DEVELOPING DIGITAL LITERACY IN THE CALCULUS SEQUENCE

Dr. Jeremy Brazas
Georgia State University
Department of Mathematics and Statistics
30 Pryor Street
Atlanta, GA 30303
jbrazas@gsu.edu

Dr. Todd Abel
Appalachian State University
Department of Mathematical Sciences
342 Walker Hall, 121 Bodenheimer Dr
Boone, NC 28608
abelta@appstate.edu

Introduction

The ever-increasing role of technology in everyday life and work prompts questions about the skills and understandings needed for effective use of that technology. The tools available for obtaining, managing, synthesizing, analyzing, and communicating information are constantly changing and adapting as the range of information and communication technology (ICT) tools grows ever greater. As technological capabilities rapidly change, those skills and understandings necessarily shift in response. Competence and knowledge with technological tools is described by and named with a variety of terms, the most prevalent of which is *digital literacy*.

This paper offers the initial results of a study designed to begin identifying characteristics of digital literacy in mathematics. As part of a Digital Literacy Initiative within the Honors Program at Georgia State University, students were provided with tablet computers and digital exercises were integrated into the Honors Calculus three-course sequence. This permitted an opportunity to investigate and understand how students chose to use ICT tools in order to solve mathematical problems and to analyze components of digital literacy for mathematics.

The specific tasks developed for and integrated into the class will be discussed below with specific illustrative examples highlighted. The aspects of mathematical digital literacy illuminated by student work will be outlined, with some initial conclusions about the nature of digital literacy in mathematics.

Digital Literacy

The term *digital literacy* was first defined by Gilster (1997) as "the ability to understand and use information in multiple formats from a wide range of sources when it is presented via computers" (p. 1). Use of the term has since expanded to often serve as an umbrella term. It is variously used to refer to procedural competence with information and communication technology (ICT) tools, the set of cognitive skills guiding effective usage of such tools, social and communication skills necessary for the integration of ICT tools into work and life, and combinations of these and other competencies (Avriam & Eshet-Alkalai, 2006; Goodfellow, 2011). Due to this proliferation in applications of the term, and the constantly changing and steadily expanding realm of ICT, there is general agreement that digital literacy involves interaction and integration of a number of dimensions, or various literacies. Digital literacy itself is integrated into a wide range of social, educational, and technological questions. The use of the word "digital" is itself far from universal, with some sources variously referring to media literacy, digital and media literacy, ICT literacy, or related specialized terms. Though many authors choose these terms in specific ways and often for specific reasons, "digital literacy" has come to envelope most, if not all, meanings. Thus, in this paper, we will use the term "digital literacy" to encompass the wide variety of terms used in order that we might draw on the valuable contributions of multiple approaches.

Emphases on digital literacy in education have grown from the recognition that traditional modes of instruction developed to prepare students for a context and culture that has dramatically shifted. The classroom should be responsive, rather than resistant, to the changing proficiency needs of students (Gutierrez & Tyner, 2012). As in other areas, the use of ICT tools is increasingly important to the work of mathematics. At the turn of the century, *Principles and Standards for School Mathematics* (NCTM, 2000) included a Technology Principle, stating that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). Though this was a major codification of the importance of technology for teaching and learning mathematics, it was hardly the first emphasis of that principle. More recently, Li & Ma's (2010) meta-analysis of the impact of computer technology on student learning showed an overall positive effect, particularly when combined with a constructivist teaching approach. ICT tools are also increasingly integrated into the work of research mathematics, not limited to computational mathematics. They are becoming increasingly valuable in pure mathematics and mathematical foundations, as well

(Monroe, 2014). ICT tools now have sufficient capabilities to not simply enhance and extend mathematical research but to shape the mathematics being done.

Despite the increased emphasis on and integration of ICT tools within mathematics, *mathematical digital literacy* is not well-defined. The competencies with ICT tools specific to mathematics would be of particular concern to educators, curriculum developers, and many other stakeholders within the field.

