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The Use of Maple in Solving Initial Value Problems

Abstract: When we solve Initial value problems occurring in science and engineering we employ some analytical methods. It is worthwhile spending time to study these methods and use paper and pencil to arrive at the solution. Here we illustrate with examples the use of Maple software to obtain solutions in seconds.

Discussion: There are problems occurring in science and engineering that can be modeled by initial value problems containing ordinary differential equations with some initial conditions. For instance the projectile motion of an object shot freely in the air can be mathematically modeled as an initial value problem containing a second order differential equation with two initial conditions. Also in another instance the study of a spring mass system the equations of motion could be illustrated by a second order differential equation with two initial conditions. To solve the second order differential equation with constant coefficients that is linear non homogeneous we have to use the method of undetermined coefficients or variation of parameters. It is indeed worthwhile the time spent on studying these analytical methods. After we master these analytical solution methods we ask ourselves sometimes whether there is a computer aided solution that can be obtained in seconds. Software developers who understand these analytical methods have succeeded in creating software that will allow us to use them as tools in solving these initial value problems. We will discuss in this paper by use of examples of mathematical models that are appearing as initial value problems and could be solved in seconds by the use of Maple software.

When we have the initial value problem that contains a single differential equation or a system of differential equations we will assign a name to the equation using the assignment operator colon followed by equal sign. If it is a system of equations that contain two or more equations the equations are entered into a computer that is separated by commas. Entering the equation(s) against the assigned name of the equation becomes the first line of Maple code that ends with a semicolon. For instance "ode1" can be assigned as a name to denote the

differential equation(s). The second line of Maple code will contain the initial conditions separated by commas entered against a pre-assigned name for the initial conditions. For instance “IC” can be assigned as a name to denote the initial conditions. The final Maple code will contain the “dsolve” command that will contain both the differential equation and the initial conditions as a set in curly brackets. We should note that as usual in computer codes these lines of Maple codes are ended by semicolon.

We will now give two examples of dynamical systems and their solutions by the use of Maple software.

Example1:

A missile is shot in the air from the tower of height 16 feet with an initial speed of 256 feet/s. Assuming that the gravitational constant is 32 feet /s², find the distance travelled by the missile after t seconds. When does the missile hit the ground?

We can realize that the problem is an initial value problem. The differential equation model that represents the problem is as follows.

$$\frac{d^2 S}{dt^2} = -32, S(0) = 16, S'(0) = 256.$$

The following maple code is written to solve the problem using Maple.

```
Ode1:=Diff(S(t),t$2) = -32 ;
```

```
IC1:= S(0) = 16, D(S)(0) = 256;
```

```
dsolve({ode1,IC1}, S(t));
```

Example2: Spring –Mass System- Driven Motion

A mass weighing 16 pounds stretches a spring 8/3 feet. The mass is initially released from rest from a point 2 feet below the equilibrium position, and the subsequent motion takes place in a medium that offers a damping force that is numerically equal to ½ the instantaneous velocity. Find the equation of motion if the mass is driven by the external force equal to $f(t) = 10 \cos(3t)$.

This is again an initial value problem. The differential equation model that represents the problem is as follows.

$$\frac{d^2 X}{dt^2} + \frac{dX}{dt} + 12X = 20 * \cos(3t), X(0) = 2, D(X)(0) = 0$$

The following maple code is written to solve the problem using Maple.

```
ode2:=Diff( X(t),t$2)+Diff( X(t),t$2)+Diff( X(t),t)+12* X(t) = 20*cos(3*t);
```

```
IC2:= X(0) = 2, D(X)(0) = 0;
```

```
dsolve({ode2,IC2}, X(t));
```

We will now illustrate these Maple codes and solve the corresponding initial value problems to show the solutions to both of our examples. The directory that we use is DEtools.

```
> with(DEtools) :
```

```
> ode1 := Diff(S(t), t$2) = -32;
```

$$ode1 := \frac{d^2}{dt^2} S(t) = -32$$

```
> IC1 := S(0) = 16, D(S)(0) = 256;
```

$$IC1 := S(0) = 16, D(S)(0) = 256$$

```
> dsolve({ode1, IC1}, S(t));
```

$$S(t) = -16t^2 + 256t + 16$$

```
> H := t -> -16*t^2 + 256*t + 16;
```

$$H := t \rightarrow -16t^2 + 256t + 16$$

```
> solve(H(t));
```

$$8 - \sqrt{65}, 8 + \sqrt{65}$$

```
> 8 + sqrt(65);
```

$$8 + \sqrt{65}$$

```
> evalf(%);
```

$$16.0622577:$$

It will take approximately 16.1 seconds for the missile to hit the ground.

```
> ode2 := Diff(X(t), t$2) + Diff(X(t), t) + 12*X(t) = 20*cos(3*t);
```

$$ode2 := \frac{d^2}{dt^2} X(t) + \frac{d}{dt} X(t) + 12X(t) = 20 \cos(3t)$$

```
> IC2 := X(0) = 2, D(X)(0) = 0;
```

$$IC2 := X(0) = 2, D(X)(0) = 0$$

```
> dsolve({ode2, IC2}, X(t));
```

$$X(t) = -\frac{64}{141} e^{-\frac{1}{2}t} \sin\left(\frac{1}{2}\sqrt{47}t\right) \sqrt{47} \\ - \frac{4}{3} e^{-\frac{1}{2}t} \cos\left(\frac{1}{2}\sqrt{47}t\right) + \frac{10}{3} \sin(3t) + \frac{10}{3} \cos(3t)$$

Conclusion:

The analytical solution of an initial value problem takes quite amount of time while the solution by using Maple will take only a few seconds. We can really appreciate the software that is used as a tool in solving a mathematical model that involves a differential equation with some prior known initial conditions.

References:

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- (3) Soma Velummylum, "*Exploring Solutions of a Dynamical System with Maple*", ICTCM Electronic Proceedings (2007) -Vol-19-C018, pp227-231