CALCULUS CLASSROOM ACTIVITIES INVOLVING GROUPS OF LEARNING CATALYTICS QUESTIONS

Przemyslaw Bogacki
Department of Mathematics and Statistics
Old Dominion University
Norfolk, VA 23529, USA
pbogacki@odu.edu

Introduction

Personal response system devices (or "clickers") can help to engage students and to gather useful feedback during a mathematics lecture. I have used such devices with considerable success since 2008. However, a major downside is that clickers either have to be acquired by students or provided by the instructor.

Learning Catalytics (http://www.learningcatalytics.com) platform offers the clickers' benefits without the hassle of dealing with dedicated physical devices. It is a web-based platform requiring a modern browser that can be accessed on smartphones, tablets, and computers. Compared to typical physical clickers, it offers additional advantages, e.g., more flexible input modes (including questions requiring students to sketch a graph) and support for peer instruction.

I have developed a number of Learning Catalytics questions for calculus. Sometimes these questions are grouped into activities which focus on a particular calculus topic; this article discusses some of these activities.

Deploying groups of Learning Catalytics questions as classroom activities

I have class-tested the activities described here in sections of Calculus I and Calculus II I taught during the years 2014-2015. In the following sections, I shall describe Learning Catalytics questions I typically deploy in the course of such activities. However, one should keep in mind that the make-up of each activity is by no means rigid:

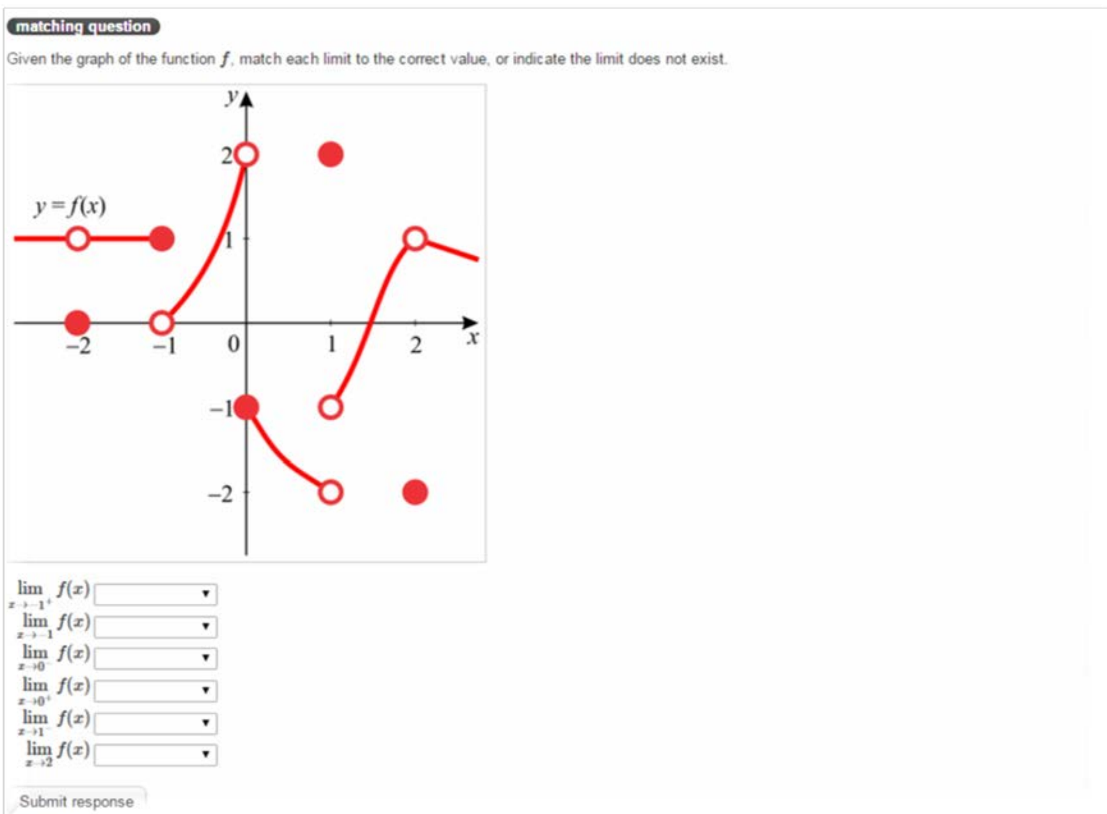
- If a particular group of students shows a mastery of topic before the pool of questions in the activity is exhausted, the instructor can easily omit some questions "on the fly".
- If student responses indicate the need for more discussion of a particular question, I would often deploy the same question again. If the initial responses were collected from individual students (which I typically prefer), such subsequent "round" is often assigned to teams of students instead. Anecdotal evidence often shows marked improvement in the quality of student responses after they participate in such group discussion. (Significantly, this improvement can still be

observed when individual students, rather than teams, are asked subsequent questions of the same type.)