One approach to discussing digital literacy is to characterize it by identifying its constituent elements. Educational Testing Service (ETS) outlined seven proficiencies with ICT tools in order to better characterize their use:

1. Define - the ability to understand and articulate the scope of an information problem
2. Access - the ability to collect and/or retrieve information in digital environments
3. Manage - the ability to organize information
4. Integrate - the ability to interpret and represent information in order to synthesize, summarize, or compare/contrast.
5. Evaluate - the ability to judge whether information satisfies needs (determine authority, relevance, etc)
6. Create - the ability to adapt, apply, design or construct information in digital environments
7. Communicate - the ability to disseminate information properly in its context (ETC, 2003)

Though designed to be general characteristics, these proficiencies served as a starting point when considering digital literacy in mathematics. Each proficiency could reasonably be applied to work within mathematics. Some examples of how these proficiencies manifest themselves within mathematics will be illustrated below.

Context and Instructional Design

The work described here took place during a single semester in three different courses: Honors Calculus I (enrollment: 22), Honors Calculus II (enrollment: 22), and Honors Calculus III (enrollment: 18). These courses covered the traditional material of the calculus sequences in a "late transcendentals" ordering. Calculus I covers limits, differentiation and its applications, and basic integral calculus. Calculus II covers exponential and logarithmic functions, integration techniques, sequences and infinite series, and polar coordinates. Calculus III covers vector calculus, calculus of multivariate functions, optimization of multivariate functions, and double and triple integrals.

Many students enrolled in the study courses had little to no experience with computational technology. While an important part of our approach to digital literacy was to allow students to freely choose the digital tools they could use, it was necessary to introduce students to some suitable options during the semester. The primary digital tools introduced to the students by the instructor were Wolfram Mathematica and the online Desmos graphing calculator.

Mishra and Koehler (2006) introduced the term technological pedagogical content knowledge (TPACK) as a framework for discussing and describing teachers' knowledge on integrating ICT tools into instruction. TPACK encapsulates the interplay between knowledge of technology, pedagogy, and content. Howland, Jonassen, and Marra (2012) identified five dimensions of learning involving ICT tools: that it be active, constructive, authentic, intentional, and cooperative. These dimensions served as a general guide for designing the integration of digital tools into assignments and exams in each course.

Class Activities

All digital content integrated into these three courses was supplementary to the traditional written course content. In each course, students still completed a graded, weekly, written homework or quiz, three semester written exams, and a written final exam. Apart from the written work, students used ICT tools to complete in-class and take-home "Digital Assignments" as well as three in-class "Digital Exams." The structure of Calculus III differed from Calculus I and II in that Mathematica was the only tool emphasized by the instructor in Calculus III and, due to time constraints, all digital assignments and exams were take-home assignments.

Digital Assignments

In relation to the rest of the course, digital assignments were primarily meant to provide students with base-line experience using digital tools to solve mathematical problems. Throughout the semester, digital assignments were typically assigned as handouts or pdf files related to the content that had recently been discussed in lecture. A set of instructions led students through the use of Mathematica or Desmos (depending on the content) to visualize and solve a set of problems. Often the instructions would require students to choose parameters to create their own individualized problems. For problems that required more advanced coding, students would be provided with a template file to edit. During in-class digital assignments, the instructor would typically provide demonstrations and move around the classroom to help students with syntax and interpretation. Students submitted their work digitally as either a Mathematica Notebook file or a link to a Desmos graph.

Digital assignments requiring students to access a specific ICT tool were designed to:

1. Give students the opportunity to actively learn to use new computational software.
2. Teach students to create and interpret their own digitally generated visualizations.
3. Expose students to sophisticated problems and computations that would not appear in a traditional calculus course.
4. Cooperatively learn syntax and debug code.
5. Encourage students to use these and/or other digital tools to support and verify work even when not required.

Though an indirect consequence of using these ICT tools may have been an increase in student understanding of content, this was not the primary goal of their incorporation into the course. The focus of digital assignments was on gaining literacy with digital tools and accessing new problems and information via their use.

