UNVEILING THE MYSTERIOUS MEANINGS OF CONFIDENCE INTERVALS USING THE STATCRUNCH PLOTTER (AND OTHER APPLET IDEAS)

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Abstract
Constructing a confidence interval using a formula is typically an easy task for students when they are given the necessary ingredient. Although understanding how to interpret the confidence interval and what the confidence interval means often remains a complex mystery to students. This paper outlines how to integrate many of the applets on StatCrunch [1] to build an understanding of confidence intervals and how to interpret them.

Introduction
There are many applets available on the internet that can be used to enhance statistical literacy, but how can these applets be effectively applied in the classroom setting? Over the years, I have used many different activities. These activities typically have the same basic approach; they start slow with students each constructing one confidence interval, then pick up the pace by collecting all the students' confidence intervals in the class and comparing them. Lastly, an applet is used to investigate what happens if more and more confidence intervals are created in the same manner.

This activity now incorporates an updated technology friendly approach using StatCrunch. First, students conduct an experiment using a spinner and create a confidence interval for the sample they generate. The results of the experiment are then gathered from the students using a survey on the StatCrunch group site. Using this raw data and the confidence interval plotter, the confidence intervals are graphed so that the students can visualize the relationship between the confidence intervals. Next, the confidence interval applet is used to determine what happens in the long run when more and more confidence intervals are constructed. Finally, as an extension to this activity, the confidence interval applet is utilized to explore what happens to the confidence intervals when the confidence level or sample size is changed.

This confidence interval lesson follows the guidelines set forth by the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report* [2]. In particular, this lesson incorporates active learning, conceptual understanding of statistical ideas, simulation of real data, and technology. The students in the classroom are assessed on their understanding of the material by verbal and written response to the underlying concept being considered.

In this paper, I outline the lesson so that the reader can incorporate the lesson in his/her own class setting making modifications as necessary to individual needs. In particular, I have included the StatCrunch commands and screen shots of the dialogue boxes for those readers who are unfamiliar with StatCrunch.

Background

The textbook used in my courses is the 12th Edition of *Elementary Statistics* [3] and the courses are conducted through MyStatLab [4]. Classes are taught in a computer lab with one student per computer although the students are placed in groups of two or three randomly each day. Typically, the class size ranges from 18 to 25 students. Classes are seventy-five minutes and the class format typically consists of a brief introduction to the material and then the students are set out on their daily activity. While students work, I interact with each group, check for understanding, and ask specific questions to access their depth of knowledge. All class meetings are conducted in this format from the first day so the students understand they must come prepared to class and ready to work.

This activity is presented to students in class after they have been introduced to the concept of confidence intervals and how to calculate them. Furthermore, in a previous activity, samples of M & M candies were examined by the students to demonstrate the distribution of sample proportions and sampling variability.

The Spinner

To start the lesson, the students are asked to conduct an experiment that involves taking samples of size 30 M & M chocolate candies and recording the proportion of orange in each of the samples. Although Mars and Company no longer posts the proportions of colors for M & M chocolate candies, it will be assumed that 20% of all M & M chocolate candies produced are orange for the experiment. Therefore, students create a spinner on StatCrunch that represents 20% Orange and 80% Not Orange.

To create a spinner on StatCrunch, first enter the necessary weights and colors:
1. In an open StatCrunch page, let var1 be Color and enter Orange in the first row and Not Orange in the second row.
2. Let var2 be Weight and enter 0.2 in the first row and 0.8 in the second row.

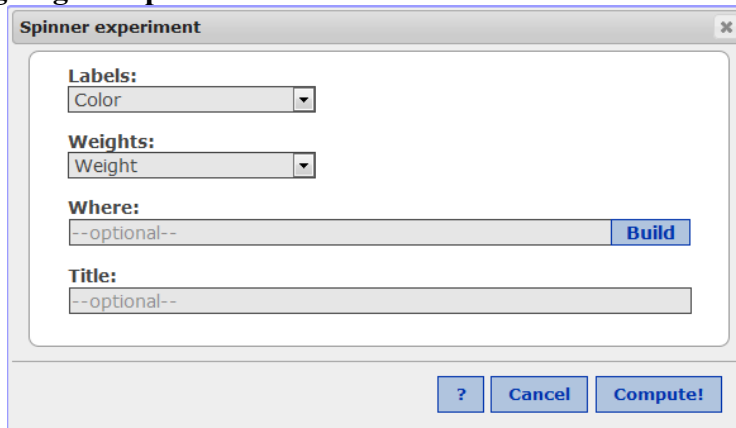**Figure 1: Entering the necessary weights in the StatCrunch spreadsheet.**