- For some activities, I have created additional questions, which I normally do not assign, but they can be assigned if an instructor so desires.

I have created a Learning Catalytics "course", which contains all activities and questions mentioned in this article. Interested faculty are encouraged to email the author (pbogacki@odu.edu) to request access so that they can use this material as a template when building their own activities.

Figure 1. Visualizing Limits - sample question involving finding limits from a graph



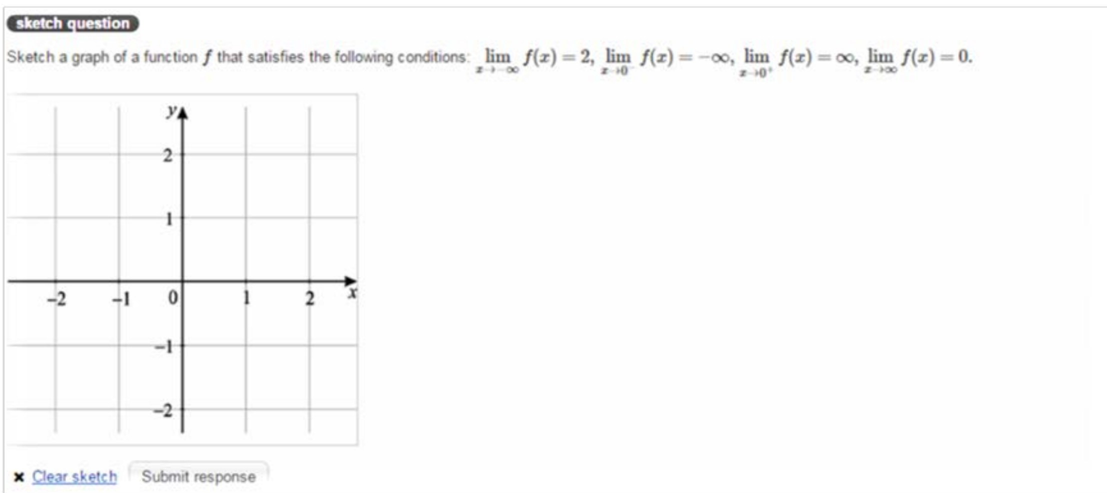
Activity: Visualizing Limits

Once the notion of a limit has been introduced, we typically discuss a variety of examples of limits. In addition to examples in which limits are illustrated numerically or symbolically it is important to help our students gain visual insight into this notion. This activity is designed to address that latter aspect.

It begins with two questions where students are asked to determine the values of limits of the function whose graph is given (Fig. 1). Three subsequent questions ask students to sketch graphs of functions whose limits have given values (Fig. 2).

Questions like these can form "stepping stones" helping improve students' skills when it comes to "reading" and composing function graphs. Those skills will be further addressed in a subsequent "Curve Sketching" activity.

Figure 2. Visualizing Limits - sample question involving sketching a graph given the limit values

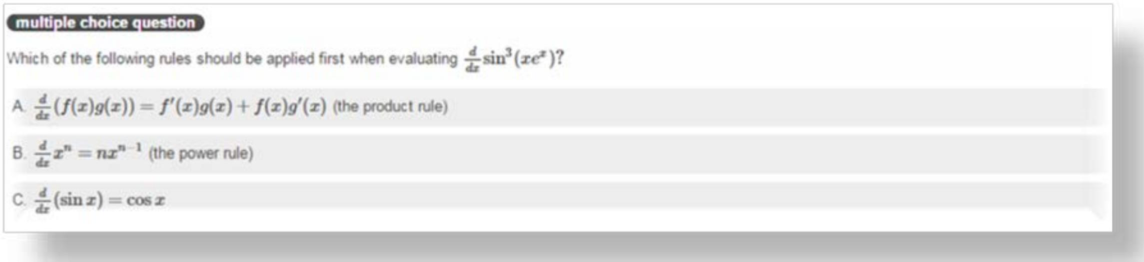


Activity: Differentiation Rules

While differentiation rules are generally not hard to learn (compared to most other calculus topic, which students tend to find more challenging), students do not always apply such rules correctly. Often, these problems are related to the student's inability to properly recognize the algebraic structure of the expression used to define the function to be differentiated.