Digital Exams

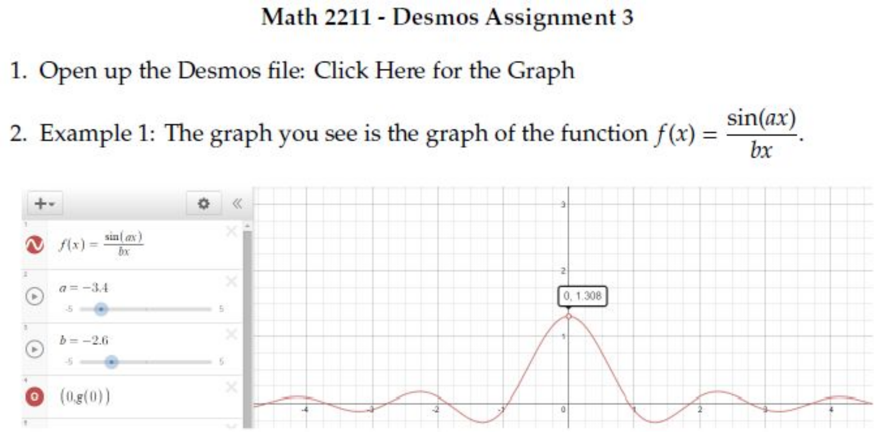
In the class period following a written exam, students completed a digital exam. The problems on the digital exam often required students to create digitally generated images/animations and to make computations that could not be completed by hand in a reasonable amount of time. These exams were designed to measure digital literacy in individual students by being “open resource” exams in the sense that students were allowed use any digital resource at their disposal. Some students also chose to use tools they were comfortable with not discussed by the instructor, e.g. Wolfram Alpha, Symbolab, eMathHelp, graphing calculators, or whiteboard software to write work done by hand. Students were only restricted in that they were not allowed to collaborate with other students or to use online mathematics question/answer forums during the exam.

Engaging with problems on the digital exams required several proficiencies with digital tools, such as identifying how ICT tools might be useful, choosing an appropriate tool, translating between mathematical and syntactical notations, interpreting digital results and communication of solutions. These proficiencies are indicative of different components of digital literacy. These will be explored in further detail after considering some specific examples.

Some specific cases

An Example of a Digital Assignment

Early in the Calculus I course, students were required to complete a digital "Desmos" assignment asking them to explore limits involving trigonometric functions. To begin the assignment, students clicked on a link embedded in the assignment pdf file for a pre-designed Desmos graph. See Figure 1.

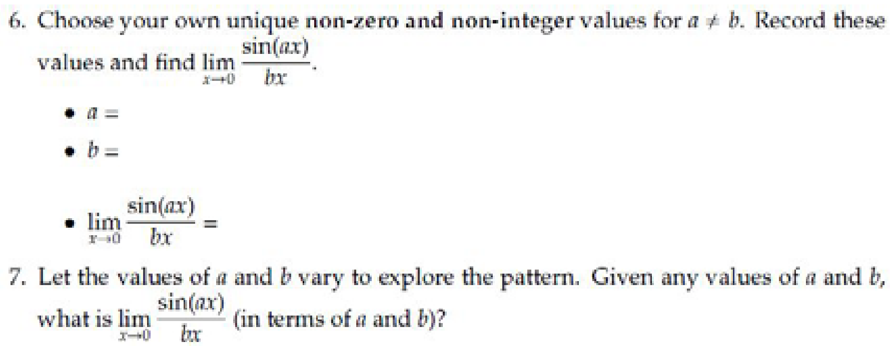

Math 2211 - Desmos Assignment 3

1. Open up the Desmos file: Click Here for the Graph

2. Example 1: The graph you see is the graph of the function $f(x) = \frac{\sin(ax)}{bx}$.

Figure 1: Start of digital assignment file.