Row	Color	Weights
1	Orange	0.2
2	Not Orange	0.8

Once the colors and weights are entered in the spreadsheet, create the spinner:
1. Go to Applets > Experiment > Spinner.
2. Let the Labels = Color
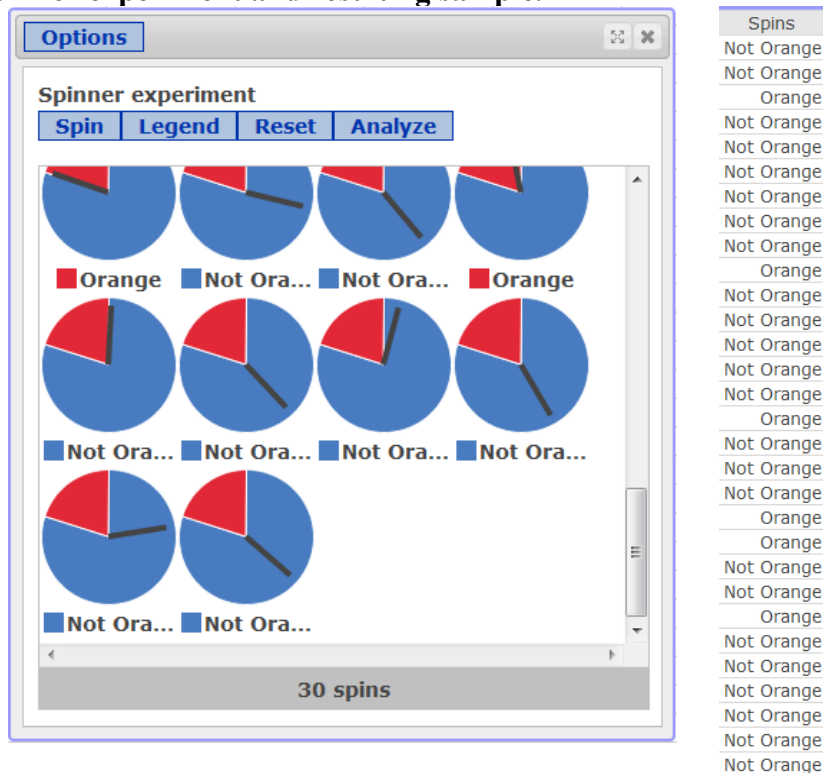3. Let the Weights = Weights

4. Select Compute!

Figure 2: Designing the Spinner

To take samples of size 30 and to save the results in the spreadsheet:
1. Spin the spinner 30 times.
2. Then select Analyze.
3. Your result of 30 spins have been saved in a column labeled Spins.

Figure 3: Spinner experiment and resulting sample.

To ensure that a minimum of 50 confidence intervals are created correctly by a small class, each student is asked to run the experiment a total of three times. In a large class, students can run the experiment once to generate enough confidence intervals. I find the

simple task of running the experiment three times reinforces the concept of sampling variability.

After each student has a sample(s), they are instructed to create an 80% confidence interval(s):

1. Go to Stat > Proportion Stats > One Sample > With Data.
2. Select the column(s) representing the results of the sample(s).
3. Let the Success be Orange.
4. Select Confidence Interval and let the level of confidence be 0.80.
5. Select Compute!

Figure 4: Creating 80% confidence intervals for the sample(s).

It should be noted by the reader that an 80% level of confidence is chosen so that there is a greater chance of creating confidence intervals that do not contain the population proportion. It is my personal experience that a 95% level of confidence could result in almost all of the confidence intervals containing the population proportion which does not shed light on the concept this lesson is designed around.

The Survey

In the second phase of this activity, the confidence intervals created by the students are gathered using a short survey on the StatCrunch class group page. The questions on the survey are asked in the same order that the data was reported in StatCrunch, so the sample proportion is the first question and not the middle. Furthermore, if the students created three confidence intervals, it is necessary that they take the survey three times to enter all three confidence intervals.

Figure 5: Gathering the confidence intervals from the students.

Confidence Intervals

1. Edit - Delete
   Sample Proportion (p-hat) (please round to 4 decimals as necessary)

   Enter a numeric response between 0 and 1 inclusive.