The activity consists of three questions asking students to identify the differentiation rule, which should be applied first (Fig. 3) - to succeed, the student has to correctly identify the expression structure (e.g., as that of a power, a product, etc.).

Figure 3. Differentiation Rules - sample question



As an optional follow-up, an instructor can also include some expression-type questions asking students to actually differentiate given functions.

Activity: Related Rates

Most calculus students find word (application) problems very challenging - modeling the word problem mathematically is often more difficult than solving the resulting mathematical problem.

In the activity devoted to related rates, we begin with the question (Fig. 4) asking students to write a formula relating the variables present in the given word problem. Note that this question is not algorithmically graded: there is no unique correct answer - for instance, students may use different names for variables.

After discussing student responses to that question in class, the second question asks the students to actually find the unknown rate of change.

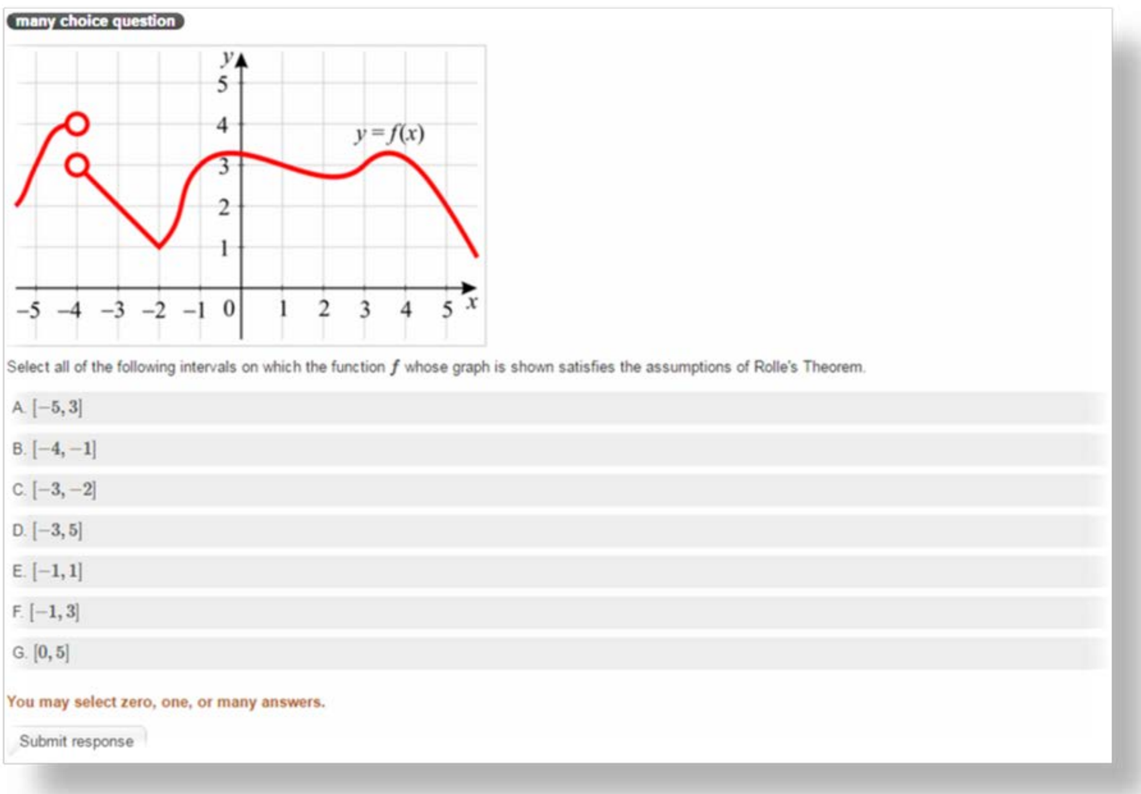
Figure 4. Related Rates - sample question

Activity: Mean Value Theorem

An important aspect of discussing Rolle's Theorem and the Mean Value Theorem in a calculus classroom is to help students understand the importance of the assumptions of these theorems.

In the first question of the related activity (Fig. 5), students are asked to identify those of the given intervals on which the function whose graph is shown satisfies the assumptions of Rolle's Theorem. The follow-up question is then deployed concerning the Mean Value Theorem (the function graph and the intervals remain the same).