The first portion of the assignment asked students to consider the limit $\lim_{x\to 0}\frac{sin(ax)}{bx}$ involving the function appearing in the Desmos file. Students used the Desmos "sliders" to evaluate the limit for various values of *a* and *b*. Eventually, students choose their own unique values to verify the pattern. The assignment included a similar exploration for 3 other common trigonometric limits: $\lim_{x\to 0}\frac{ax}{sin(bx)}$, $\lim_{x\to 0} ax\ cot(x)$, and $\lim_{x\to 0}\frac{1-cos(ax)}{bx}$.


6. Choose your own unique non-zero and non-integer values for $a \neq b$. Record these values and find $\lim_{x\to 0}\frac{\sin(ax)}{bx}$.
   - $a =$
   - $b =$
   - $\lim_{x\to 0}\frac{\sin(ax)}{bx} =$

7. Let the values of a and b vary to explore the pattern. Given any values of a and b, what is $\lim_{x\to 0}\frac{\sin(ax)}{bx}$ (in terms of a and b)?

Figure 2: Students searched for patterns in Desmos using sliders

Apart from the digital assignment, students were provided with a rigorous proof of the first computation. As with many digital assignments, this one provided exploration, experience, and visualizations that would later support formal computations, theorems, and proofs. This particular assignment also provided students with early experience using

Desmos. During the assignment, the instructor reminded students that the use of Desmos itself as a tool was only the secondary purpose of the assignment. The primary purpose of this assignment was to encourage students to begin using technology when presented with an apparently intractable problem.

<u>An Example of a Digital Exam</u>

Digital Exams were designed to assess student's ability to apply ICT tools to challenging problems that may not be executed by hand within the time constraints of the class. These exams were "open note" in the sense that students could use the internet, all course notes, all course files, and links previously used, etc. All submissions were digital, even if they included hand-written work completed on a tablet. Justification of submitted solutions typically included proof of correct use of a reliable ICT tool, e.g. Mathematica notebook (.nb) file, link to a Desmos file, link to Wolfram Alpha computation, screenshot, etc. In Figure 3, two Calculus II Digital Exam problems are shown that include problems on Taylor series which are impractical to solve by hand. To illustrate the ways in which students used digital tools to solve problems and communicate their solutions, samples of student work for the above these two problems are included in Figure 4.

3. (5 pts) $g(x) = \int \sin(x^2)\,dx$ is not an elementary function but still has a convergent Taylor series. Assume $g(0) = 0$. Find the 15th Taylor polynomial centered at $x = 0$ of this funciton. Use this polynomial to approximate $g(3/2)$ (even though we don't have a formula for g to "plug in").

4. (5 pts) Create an animation that illustrates the convergence of the power series

$$\arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$$

Figure 3: Two Calculus II Digital Exam Problems

Student 1 submission of #3	Student 2 submission of #3

In[1]:= `(* Problem 3 *)`
`p15[x_] = Integrate[Normal[Series[Sin[x^2], {x, 0, 15}]], x]`
`N[p15[3/2]]`
Out[1]= $\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75600}$
Out[2]= 0.777928
	`(*3*)`
	`Clear[f, a];`
	`f[x_] := Sin[x^2]`
	`a = 0; (*center*)`
	`Integrate[Normal[Series[f[x], {x, a, 15}]], x]`
	`p8[x_] = f[a] + Integrate[Normal[Series[f[x], {x, a, 15}]], x]`
	`(*we must add the constant term which is f(center)*)`
	`N[p8[3/2]] (*approximation for g(1)*)`
	$\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75600}$
	$\frac{x^3}{3}-\frac{x^7}{42}+\frac{x^{11}}{1320}-\frac{x^{15}}{75600}$
	0.777928

Figure 4: Samples of student work on Calculus II Digital Exam problem #3

Both students chose Mathematica for this particular problem. This is not surprising given the nature of the problem and the tools with which most students were comfortable. What is interesting is the difference in their processes. Student 1 submitted a concise and correct solution. Student 2 also submitted a correct solution, however, the student copied previously used code provided by the instructor to find the 8th Taylor polynomial of a given function. Note that Student 2 did not bother to change p8 to p15 even though the problem is to find the 15th Taylor polynomial. A side-by-side comparison suggests that Student 1's solution exhibited greater digital literacy since they were comfortable enough with the content and syntax to simplify their code whereas Student 2 attempted to simply mimic a previous application of digital tools.