2. Edit - Delete
   Lower Limit of Confidence Interval (please round to 4 decimals)

   Enter a numeric response between 0 and 1 inclusive.

3. Edit - Delete
   Upper Limit of Confidence Interval (please round to 4 decimals)

   Enter a numeric response between 0 and 1 inclusive.

The Plotter

The Plotter applet allows the user to plot confidences intervals horizontally by stacking them on top of one another. It allows the students to visualize what would happen if multiple confidence intervals were created for different samples that are gathered in the same manner. The plotter also reports the proportion of the confidence intervals that contain the true population parameter.

The reader should note that although I would hope that my students at this point can create a confidence interval correctly and answer a survey correctly, it will be necessary to accommodate for errors in setting up the applet. This will be explained in the set up.

After all of the students have completed the survey, open the results of the survey into a spreadsheet. With the data open in another tab, the plotter is use to visual the relationship between the confidence intervals and the population parameter:
1. Go to Applets > Confidence Intervals > Plotter
2. Let the Lower Limit be the Lower Limits
3. Let the Upper Limit be the Upper Limits
4. Since some student may have incorrectly entered their confidence interval result, select Build under the Where condition.
   a. In the Column list, select Lower Limit and then Add Column
   b. Select < (the less than symbol) on the right
   c. In the Column list, select Upper Limit and then Add Column
   d. The condition of "Lower Limit" < "Upper Limit" has been added to the plotter. Although this condition does not guarantee that the student created the confidence interval correctly, it does guarantee that the plotter will plot the results and not return an error.
5. Enter 0.80 for the Target Confidence level
6. Enter 0.20 for the True parameter.
7. Select Compute!

Figure 6: Setting up the plotter

The results of the confidence interval plotter illustrate the number of confidence intervals that contain the true parameter, the total number of confidence intervals, the proportion of the confidence intervals containing the true parameter, and the target level of confidence. Selecting one of the confidence intervals reveals the lower and upper limit of that confidence interval. All of the confidence intervals presented in red do not contain the true parameter while those in green do contain the true parameter. Points to make to your students include the relationship between the proportion containing the true parameter and the target level as well as investigating the sample proportion used to create the confidence intervals that do and do not contain the true parameter. The reader should note that 100 confidence intervals are displayed per screen.

Figure 7: The results of the plotter
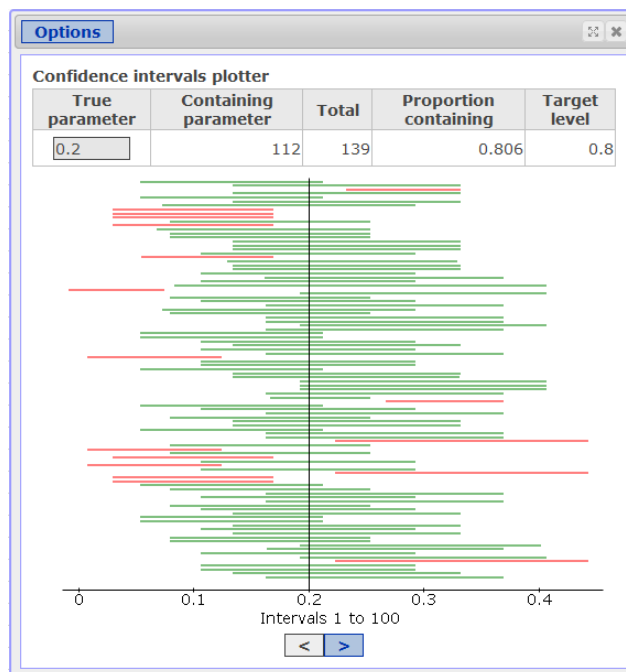
The Confidence Interval Applet

The confidence interval applet allows the students to generate more and more confidence intervals with ease and visualize their relationship. In this phase of the activity, students use the confidence interval applet to investigate two concepts; first, what happens when more and more confidence intervals are created and second, what sample proportions result in confidence intervals that do and do not contain the true parameter.

The student will continue to simulate data based on a sample of M & M chocolate candies that are classified as either orange or not orange. Therefore, they will assume that 20% of all M & M chocolate candies produced are orange and 80% are not orange.

For the initial set up of the applet:
1. Go to Applets > Confidence Intervals > For a Proportion
2. Let p = 0.2 (the proportion of orange created by Mars),
3. Let the initial confidence level = 0.80,
4. Let the initial sample size = 30,
5. Select Standard-Wald under Interval
6. Select Compute!
7. One hundred confidence intervals will be generated.

