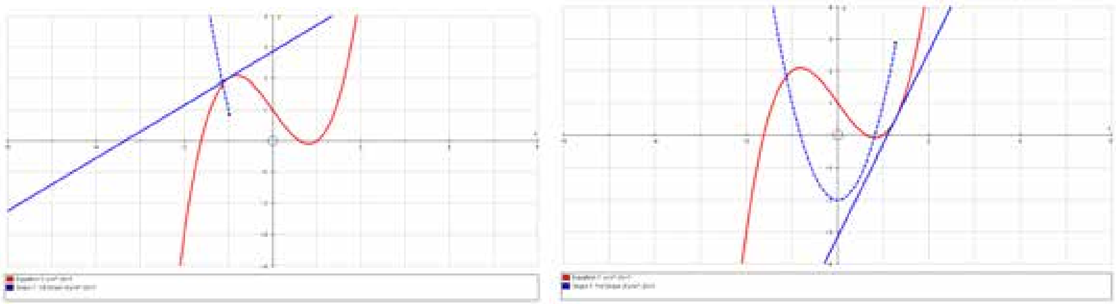


Figure 2. Two snapshots of dynamic illustrations showing how tangents are plotted while at the same time displaying the derivative function

As mentioned earlier, the teaching approaches and textbooks used were not identical in the control and experimental sections. Figure 3 below summarizes how the teaching methods varied between the two groups. The figure shows that the lecture method was most frequently used in the control group (58.8%), followed by individual work (22%), while technology and group work were never used. The remaining time (19. 2 %) was devoted to Q/A sessions at the beginning of every class. On the contrary, the lecture method was the least used in the experimental group (12%) and only 17.7% of the time was used for individual work. Instead, during 33% of the time students completed paper-and-pencil activities working in groups and 24.3 % of the time was spent using technology. Due to time limitation however, the Autograph based activities were conducted by the instructor, and students were asked to observe, analyze, make conjectures and interpret the problem until an appropriate conclusion was reached. As in the control group, the remaining time was spent on Q/A sessions.

The Post Test

A 60-minute test on derivatives was administered to all students in both the control and experimental groups after the implementation of the derivative unit (see Appendix B). The purpose of the test was to identify the types of difficulties learners face with the notion of derivative, and to assess their ability to move back and forth between the different representations of this concept. Contrary to a traditional test on derivatives usually dominated by the symbolic approach, the five questions on this test were characterized by the use of graphs and tables of values of functions without any algebraic expression. To ensure fairness between the two groups, students in the experimental group had not solved in class any question similar to the ones found on the test.