Figure 5 shows two student submissions for Problem #4. Note that Student 1 chose to use Desmos to create their animation for the convergence of the Taylor series of the arctangent function. This student completed many of the digital exam problems in Mathematica but decided they felt more comfortable using Desmos on this particular problem. The ability to weigh options among different tools and make choices based on their relative strength is indicative of at least one component of digital literacy. Unfortunately, the solution is incorrect because the student forgot to include the alternating term in the series and the submitted animation does not illustrate any convergence of curves. All students had experience viewing and creating such convergence animations. Student 1 was not able to correctly identify that their digital work was incorrect despite the difference to similar experiences - translating mathematics between digital and non-digital environments was problematic. This example illustrates that students may exhibit some aspects of digital literacy while showing less literacy in other aspects.

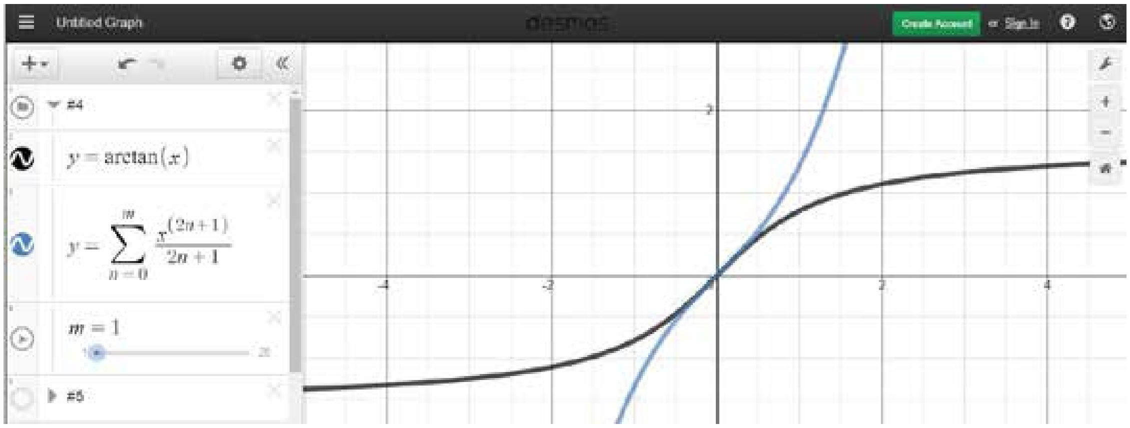


Figure 5: Student 1 submission of Problem #4

Student 2, whose solution appears in Figure 6, felt more comfortable completing Problem #4 in Mathematica. Upon evaluating this cells, the viewer sees the approximating polynomials disappear from the plot after step n=8. This failure of quality that does not arise in Desmos. Even though Student 2 chose an ICT tool that they could implement effectively, they were not necessarily able to recognize which would provide the clearest results.

```
In[15]:= Number 4;
         Clear[f, a, n];
         a = 0;
         f[x_] := ArcTan[x]
         Manipulate[Show[Plot[f[x], {x, -10, 10}],
           Plot[Evaluate[Normal[Series[ArcTan[x], {x, a, n}]]], {x, -10, 10},
            PlotStyle → Red]], {n, 0, 20, 1}]
```