Figure 8: Setting up the confidence interval applet
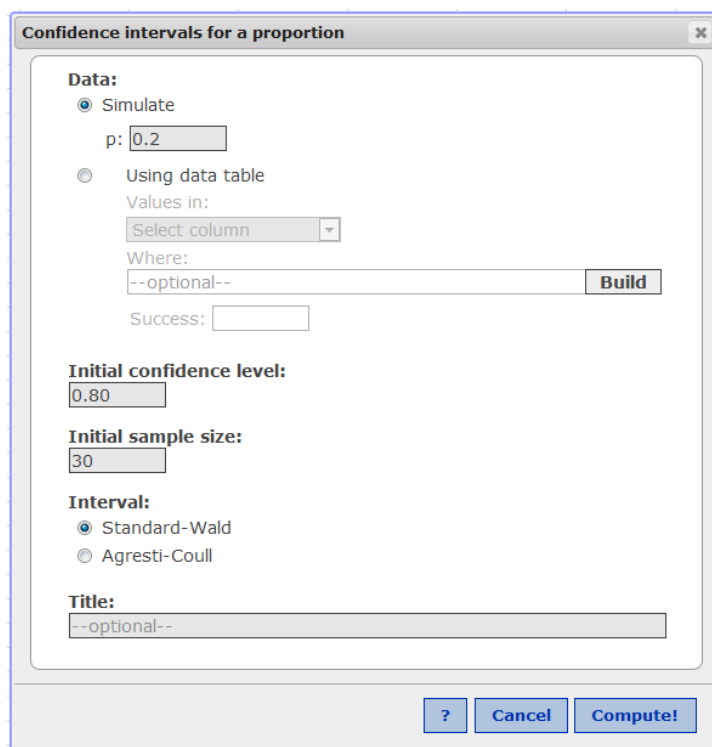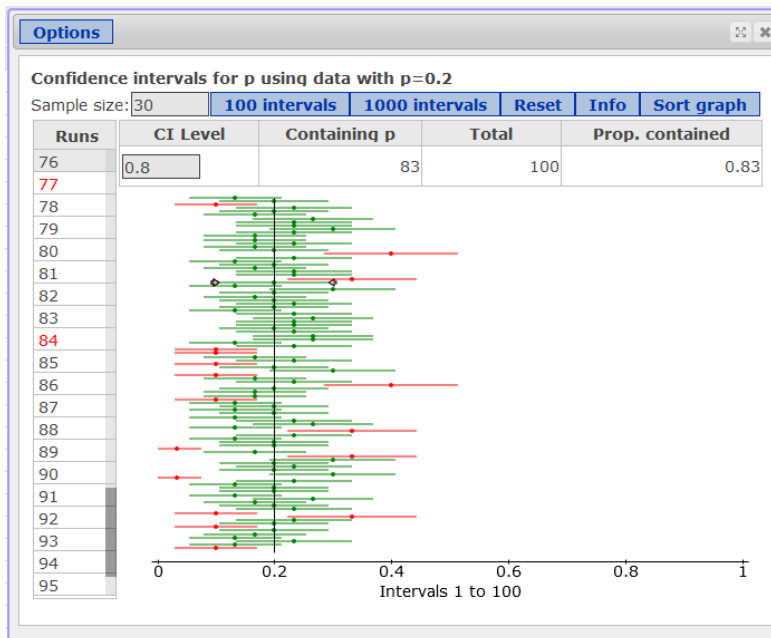
Figure 9: One hundred confidence intervals

After inspecting the first one hundred confidence intervals, students continue to take more and more samples until they have created a minimum of 1000 confidence intervals.

They will notice that as the number of samples increase, the proportion of confidence intervals that contain the population parameter approaches the level of confidence.