Figure 5. Mean Value Theorem - sample question

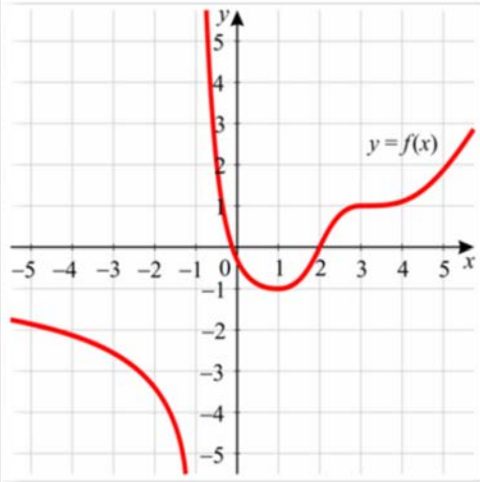


Activity: Curve Sketching

Similarly to the activity on visualizing limits, this activity contains two groups of questions. The first two questions ask the student to indicate the sign of the first and second derivative on the given intervals (Fig. 6). These are followed by three questions in which students are asked to sketch a graph of a function satisfying the given conditions (Fig. 7).

Figure 6. Curve Sketching - sample question involving "reading" a graph

many choice question



Which of the following are true statements for the function f whose graph is shown?

A. $f'(x) > 0$ if $x < -1$

B. $f''(x) > 0$ if $x < -1$

C. $f'(x) > 0$ if $-1 < x < 1$

D. $f''(x) > 0$ if $-1 < x < 1$

E. $f'(x) > 0$ if $1 < x < 2$

F. $f''(x) > 0$ if $1 < x < 2$

G. $f'(x) > 0$ if $2 < x < 3$

H. $f''(x) > 0$ if $2 < x < 3$

I. $f'(x) > 0$ if $3 < x$

J. $f''(x) > 0$ if $3 < x$

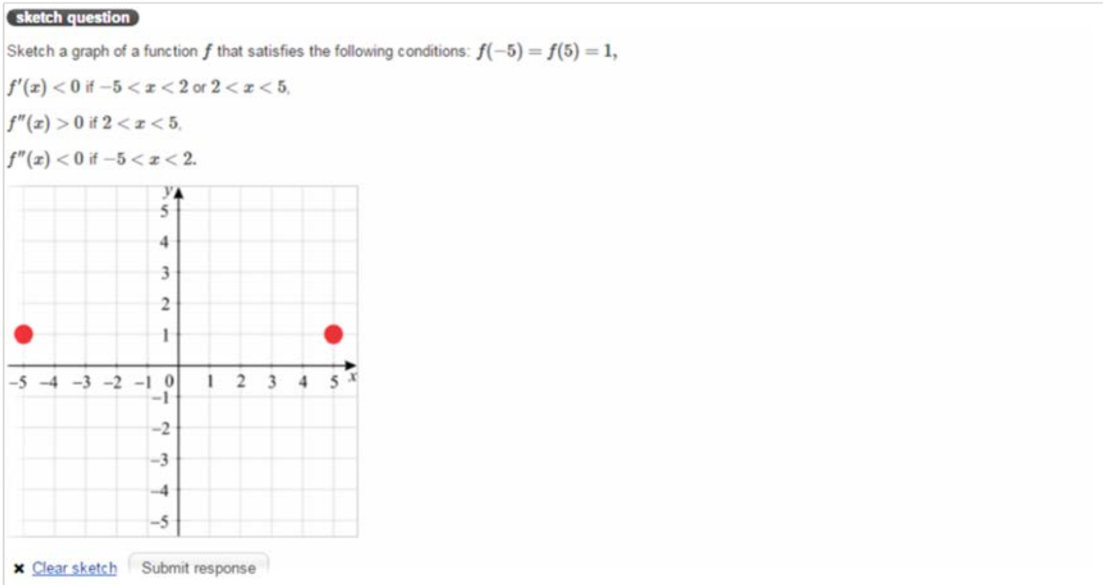
You may select zero, one, or many answers.

Submit response

After collecting student responses for each of the graphing questions, I would conduct a discussion of all the graphs submitted by the students. In particular, for the incorrect graphs, it is important to highlight the reasons why they fail to meet the required conditions.

Having conducted curve sketching activities of this type in a number of my Calculus I sections, I noticed a marked improvement in the quality of graphs students produced on their exams. All evidence suggests that this improvement can be attributed to the related Learning Catalytics activities. The platform's ability to support student-created graphs as answers makes it superior to physical clickers in this regard.