Out[19]=

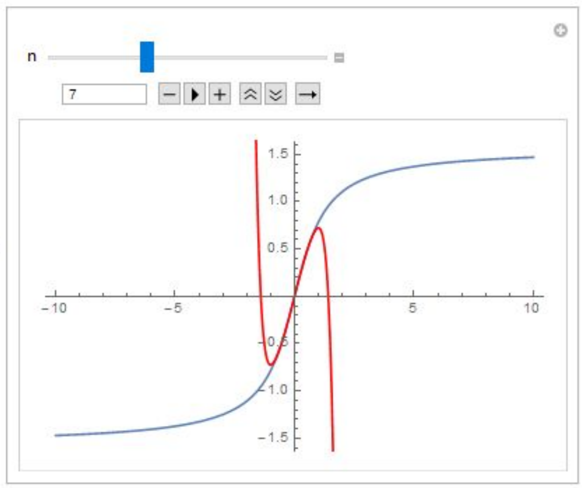


Figure 6: Student 2 submission of Problem #4

Impacting Student Digital Literacy

At the conclusion of the semester, all students were invited to complete an online survey designed to explore their experiences with digital literacy. The 35 respondents reported increased comfort with digital tools and computational technology and positive attitudes toward their integration. When asked to rate their comfort level with digital tools at the beginning and end of the semester on a scale of 1 (not comfortable at all) to 10 (very comfortable), every student reported the same or greater confidence, as shown in Figure 7, a scatterplot of student ratings of their comfort level at the beginning and end of the class.

Figure 8 shows the frequency of each response for the beginning and end of the class. These two graphs illustrate the reported increase in comfort level of the class as a whole.

At the beginning of the class, 11 students (31.4%) reported a comfort level of 8 or higher. At the end of the class, 25 students (71.4%) reported a comfort level of 8 or higher. The mean change in self-reported comfort level was 1.6 with a median change of 1.

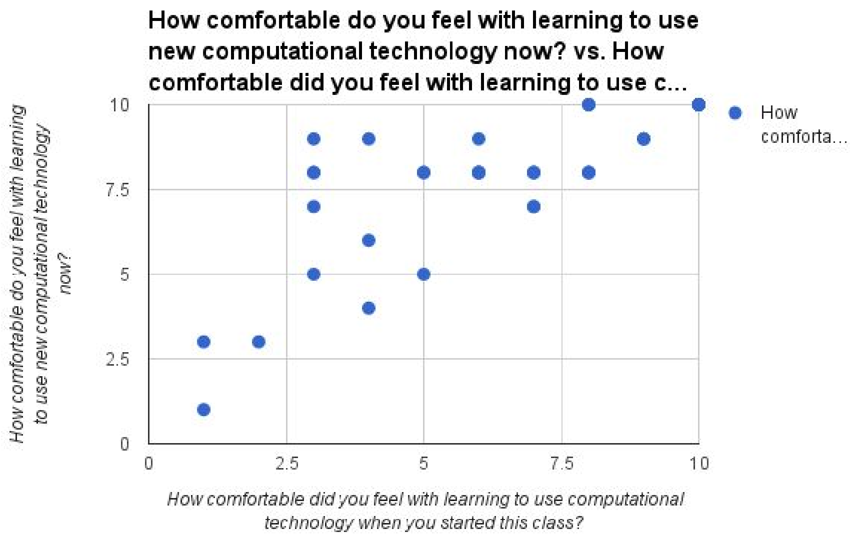


Figure 7: Scatterplot of student self-reported comfort level with computational technology at the beginning and end of the class.

