COMMERCIAL VERSUS FREE ON-LINE CAS SYSTEMS: COMPARE AND CONTRAST

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Abstract
In this paper, we will explore the advantages and the shortcomings of commercial CAS systems as compared to the free online software. We will solve a variety of pre- and post-calculus problems using Maple and compare and contrast the syntax, ease of use, and results with the same problem solved using WolframAlpha.

Keywords: WolframAlpha, Maple, math Web resources

Introduction

Commercial CAS systems, such as Maple and Mathematica, have been in the market since mid-1980s and during this time have gone through major improvements. More recently, several free on-line sources have appeared which are powerful alternatives to the commercial CAS. Sage and WolframAlpha are well known free on-line resources. Sage http://www.sagemath.com is a mathematics software developed by William Stein from the University of Washington in 2005 as an open source alternative to the more traditional CAS. Most of the code is written in Python and it is constructed using over 100 available open source packages ranging from basic mathematics to more advanced topics such as number theory and abstract algebra. *WolframAlpha* http://www.wolframalpha.com is a popular application which was launched several years ago by Steven Wolfram (who wrote the popular software Mathematica). WolframAlpha is an all-purpose computational knowledge engine which uses built-in knowledge created by experts to compute on the fly responses to a specific question. One might think of it as a dynamic Wikipedia for computing and statistics. More recently a more complete version of this software called WolframAlpha|PRO has become available for subscribers who pay a nominal fee. For mathematics the distinguishing feature of WolframAlpha is its natural language interface. As we will demonstrate in the following examples, the syntax used with WolframAlpha is, compared to Maple, very forgiving.

Computer algebra systems are powerful investigative tools to help develop students' understanding of topics in typical undergraduate mathematics curricula [1],[2]. In this paper we will solve a variety of pre and post-calculus problems using Maple (representing commercial CAS) and compare the results with the same problem solved using WolframAlpha (representing the free, on-line CAS).We will start our paper by introducing a list containing the most common CAS commands and compare the syntax and ease of use between Maple and WolframAlpha. We will then demonstrate the commands used to perform basic computations and symbolic manipulations for sample problems focused on topics from pre and post-calculus courses. We will conclude our talk by illustrating some other features of WolframAlpha (WA). Our paper is intended for mathematics and statistics educators with interest in using CAS and web resources in their classroom teaching.

Comparing Maple and Wolfram alpha

In the following table we compare Maple and WA in ten categories. Please note that the comparisons are made from the point of view of a novice student user. Also, we must emphasise that Maple is a software, whereas WA is simply a platform which utilizes another major CAS, namely Mathematica. As the table indicates, Maple is clearly more suitable for multi-step tasks or projects beyond calculus which involve using specific packages. A Maple user has more control over parameters to produce more meaningful output. Furthermore, Maple as software can be loaded on any computer and does not require Internet connection. One of the main advantages of using WA is flexibility of the syntax. The rigid syntax in Maple and other CAS is a source of frustration for most of the first-time users. WA also frequently returns more information that what the user requested. In majority of cases this is considered a plus. For example when asked to solve an equation, WA returns all of the solutions (real and complex) and other useful information such as the graph of the function.

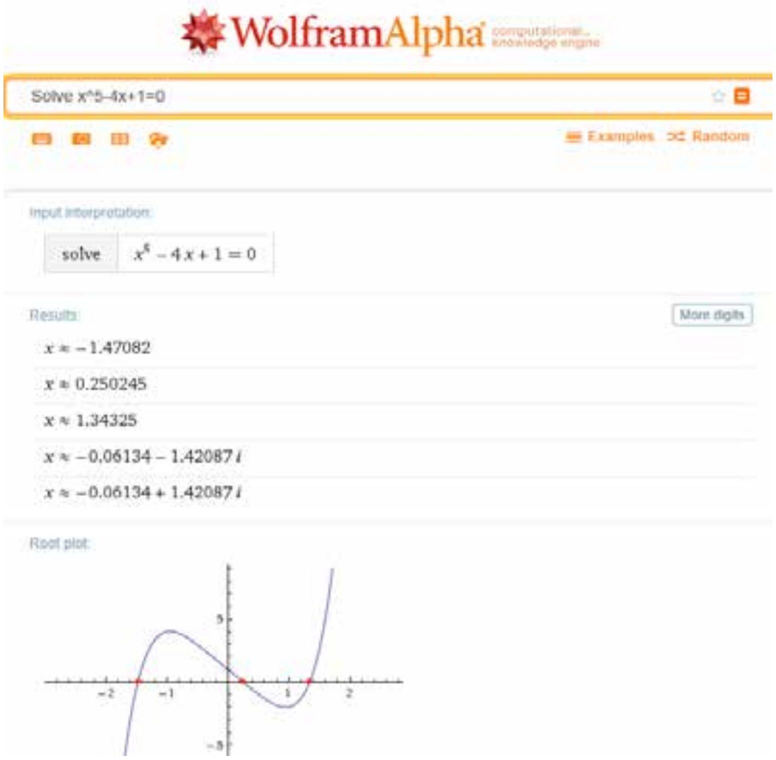
	Control	Needs Internet	Rigidity of syntax	Appearance	Information returned
WA		X	Relaxed	X	X
Maple	X		Rigid		
	Mult step computation	Through Calculus	Beyond Math/Stat	Targeted Packages	Applications Beyond Math
WA		X			X
Maple	X		X	X	X