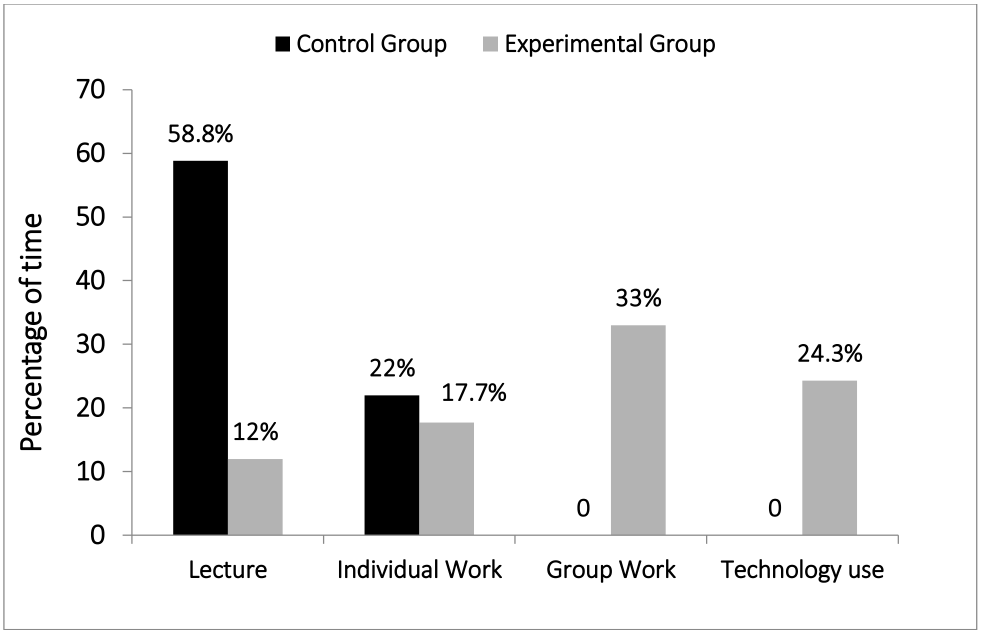


Figure 3. Comparisons of different types of teaching methods used in the two groups

In brief, Question I tested in sub-part 2 the relationship between the derivative of a function at a point and the slope of the tangent line at that point, while in sub-part 3 it tested the concept of linearization, all in the absence of a formula for the function (sub-part 1 was a mere evaluation of the function at a given point). Question II tested students' understanding of the relationship between the derivative of a function at a point and the rate of change of the function. In Question III the graph of a derivative function G' was given and students were asked about the properties of the function G (sense of variation, critical points, maxima and minima). Question IV required that students prove graphically that the derivatives of two functions that only differ by a constant are equal. Finally Question V tested whether students can read a table of values and estimate the derivative at a point numerically. Figure 4 summarizes and shows a comparison between the numbers of correct responses in the two groups. The figure also shows that students in the experimental group outperformed their counterparts on the test.

In summary, results show that many students in the control group have deficiencies in their graphical understanding of the derivative as the slope of the curve or the slope of the tangent line (Question I, sub-part 2), and only few of them acquired a good comprehension of the linearization concept (Question I, subpart 3). In addition, only 3 students in the control group (compared to 15 in the experimental group) succeeded in justifying why two functions that differ by a constant have equal derivatives (Question IV) without having to first define the function with an equation. Figures 5 and 6 are samples of students' work from the experimental section on the linearization question and on Question IV respectively. Even though in the latter question the student considered a special case of a function ($y = x^2$), yet the argument is valid for any function.

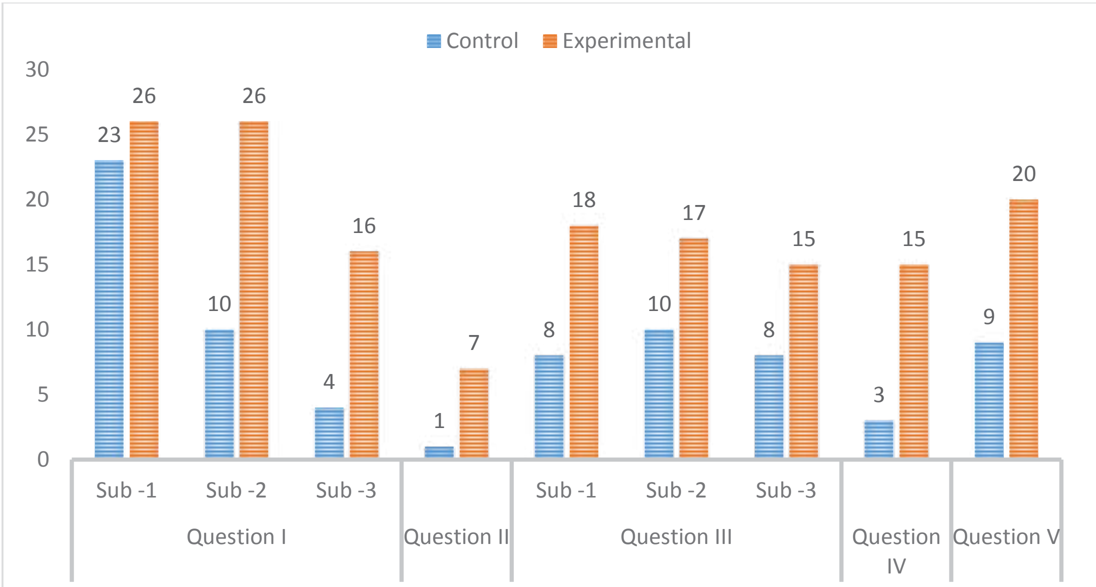


Figure 4. Comparon of the Number of correct responses on each question/ sub-question of the test, in the two groups.