Figure 7. Curve Sketching - sample question involving composing a graph



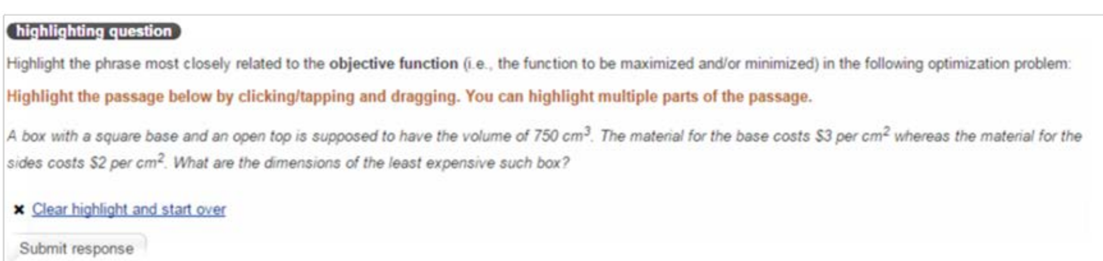
Activity: Optimization

Just like related rates, this topic involves word problems, which many calculus students find difficult.

The first question (Fig. 8) in the optimization activity asks the student to highlight the phrase most closely related to the objective function in the given optimization problem. The rationale for starting the activity with this question was that correctly identifying the objective function is critical to successfully solving an optimization problem, but it's also something that students often miss.

The second question in this activity asks the student to specify the expression corresponding to the objective function in the same optimization problem.

Figure 8. Optimization - sample question

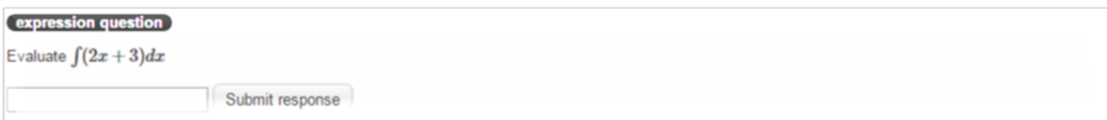


Activity: Antidifferentiation

This activity is typically conducted shortly after the basic antidifferentiation rules have been introduced. It is composed of five questions asking the students to evaluate the given indefinite integrals. In addition to the first question shown in Fig. 9 we also ask students to evaluate $\int x(9x-4)\,dx$, $\int \frac{1}{\sin^2 x}\,dx$, $\int \frac{x+1}{x^2}\,dx$, and $\int \sqrt[4]{x^7}\,dx$. Some of these integrals are intended to create a "teachable moment" in the classroom (e.g., some students may invent a nonexistent "quotient rule" for integration - when reviewing the results, these kinds of mistakes should be emphasized).

After the conclusion of this Learning Catalytics activity, I distribute a solution key which contains detailed step-by-step solutions for all these problems.

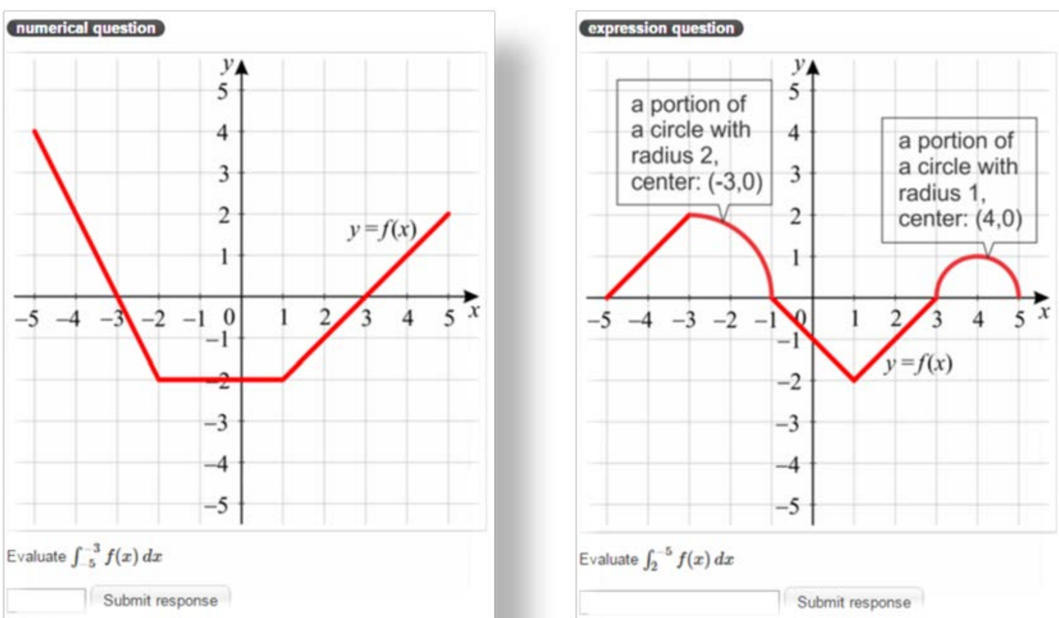
Figure 9. Antidifferentiation - sample question