Table 1: comparing Maple and WA in ten categories

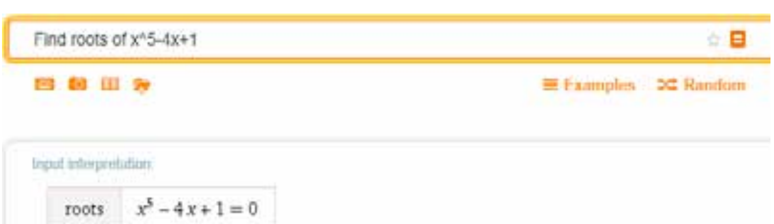
In the following sections, we will be presenting examples solved using Maple and WA to demonstrate some of the points summarized in table 1.

Examples from Algebra and Pre-Calculus

One of the strengths of WA is the ability to parse its input. Instead of looking for a rigid syntax WA looks for possible meanings behind your entry. This is often referred to as a natural language interface. The following example seeks zeros of a fifth degree polynomial. We will show several choices of syntax for solving this problem.



To demonstrate the flexibility of syntax in WA, we will solve the same problem using several other formats. WA interprets all of these forms as solving an equation and returns the same solutions:



Real solutions: Exact forms More digits

$x \approx -1.47082$

$x \approx 0.250245$

$x \approx 1.34325$

Complex solutions: Exact forms More digits

$x \approx -0.0613366 - 1.42087\,i$

$x \approx -0.0613366 + 1.42087\,i$

Solving the same problem in Maple:

> $solve(x^5 - 4\cdot x + 1, x)$

$$RootOf(\_Z^5 - 4\_Z + 1, index = 1), RootOf(\_Z^5 - 4\_Z + 1, index = 2), RootOf(\_Z^5 - 4\_Z + 1, index = 3), RootOf(\_Z^5 - 4\_Z + 1, index = 4), RootOf(\_Z^5 - 4\_Z + 1, index = 5)$$

> $evalf(\%)$

$$0.2502453409,\ 1.343246089,\ -0.06133661254 + 1.420873558\,I,\ -1.470818205,\ -0.06133661254 - 1.420873558\,I$$

Altering the syntax in Maple will not produce any roots. There are some commands in Maple which may produce partial solutions. For example, if we open the “student” package, we can use the “intercept” command to get one of the three real solutions:

> *# "intercept" command in the student package only returns one intercept*

> $with(student):$

> $intercept(y = x^5 - 4\cdot x + 1, y = 0)$

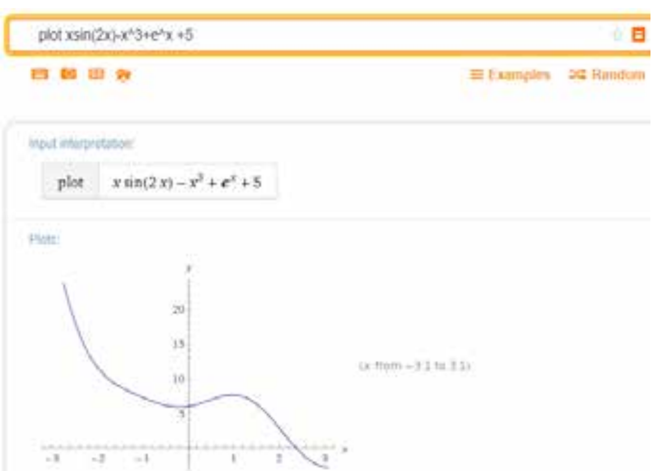
$$\{x = RootOf(\_Z^5 - 4\_Z + 1), y = 0\}$$

