VIRTUAL VECTOR CALCULUS TUTORS

By R. Howard Henley

JSRCC

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Virtual Calculus Tutors I and II, and Virtual Vector Calculus Tutors are collections of 60 state-of-the-art Camtasia videos covering selected topics in three semesters of calculus.

These tutors are especially useful for students who

a. miss lectures,
b. need to review topics for a project or a test,
c. need to strengthen their foundation in a course where calculus is a prerequisite, and
d. need support in distance learning environments.

These tutors are brief and interesting. They include thousands of fabulous Mathematica and Maple Graphs and embellishments, charts and animation to make maximum use of visual imagery in order to enhance learning and retention of fundamental concepts of each calculus course.

Let's take a look at the menu of 20 videos for Vector Calculus.

Menu / Table of Contents

	Virtual Vector Calculus Tutors	Time (minutes)
	Vectors and Geometry of Space	
1.	Vectors Part 1: Dot Product	16
2.	Vectors Part 2: Cross Product	21
3.	Vectors Part 3: Surfaces in Space	8
	Vector-Value Functions	
4.	Vector-Value Functions Part 1: Differentiation and Integration	15
5.	Vector-Value Functions Part 2: Velocity and Acceleration	7
6.	Vector-Value Functions Part 3: Tangent, Normal Vectors, & other	17
	Functions of Several Variables	
7.	Functions of Several Variables 1: Limits, Continuity, Partials	15
8.	Functions of Several Variables 2: Chain Rules, Directional Derivatives, Gradients	11
9.	Functions of Several Variables 3: Tangent Planes, Extrema	11
	Multiple Integration	
10.	Multiple Integration Part 1: Iterated Integrals, Area	7
11.	Multiple Integration Part 2: Double Integrals, Volume	10
12.	Multiple Integration Part 3: Change of Variables Polar, Rectangular Coordinate System	12
13.	Multiple Integration Part 4: Center of Mass, Surface Area	7
14.	Multiple Integration Part 5: Triple Integrals, Cylindrical and Spherical Coordinate Systems	10
	Vector Analysis	
15.	Vector Analysis Part 1: Vector Fields	13
16.	Vector Analysis Part 2: Line Integrals	17
17.	Vector Analysis Part 3: Green's Theorem	7
18.	Vector Analysis Part 4: Surface Integrals	12
19.	Vector Analysis Part 5: Divergence's Theorem	10
20.	Vector Analysis Part 6: Stokes' Theorem	13

Total time $\approx$ **4 hours**

Virtual Vector Calculus Tutors; Page **2** of **3**

Please answer the two questions on the handout after watching the sample Camtasia video.

ICTCM

International Conference on Technology in Collegiate Mathematics

Las Vegas, NV March 12 - 15, 2015

Innovative Use of Technology: Virtual Vector Calculus Tutors by R. Howard Henley

Please Print:

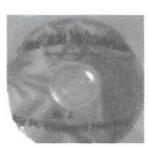
Name ________________________ email ______________________________

1. **Over All Rating of Presentation: (check only one)**

   Outstanding ___ Good ___ Fair ___ Poor ___

2. **Which product would you like to have? (check all that apply)**


DVD ___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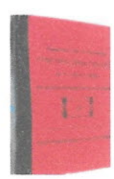
Hard Copy Manual ___

International Conference on Technology in Collegiate Mathematics (ICTCM)

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Innovative Use of Technology

Virtual Vector Calculus Tutors

By R. Howard Henley

Introduction

Greetings fellow mathematicians. My name is Ruth Henley.

Vector Calculus Tutors I and II were presented at AMATYC in Anaheim, Ca. The third volume of this three volume series will be introduced here today.

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Vector Calculus

This presentation contains an excerpt from Virtual Vector Calculus Tutors, which is a collection of 20 state-of-the-art Camtasia videos.

These tutors are **unique** among online help resources in that they include thousands of fabulous Mathematica and Maple graphics and embellishments, charts and animation.

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3

Vector Calculus

Success with these tutors requires a knowledge of the fundamental of single variable calculus.

The example selected for today's presentation demonstrates early in the course techniques for organizing solutions, and shows that learning can be easy and interesting while still preparing students to succeed beyond this course. Let's take a brief look.

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4

Acceleration

Acceleration can be broken into two components: Tangential and Normal components.

The **tangential component of acceleration,** a_{T} acts in the line of motion, and the **normal component of acceleration,** a_{N} acts perpendicular to the line of motion.

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Tangential and Normal Components of Acceleration

$\mathbf{r}(t)$ is a position vector.

$$a_{\mathrm{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{\|\mathbf{r}'(t)\|}$$

Tangential component of acceleration.

$$a_{\mathrm{N}} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}$$

Normal component of acceleration.