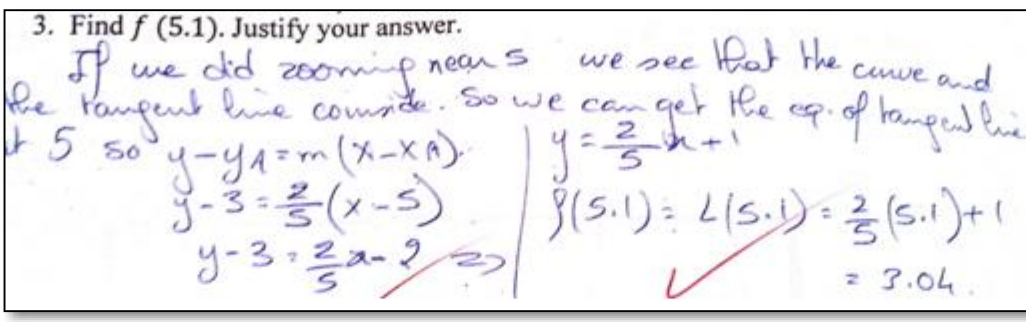


Figure 5. A sample answer from a student in the experimental group to sub-question 3 of Question I revealing a good understanding of the linearization concept

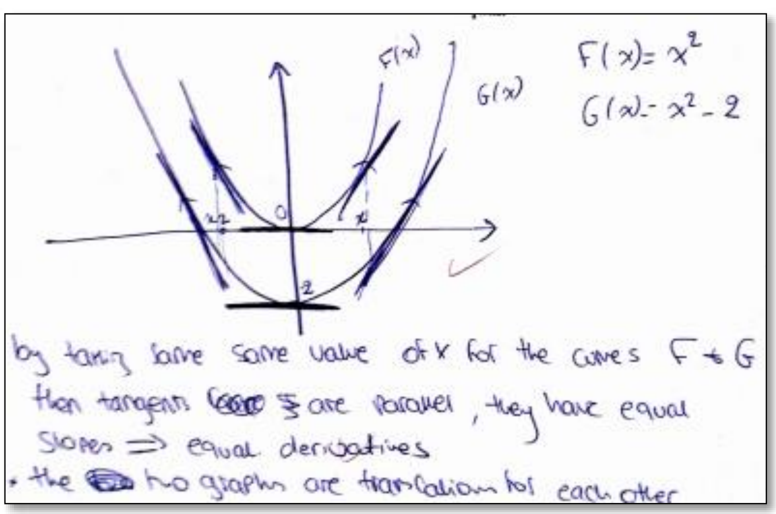


Figure 6. A sample answer from a student in the experimental group to Question IV taking polynomials as an example of functions

The Interviews

Twelve students, six students from each of the control and experimental groups, were interviewed individually by the end of the unit. At the beginning of the interviews, students were asked to freely express their opinion about math in general, and about Calculus I in particular. On the teaching approach, five out of the six of the control group interviewees said that it was neither motivating nor interactive. A student with an A average for instance stated that "*one can pass the course easily by just studying the rules and at the last minutes*", while another (a student with a B average) thought that the course "*is complicated and it is all about formulas and rules to memorize.*" On the other hand, five out of the six students in the experimental group thought otherwise. They argued that technology and animation provided connections between the graphical representation, the table of values, and the algebraic expressions of functions. For instance, one student with a B average from the experimental group stated: "*I am repeating this course for the second time. I don't remember anything from the first time because it was all rules and equations..., but now I feel I can remember more because it includes more graphs, more visual components…*" A student with a C average from the same group added "*I used to hate math a lot and I used to fail in all my exams. Now, I am happy. I am passing my exams. I feel that I am more confident and I can understand because of the visual element of the course.*"

Later students were asked about their understanding of the meaning of the derivative. Three out of the six interviewed students from the control group mentioned only the symbolic representation of derivative, including rules of differentiation and the formal definition of derivative; two students mentioned both the symbolic and the graphical representation as slope of the tangent line, and only one student mentioned in addition the numerical representation. On the contrary, all students in the experimental group mentioned the three types of representations in their definition, and five of them even spoke of real life applications.