> $evalf(\%)$

$$\{x = 0.2502453409, y = 0.\}$$

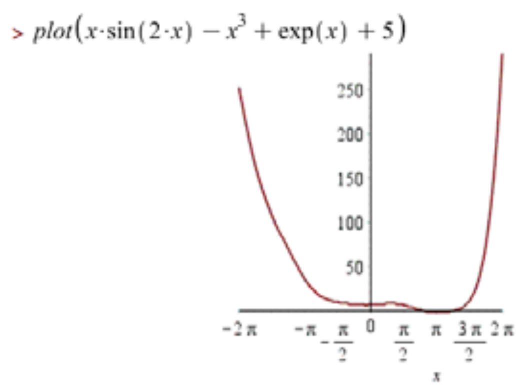
>

The following example shows the flexibility of WA syntax in graphing a function. For the sake of brevity, we will show only a portion of the output:

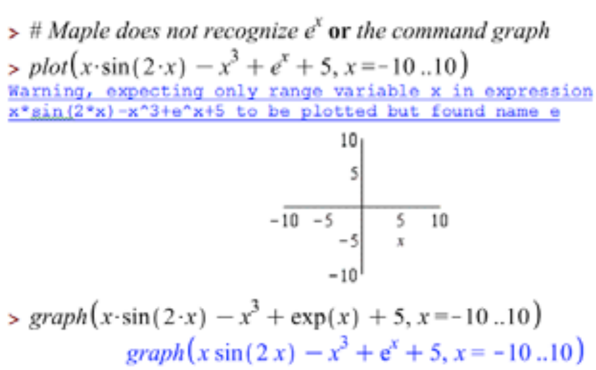


The same result is obtained using several different commands with WA interpretation:

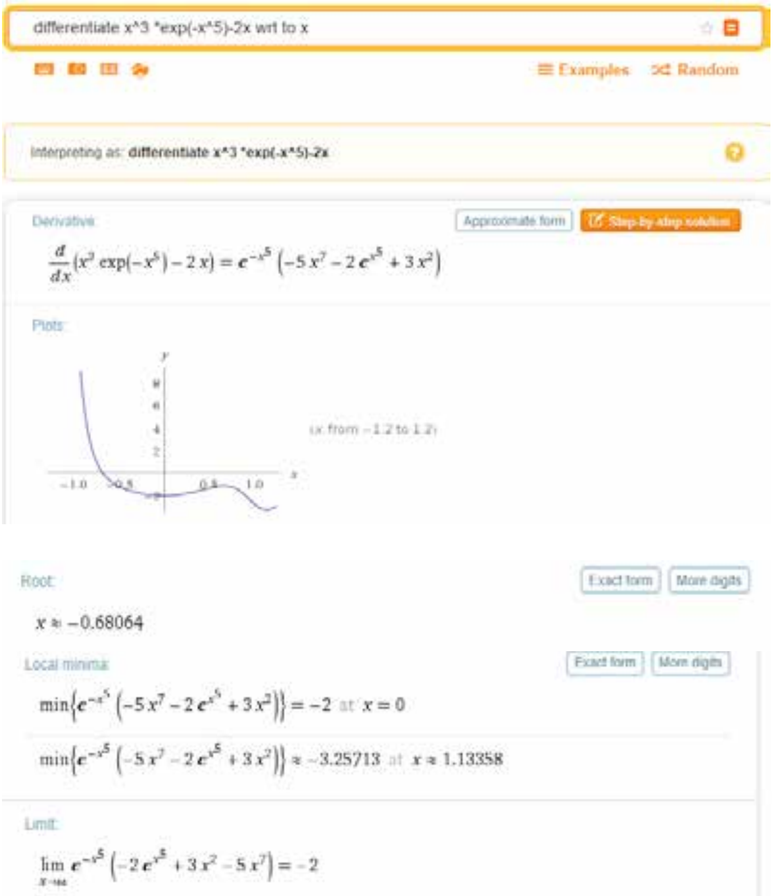
We can get the same graph using Maple:



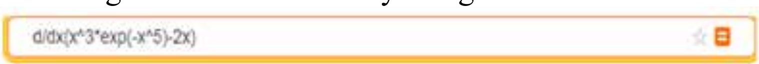
However note that the command extremely rigid. For example, most novice users will use e^x in place of exp(x) which results in an error message:



In the next example we asked WA to differentiate a function. Note that WA gives a lot more information than (only a portion of the output is presented here) we asked:



We will get the same results by using other forms of the same command:



There are several ways to find a derivative in Maple, however the syntax has to be followed exactly:

> $diff\left(x^3 \cdot \exp(-x^5) - 2 \cdot x, x\right)$

$$3\,x^2\,e^{-x^5} - 5\,x^7\,e^{-x^5} - 2$$

> $f := x \rightarrow x^3 \cdot \exp(-x^5) - 2 \cdot x$

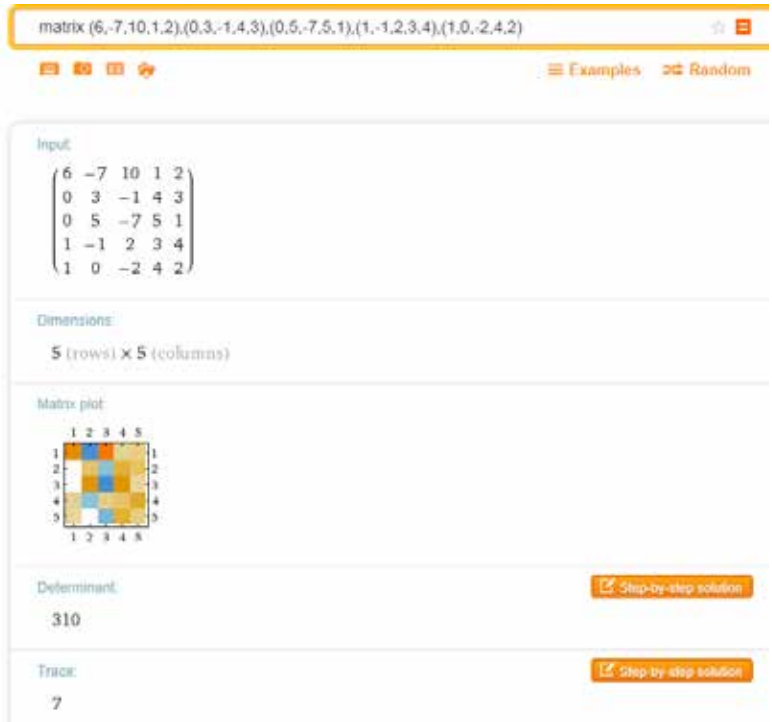
$$f := x \rightarrow x^3\,e^{-x^5} - 2\,x$$

> $\mathrm{D}(f)$

$$x \rightarrow 3\,x^2\,e^{-x^5} - 5\,x^7\,e^{-x^5} - 2$$

Post-Calculus Examples

One can use WA to find Taylor series, solve differential equations, perform matrix operations, and find eigenvalues and eigenvectors and most other tasks which a student encounters in post calculus classes. For the sake of brevity we will just do one example from matrix algebra. Note in the following example we simply define a 5 x 5 matrix and WA reads our mind! and gives us an avalanche of information about that matrix including the eigenvalues, eigenvectors, the inverse, condition number, etc.



We can get the same information using Maple. However, we need to open the linear algebra package and ask specifically for the output we desire.

```
> with(LinearAlgebra) :
> A := Matrix([[6,-7, 10, 1.0, 2], [0, 3,-1, 4, 3], [0, 5,
    -7, 5, 1], [1,-1, 2.0, 3, 4], [1, 0,-2, 4, 2]])
```

$$A := \begin{bmatrix} 6 & -7 & 10 & 1.0 & 2 \\ 0 & 3 & -1 & 4 & 3 \\ 0 & 5 & -7 & 5 & 1 \\ 1 & -1 & 2.0 & 3 & 4 \\ 1 & 0 & -2 & 4 & 2 \end{bmatrix}$$

```
> Determinant(A)
```

$$310.00$$

```
> Trace(A)
```

$$7$$

```
> CharacteristicPolynomial(A, x)
```

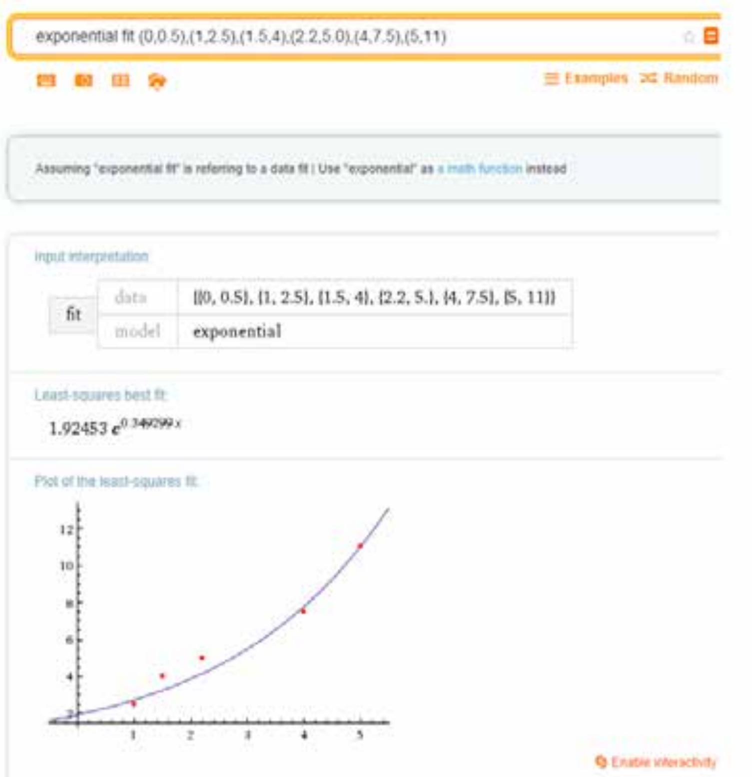
$$-47.0\,x^3 + 400.0\,x^2 - 537.00\,x - 310.00 - 7\,x^4 + x^5$$