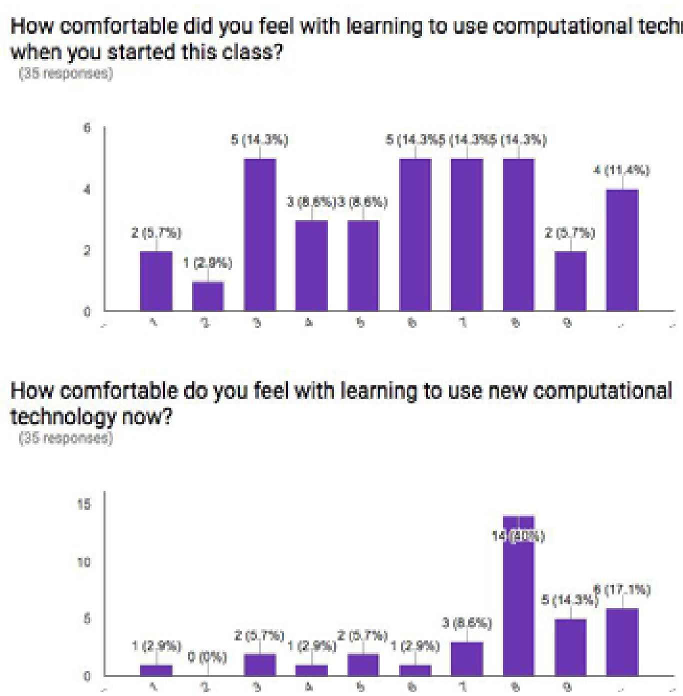


Figure 8: Frequency bar graphs showing students' self-reported comfort level with computational technology at the beginning and end of the class.

All students reported using some technology outside of the digital assignments and course requirements (See Figure 9). Unsurprisingly, Mathematica and Desmos were the most popular. Wolfram Alpha and graphing calculators were also utilized by a significant portion of students.

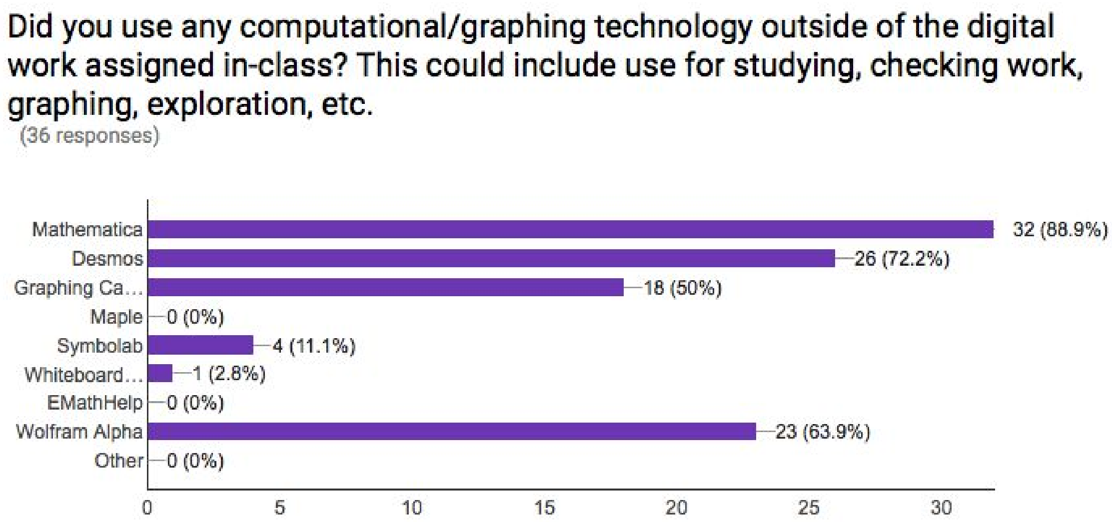


Figure 9: Digital tools used by students outside of work assigned in class

When students were asked to describe how they used digital tools in the class, the most common response was for visualization. In particular, students noted the value of Desmos for graphing equations and of Mathematica for graphing three-dimensional solids. They also valued the ability to quickly perform calculations and to check answers, though many noted that learning the syntax for Mathematica was difficult, at least initially.

Digital literacy includes not just facility with digital tools, but also their use in order to organize knowledge, and to explore and organize concepts. The degree to which incorporating digital tools impacted student conceptual understanding is unclear without the presence of a comparison group. However, it appears that students *felt* digital tools were valuable to their understanding, with one student stating, “I felt like the benefits gained from using technology in [con]junction with math, offset the uneasiness at the beginning of the semester and opened me up to different ways of approaching math.” Another student indicated that the nature of mathematical work particular to digital environments contributed to understanding:

> Working through Mathematica problems helped to figure out the inner workings of how the math actually works. If one code was mistyped then

you were forced to go back and see why it was wrong and how it can be corrected.

Another student commented that using tools to do operations learned in previous courses was "helpful, as it allowed me to focus on concepts I was learning in this specific class."