Interviews then revolved around the five questions found on the Post Test. It was hoped that this way researchers would obtain a clearer, more explicit, and better picture of students' conceptual understanding of the derivative. While we shall not report in details on the latter part of the interviews, preliminary analyses show that the approach used in the experimental group seems to have positive effects on students' conceptual understanding. The interviewees in the experimental group were more flexible and comfortable working with the different representations of the derivative. They outperformed students in the control group, and demonstrated a good understanding of the derivative concept. They were able to explain the relationship between the derivative of a function at a point with the slope of the tangent line and the instantaneous rate of change at that point. Interviewees from the control group however were not comfortable working with functions without their algebraic expressions and some expressed their frustrations each time a function was defined differently.

Preliminary Conclusions

The findings obtained from the observations, the Post Test, and the preliminary results of the interviews reveal that students in the experimental group showed better understanding of the derivative concept than students in the control group. As mentioned before, for some students, subjects of this study, the derivative concept as presented in Calculus I was not their first exposure to the topic since it was discussed in their high school years but with emphasis placed on the rules of differentiation. The intervention that was implemented in the experimental group when teaching derivatives enriched and deepened students' conceptual understanding of this concept. The intervention successfully allowed the experimental group instructor to instill in students' minds many key aspects of the derivative concept such as the slope of the tangent line, the derivative as the instantaneous rate of change, the derivative function, and the relation between a function and its derivative, and others. Also, the use of technology (Autograph) has improved students' abilities to interpret graphs and make connections and associations between the properties of a function and its derivative. In the control group however, many students were not comfortable working with functions without the knowledge of their algebraic expressions; some students even expressed frustrations that were revealed during the interviews. Students in the control group showed deficiencies in the understanding of derivatives, and their thinking was dominated by its procedural symbolic representation.

Based on the interviews it is noticed that the teaching methods used in the two groups have affected students' attitudes and motivation toward calculus. Initially many students in the experimental group were not motivated; they even resisted the approach used and found it difficult. However, at the end of the instructional treatment, most students in the experimental group liked the visual part and agreed that technology helped them make connections between the graphical representation, the table of values, and the algebraic expressions of functions. Those results are consistent with previous studies conducted on the positive relationship between the use of multiple-representations and students' attitudes toward mathematics (Tseng, Chang, Lou, & Chen, 2013; González & Rodríguez, 2011). There remained however few students in the experimental group who did not like the approach and commented that it was challenging. One student noted that "*working with graphs is not an easy job, we have to interpret the graphs, extract information and then relate them to the function and its derivative. However, rules are easy; we just need to memorize them*". One interpretation of such attitude may be that students needed more time to assimilate the new approach, specifically because the traditional instructional method of teaching carries over from school years; consequently, the sudden shift to a teaching method that required visualization, critical thinking and analysis was not easy for students. This is in line with earlier research results found for instance in Eisenberg and Dreyfus, (1991), and Habre (2001). On the contrary, most students in the control group were not motivated and were passive learners; they agreed that math is about rules and formulas that need to be memorized.

In conclusion, the preliminary results of this study suggest that the pedagogy used in the experimental group (group work, Autograph, and activities that emphasize multiple representations of the concept) is effective in helping students develop a better understanding of the derivative concept. The approach used in the experimental group had positive effects on students' conceptual understanding, and they were more flexible and comfortable working with the different representations of the derivative than those of the control group. For the latter group, formulas and equations come first, but students of the former group were able to explain the relationship between the graphical, the numerical and the symbolic representations of derivative, indicating an understanding of the derivative concept, and not just mere knowledge based on memorized facts.

References

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. (1997). The development of students' graphical understanding of derivative. *Journal of Mathematical Behavior*, *16*(4), 399-431.