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Tangent and Normal Components of Acceleration

The position vector of a moving point at time t is

$$\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k} \text{ for } 1 \le t \le 5.$$

a) Find the tangential component of acceleration at time t.

b) Find the normal component of acceleration at time t.

c) Find three-decimal-place approximations for $\|\mathbf{v}(t)\|$, a_{T} and a_{N}, over the interval, and describe the motion of the point.

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Tangential Component of Acceleration

a). Find the tangential component of acceleration at time t. $\quad \mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}$

Solution:

$\mathbf{v}(t) = \mathbf{r}'(t) = \mathbf{i} + 4t\mathbf{j} + 3t^2\mathbf{k}$ Velocity vector.

$\mathbf{a}(t) = \mathbf{r}''(t) = 4\mathbf{j} + 6t\mathbf{k}$ Acceleration vector.

$\|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \left(1 + 16t^2 + 9t^4\right)^{1/2}$ Speed

$$a_{\mathrm{T}} = \frac{r'(t) \cdot r''(t)}{\|r'(t)\|} = \frac{16t + 18t^3}{\left(1 + 16t^2 + 9t^4\right)^{1/2}}$$ Tangential component of acceleration.

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Normal Component of Acceleration

b) Find the normal component of acceleration at time t.

Solution: $\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}$, $\mathbf{v}(t) = \mathbf{r}'(t) = \mathbf{i} + 4t\mathbf{j} + 3t^2\mathbf{k}$

$\mathbf{a}(t) = \mathbf{r}''(t) = 4\mathbf{j} + 6t\mathbf{k}$, $\|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \left(1 + 16t^2 + 9t^4\right)^{1/2}$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4t & 3t^2 \\ 0 & 4 & 6t \end{vmatrix} = 12t^2\mathbf{i} - 6t\mathbf{j} + 4\mathbf{k}$$

Cross product of velocity and acceleration vectors.

$$a_N = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|} = \frac{\left(144t^4 + 36t^2 + 16\right)^{1/2}}{\left(1 + 16t^2 + 9t^4\right)^{1/2}} = 2\left(\frac{36t^4 + 9t^2 + 4}{9t^4 + 16t^2 + 1}\right)^{1/2}.$$

Normal component of acceleration.

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Technology: Collect and Organize Data

Solution continued:

Technology:

$Y_1 = \sqrt{1+16X^2+9X^4}$
$Y_2 = (16X+18X^3)/Y_1$
$Y_3 = 2\sqrt{36X^4+9X^2+4}/Y_1$

X	Y1	Y2	Y3
1.000	5.099	6.668	2.746
2.000	14.457	12.174	3.434
3.000	29.563	18.063	3.706
4.000	50.606	24.029	3.824
5.000	77.627	30.015	3.884

Understanding how to collect, organize, and analyze data is essential to the efficient processing of many in-depth problems. Here we apply technology to populate the following charts.

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Describe the Motion of the Point

c) Solution continued: $\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}$

t	1	2	3	4	5
Position of the point $(t, 2t^2, t^3)$	(1, 2, 1)	(2, 8, 8)	(3, 18, 27)	(4, 32, 64)	(5, 50, 125)
Speed $\lVert \mathbf{v}(t) \rVert$	5.099	14.457	29.563	50.606	77.627

As t increases from 1 to 5, the point moves along C from (1, 2, 1) to (5, 50, 125), gaining speed rapidly.

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Describe the Motion of the Point

c) Solution continued:

t	1	2	3	4	5
Position of the point $(t, 2t^2, t^3)$	(1, 2, 1)	(2, 8, 8)	(3, 18, 27)	(4, 32, 64)	(5, 50, 125)
a_T	6.668	12.174	18.063	24.029	30.015

The tangential component, a_T, increases at approximately 6 units per unit change of time.

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Describe the Motion of the Point

c) Solution continued:

t	1	2	3	4	5
Position of the point $(t, 2t^2, t^3)$	(1, 2, 1)	(2, 8, 8)	(3, 18, 27)	(4, 32, 64)	(5, 50, 125)
a_N	2.746	3.434	3.706	3.824	3.884

The normal component of acceleration, a_N approaches 4. $\lim_{t\to\infty} a_N = 4$

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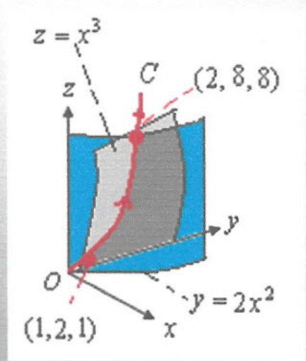
Tangential and Normal Components of Acceleration

$$\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}$$



The point begins at $t = 1$, and as t increases, the curve C is traced out through the points given in the charts.

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Vector Calculus Technology Workshop

R. Howard Henley, Associate Professor of Mathematics 15

The End

To learn more about these tutors, go to:

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R. Howard Henley, Associate Professor of Mathematics 16