Thus, these preliminary results indicate that integration of digital assignments and digital exams increased students' comfort with using ICT tools and could also have contributed to greater understanding of mathematical concepts.

Toward an Understanding of Mathematical Digital Literacy

As noted in the introduction, digital literacy for mathematics has not been fully characterized. Student work and feedback on digital assignments and exams described above do, however, provide some initial information toward understanding mathematical digital literacy. Students used digital tools in a variety of ways, some of which students engaged in concurrently on a particular problem. It should also be noted that not all of these uses were exhibited on any particular problem.

1. *Determine which tool should be used to solve a given problem*. Outside of some digital assignments that prescribed certain approaches or required work with certain tools, students were not required to use any given tool. The first step, then, was to determine how digital tools might contribute to their understanding or solution of a problem, then decide which tool best meets those needs. That is, one component of digital literacy is determining when and how a certain tool might be useful in order to choose among various options.
2. *Learn and apply syntax of technological tool (sometimes based on template)*. This was a particular issue with learning the programming language of Mathematica, but all ICT tools have a particular syntax that takes time to learn. Desmos has certain input protocol, as do graphing calculators (depending on the operating system, these can differ even within a given manufacturer's products).
3. *Decide how to translate mathematics into input in chosen tool*. Once students understood the syntax and notation of any given ICT tool, they had to determine how to translate mathematical concepts and notation into that syntax and notation. Translation in the other direction was also a key use.
4. *Interpret technological results to find a proposed solution.* Students often needed to interpret digital information within a mathematical context and communicate it using traditional means. For example, if they created a graph, they had to

understand how to use that graph to draw mathematical conclusions; or, they had to translate digital output to draw mathematical conclusions.

5. *Use technology to justify that a proposed solution is correct*. One frequently-cited use of ICT tools was to "check answers". Students had to understand the relationship between the solution and solution strategy and the capabilities and output of whatever tool they chose to use.
6. *Display and submit answer and supporting work digitally.* In addition to using digital tools to explore mathematics, and produce and check solutions, students also used digital tools to communicate solutions and solution strategies.

Though there was much variation in the particular ways students engaged in these activities with ICT tools, they fell into these six main categories of use. Such a categorization permits some initial conjectures about components of mathematical digital literacy.

	Components of Mathematical Digital Literacy
Component 1	Ability to assess and choose tools based on potential use along multiple proficiencies
Component 2	Translation between digital and mathematical contexts ● Navigating multiple representations (notational, graphical, syntactical) ● Digital and mathematical trouble-shooting
Component 3	Using ICT tools to enhance or complement (rather than replace) mathematical understanding
Component 4	Using ICT tools to communicate mathematics and mathematical understanding

Table 1: Components of mathematical digital literacy

These components are related to the seven proficiencies with ICT tools described by ETS (2003), but are specific to mathematics in a way that the ETS proficiencies were not designed to be. Some components are directly analogous to ETS proficiencies. For example, each emphasizes using ICT tools to communicate and to create. However, mathematical digital literacy must necessarily also include some components that are not present in a description of proficiencies with particular tools. For example, component 1 is related to the "define" proficiency, which describes the need to understand the scope of a problem. However, component 1 also considers the need to weigh options and choose among various tools. Component 2 might be construed as related to some of the ETS proficiencies, but is not directly analogous to any of them. It is particular to the work of mathematics.

Conclusion

Work remains to be done to fully characterize digital literacy for mathematics. However, this preliminary study, placed within the context of existing research, indicates that a focus on learning and doing mathematics within digital environments increases student facility and comfort with ICT tools. Moreover, there is some indication that these particular assignments helped increase the conceptual understanding, or at least confidence in conceptual understanding, for at least some students. Finally, the ways students utilized digital tools provides some initial indications of important components of digital literacy, including the ability to assess and choose tools, to translate among environments, to enhance understanding, and to communicate.

As access to digital tools and the tools themselves continue to evolve, the ways we might engage mathematically within them will continue to evolve, as well. A fuller understanding of digital literacy within mathematics can help us understand how new technologies can impact classrooms and students.

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