Activity: Definite Integrals and Areas

After the definite integral is introduced as a limit of Riemann sums, it is important to investigate the relationship between that notion and the area between the graph of a function and the x-axis.

Figure 10. Definite Integrals and Areas - two sample questions



The activity consists of six questions (two of which are shown in Fig. 10), where the student is asked to deduce the value of the definite integral from the corresponding graph.

Activity: Integration by Substitution

This activity begins with six questions asking the student to specify the most helpful substitution for evaluating the given indefinite integral. In addition to the question shown in Fig. 11, questions also involve the following integrals: $\int e^x \sin e^x \; dx$, $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$, $\int (x^2+2)(x^3+6x)^5 dx$, $\int \frac{\sin x}{\sqrt{\cos x}} dx$, and $\int \frac{x+e^{2x}}{x^2+e^{2x}} dx$.

Figure 11. Integration by Substitution - sample question

After each question, we discuss student responses, making sure that the typical mistakes are addressed and all student questions are answered. Hopefully, at this time, we are approaching a consensus on what the best substitution really is for each of those problems.

Once that part is concluded, I distribute a paper worksheet, which lists the same six integrals along with the best substitution we agreed on using earlier. Students are then asked to actually perform all of the integration steps in detail. I design the handout to be single-sided and ask each student to sign their name on the back side of the worksheet. After, say, 20 minutes, I collect their work and I display some of the student's solutions to the entire class using the document camera. This includes both solutions that contain errors as well as correct ones. (I do not actually grade these for correctness, just give students some completion credit. I then return these handouts to the students.)

Activity: Techniques of Integration

The final activity mentioned in this article would be conducted early in Calculus II after students have learned integration by parts and trigonometric integrals (but before discussing trigonometric substitutions or integration by partial fractions). It is made up of eleven questions asking the student to identify the best initial step when evaluating the given indefinite integral. In addition to the three questions shown in Fig. 12, others involve the following integrals: $\int x^2 \sin x \; dx$, $\int x \cos x^2 \; dx$, $\int \sin^5 x \cos^5 x \; dx$, $\int \sin^2 x \cos^2 x \; dx$, $\int \sec^3 x \tan^3 x \; dx$, $\int \sec^3 x \; dx$, $\int \sec^4 x \tan^4 x \; dx$, and $\int \frac{\tan x}{\sec x} dx$.

Figure 12. Techniques of Integration - three sample questions

multiple choice question

Which of the following is the best initial step when evaluating $\int \frac{\ln x}{x^3}\,dx$?

A. Substitution: $u = \ln x$

B. Substitution: $u = \frac{1}{x}$

C. Integration by parts: $u = \ln x,\ dv = \frac{1}{x^3}\,dx$

D. Integration by parts: $u = \frac{1}{x^3},\ dv = \ln x\,dx$

multiple choice question

Which of the following is the best initial step when evaluating $\int \frac{(\ln x)^4}{x}\,dx$?

A. Substitution: $u = \ln x$

B. Substitution: $u = \frac{1}{x}$

C. Integration by parts: $u = (\ln x)^4,\ dv = \frac{1}{x}\,dx$

D. Integration by parts: $u = \frac{1}{x},\ dv = (\ln x)^4 dx$

multiple choice question

Which of the following is the best initial step when evaluating $\int \cos^7 x\,dx$?

A. Substitution: $u = \sin x$

B. Substitution: $u = \cos x$

C. Applying power-reducing formulas

D. Integration by parts

Conclusions

The author developed calculus activities based on Learning Catalytics questions, but also involving paper handouts and worksheets. These activities focus on some of the areas of single-variable calculus that students find challenging, e.g., techniques of integration, curve sketching, and optimization. Early results indicate improved student learning and better performance with respect to some of these critically important topics (in particular, curve sketching).