Bezuidenhout, J. (1998). First-year university students' understanding of rate of change. *International Journal of Mathematical Education in Science & Technology*, *29*(3), 389-399.

Butler, D. (2013). Vectors and Differential Equations: A visual approach using Autograph. In S. Habre (Ed.), *Enhancing mathematics understanding through visualization: The role of Dynamical (pp. 113 – 126)*. IGI Global, 77-88.

Douglas, R. G. (Ed.) (1986). *Toward a lean and lively calculus.* (MAA Notes No.6).Washington, D.C.: Mathematical Association of America.

Eisenberg, T., & Dreyfus, T. (1991). On the reluctance to visualize in mathematics. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning mathematics* (pp. 25-37). Washington, DC: Mathematical Association of America.

Ellison, M. (1993). The effect of computer and calculator graphics on students' ability to mentally construct calculus concepts. *Dissertation Abstracts International*, 51(11), 4020.

Ferrini- Mundy, J. & Graham, K. (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals. In J. Kaput and E. Dubinsky (Eds.), *Research issues in undergraduate mathematics learning: Preliminary analysis and results, (pp. 29-45).* Washington, D.C.: Mathematical Association of America.

Fey, J. T. (1989). Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics*, *20*(3), 237-272.

Gonzalez, C. C., & Rodriguez, J.B. (2011). New technologies and education: Constructive, geometric and dynamic introduction of the derivative concept. *International Journal for Technology in Mathematics Education, 18*(4), 203-208,

Habre, S. (2001). Visualization in multivariable calculus: The case of 3D-surfaces. *Focus on Learning Problems in Mathematics*, *23*(1), 30-48.

Habre, S., & Abboud, M. (2006). Students' conceptual understanding of a function and its derivative in an experimental calculus course. *The Journal of Mathematical Behavior*, *25*(1*),* 57- 72.

Herceg, D. (2010). Numerical integration with GeoGebra in High School. *International journal of Technology in Mathematics education 17*(4), 205-210.

Kaput, J. (1987). Representations systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Kaput, J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). Reston, VA; New York: National Council of Teachers of Mathematics; Macmillan.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston VA: The Council.

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, *15*, 235-250.

Porzio, D. (1999). Effects of differing emphases in the use of multiple representations and technology on students' understanding of calculus students. *Focus on Learning Problems in Mathematics*, *21*, 1-29.

Sacristan, A., Calder, N., Rojano, T., Santos-Trigo, M., Friedlander, A., & Meissner, H. (2010).The influence and shaping of digital technologies on the learning - and learning trajectories - of mathematical concepts. In C. Hoyles, & J. Lagrange (Eds.), *Mathematics education and technology - Rethinking the terrain: The 17th ICMI Study* (pp. 179-226). New York, NY: Springer.

Steen, L.A. (Ed.) (1987). *Calculus for a new century: A pump, not a filter*. Washington, D.C.: Mathematical Association of America.

Tseng, K-H., Chang, C-C., Lou, S-J., & Chen, W-P. (2013). Attitudes towards science, technology, engineering and mathematics (STEM) in a project- based learning environment. *International Journal of Technology and Design Education*, *23*(1), 87- 102.

Vinner, S. (1982). Conflicts between definitions and intuitions: The case of the tangent. In E. Vermandel (Ed.), *Proceedings of the Sixth International Conference for the Psychology of Mathematics Education* (pp. 24-28). Antwerp, Belgium: Universitaire Instelling Antwerpen.

Vinner, S. (1989). The avoidance of visual consideration in calculus students. *Focus on Learning Problems in Mathematics*, *11*, 149-156.

Zandieh, M. (1998). *The evolution of student understanding of the concept of derivative* (Doctoral dissertation). Oregon State University.