Also the output is not as attractive as WA output:

```
> MatrixInverse(A)
    [[0.206451612903226, -0.696774193548387,
      0.838709677419355, 0.658064516129032,
      -0.896774193548387],
      [0.0709677419354839, 0.135483870967742,
      0.225806451612903, 0.0387096774193548,
      -0.464516129032258],
      [0.0354838709677419, 0.567741935483871,
      -0.387096774193548, -0.480645161290322,
      0.267741935483871],
      [-0.0129032258064517, 0.793548387096774,
      -0.677419354838710, -0.916129032258064,
      0.993548387096774],
      [-0.0419354838709677, -0.670967741935484,
      0.548387096774194, 1.02258064516129,
      -0.770967741935484]]
```

Examples from Statistics and Curve Fitting

WA can perform routine descriptive statistics and regressions tasks. Surprisingly WA, using least squares, produces a nonlinear exponential fit. Whereas the "curve fitting" package in Maple can only handle cases where the parameters of the model are linear as shown in the following example:



Now using Maple's curve fitting command, we get an error message when we attempt an exponential model of the form $y = ae^{bx}$, since in this form, the least squares method results in non-linear eqautions for a and b.

> with(CurveFitting) :

> mydata := [[1.0, 4], [−2, 3], [0, 5], [2, 6], [3.0, 4], [5, 11]]

$$mydata := [[1.0, 4], [-2, 3], [0, 5], [2, 6], [3.0, 4], [5, 11]]$$

> LeastSquares(mydata, x, curve = a·x + b)

$$4.10169491525424 + 0.932203389830508\, x$$

> LeastSquares(mydata, x, curve = a·x² + b·x + c)

$$3.58556588299617 + 0.351558228540186\, x + 0.193548387096774\, x^2$$

> LeastSquares(mydata, x, curve = a·exp(b·x))

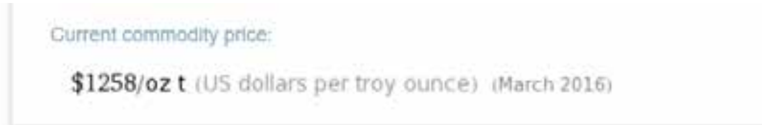
Error, (in CurveFitting:-LeastSquares) curve to fit is not linear in the parameters

> LeastSquares(mydata, x, curve = a·exp(x))

$$0.0788471189\, e^x$$

WolframAlpha as a Knowledge Engine

Wolfram alpha, in addition to being a computational tool, is a knowledge engine. One can use WA to find histograms and trends for various variables. We will demonstrate this amazing capability of WA in the following example concerning the price of gold over the past hundred years.



Conclusions
Both WA and Maple give correct answers to simple queries. Maple tends to limit its response to the specific question, whereas WA is more verbose and returns many results that you "may" be interested. Maple has rather rigid syntax and confusing error messages. Unlike Maple WA is very forgiving in its syntax requirements. And finally Maple is "for fee" and WA is for free. For routine calculations WA is the preferred tool but for more complicated multi-step projects in post-calculus courses Maple is a better tool.

Acknowledgements
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References
[1]] R. O. Abbasian and John T. Sieben, *A Survey of the Use and effectiveness of Technology-based mathematics Instruction among Texas High schools, Colleges and Universities: A preliminary report*, proceedings of the International Conference on Technology in Collegiate Mathematics, Chicago, USA, March 2010.

[2] R. O. Abbasian and John T. Sieben, *Shortcomings and Misuses of Technology (Maple, TI calculators) in Undergraduate Mathematics*, proceedings of the International Conference on Technology in Collegiate Mathematics, Chicago, USA, November 2003.