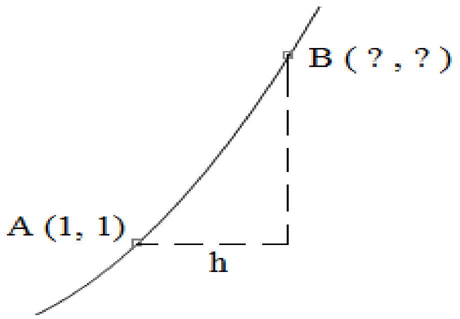
Zimmermann, W. (1991). Visual thinking in mathematics. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning mathematics* (pp. 127-137). Washington, DC: Mathematical Association of America.

Appendix A

Sample of Classroom Activities Using Autograph

Sample Activity 1

The diagram shows the graph of $y = x^2$ near the point $A(1,1)$. The point B is a at horizontal distance h from A.



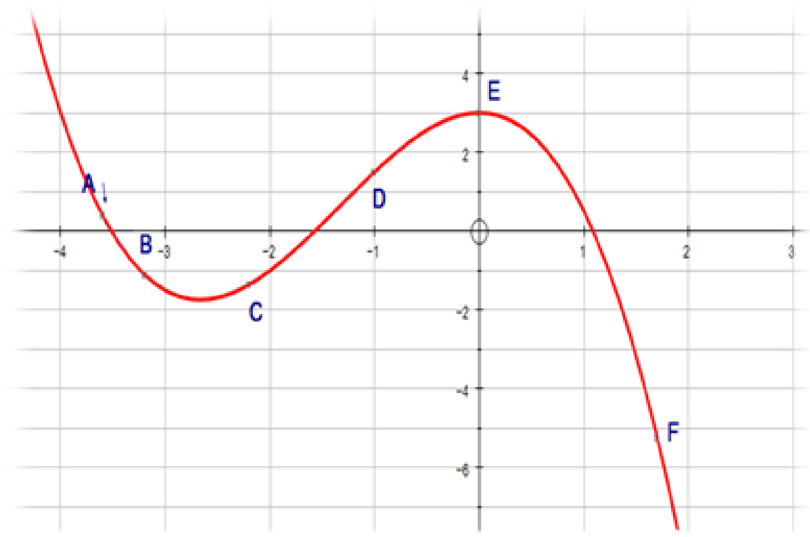
a) Find the coordinates of B in terms of h.

b) Find the slope of the secant line passing through A and B, in terms of h. Simplify your answer.

c) Explain what might happen to the secant line as B gets closer and closer to A? What value does the slope obtained in part b approach?

Sample Activity 2

Below is the graph of $y = f(x)$.



a) Match the points labeled on the curve with the given slopes of the curve in the table below.

Slopes	Points
- 8	
- 5	
-2.5	
0	
1.5	
2.5	

b) Sketch the graph of the derivative of $y = f(x)$ in the same system.

Appendix B

The Derivative Test

I. (L) is a straight line tangent to the graph of the function $y = f(x)$ at the point (5, 3), as shown in Figure 1.

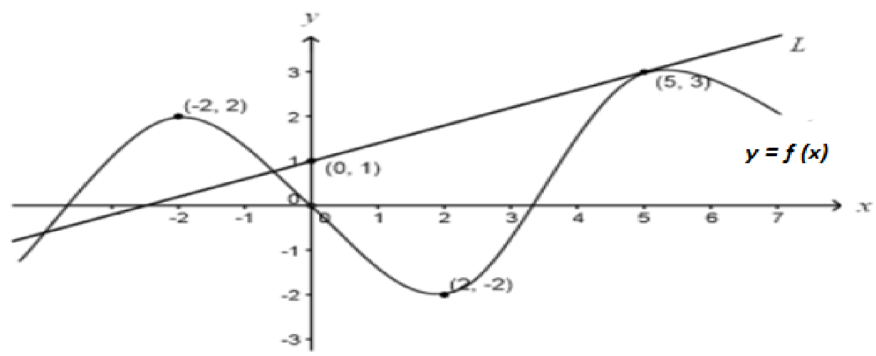
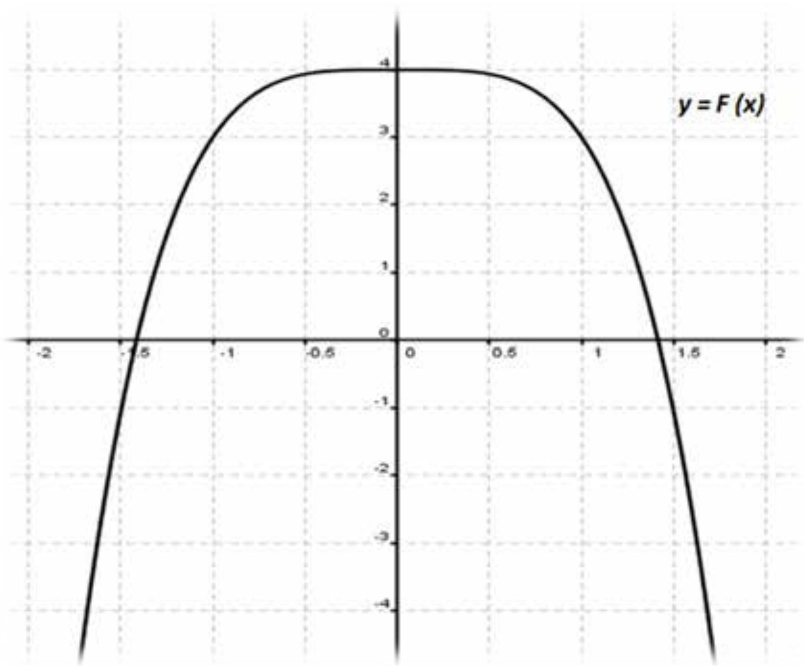


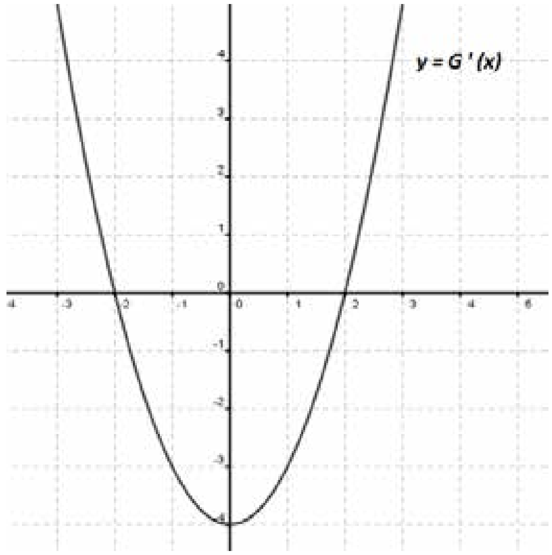
Figure 1: A straight line and a function.

1. Find $f(5)$. Justify your answer.
2. Calculate the value of $f'(5)$. Justify your answer.
3. Estimate $f(5.1)$. Justify your answer.

II. Given the function $F(x)$ below defined on the interval (- ∞, ∞). Is its derivative increasing, decreasing or both? Discuss using rate of increase and decrease of the function.



III. The graph of the function $\boldsymbol{G'(x)}$ (derivative) is shown below.



a. On what interval is $G(x)$ increasing? decreasing? Justify your answer.
b. At what point(s) does $G(x)$ have critical points? Justify your answer
c. Which critical point is a local maximum / local minimum? Justify your answer.

IV. Let $F(x)$ be any function and let $G(x)$ be another function defined by $G(x) = F(x) + C$, where C is a constant. Clearly $G' = F'$ since $\frac{d}{dx}[C] = 0$ (the derivative of a constant = 0). Explain geometrically why the two derivatives are equal.

V. Suppose the table below gives the concentration (mg/cc) of a drug in the bloodstream at time t (min).

t (min)	0	0.1	0. 2	0. 3	0. 4	0. 5	0. 6	0. 7	0. 8
$C(t)$	0.84	0.89	0.94	0.98	1	1	0.9	0.79	0.63

Fill the table below by finding the estimated values for $C'(t)$, the derivative of $C(t)$ with respect to time. Explain and justify your answers.

t (min)	0	0.1	0. 2	0. 3	0. 4	0. 5	0. 6	0.7
C ' (t)								