Figure 10: One thousand confidence intervals
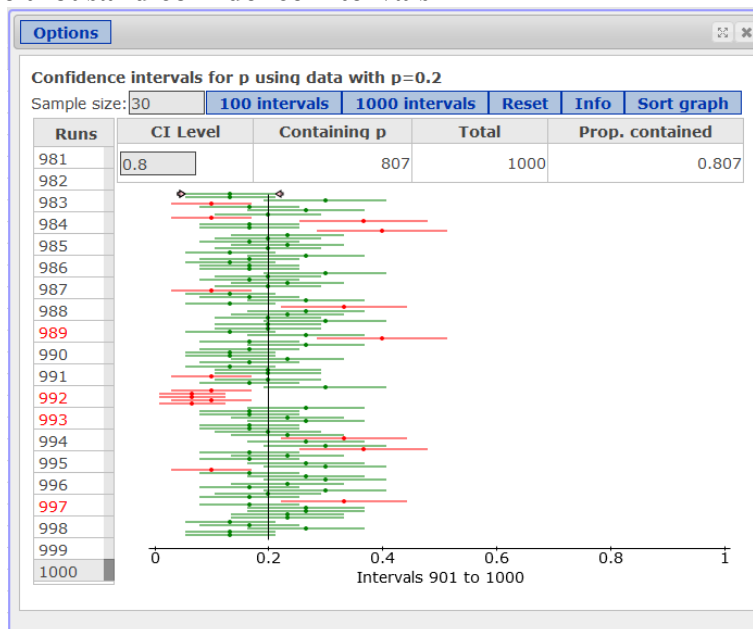
To understand the characteristics of the sample proportions that create confidence intervals that do and do not contain the true parameter, the plotter is sorted:

1. Select Sort graph on the top right of the dialogue box.
2. The confidence intervals are sorted from the largest to the smallest sample proportion.

Figure 11: Confidence intervals sorted.
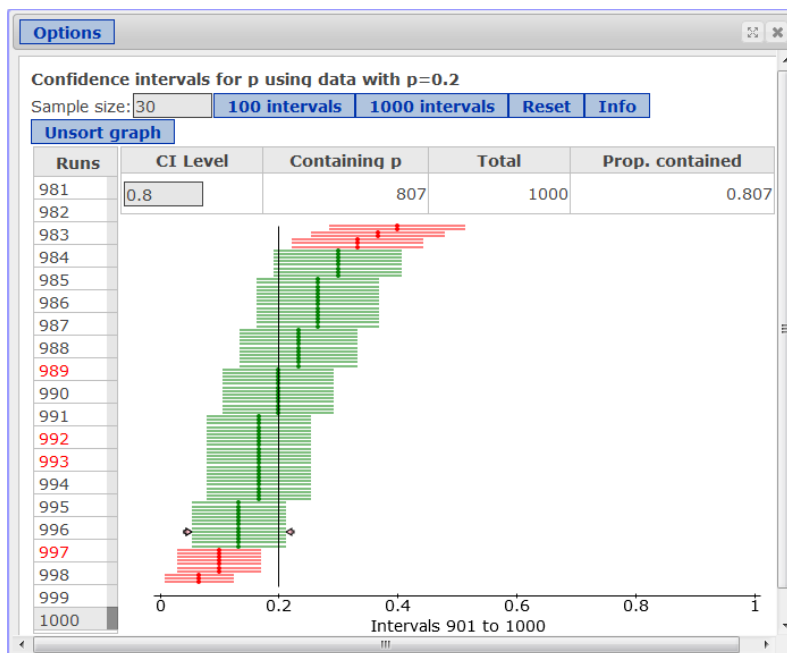
Looking at the graph, students can easily notice that the sample proportions that are in the tails of the sampling distribution do not create confidence intervals that contain the true parameter. While sample proportions that are in the center of the sampling distribution do create confidence intervals that contain the true parameter.

Extensions

To complete the activity, students investigate two more important concepts of confidence intervals; the relationship between the confidence interval and the confidence level as well as the relationship between the confidence interval and the sample size. Images for these extensions have not be included in the paper since the reader now has visual images of the confidence interval applet.

Investigating what happens when the level of confidence increases and all other properties stay the same is very visual with the help of the applet. Students maintain the results of the 80% confidence level and then run the applet for both 90% and 95% confidence levels. With the sorted results of these three simulations visible, the students can determine that the greater the level of confidence, the larger the margin of error and the wider the confidence interval. So, the students understand that the more confident you want to be the less you have to be confident about.

Lastly, the students examine what happens when the sample size increases and all other properties stay the same. Students are asked to generate 95% confidence intervals for samples of size 30, 300, and 3000. With the sorted results of these three simulations visible, the students can determine that the greater the sample size, the smaller the margin of error and the narrower the confidence interval. So, it is easy to see that the larger the sample size, the smaller the error in estimating the population parameter.

Conclusion

This technologically enriched version of an old activity has brought insight to students' understanding that may not have always been clear in the past. Having a common theme of M & M chocolate candies for all levels of this activity has built connection within the lesson. In the past, students did not always understand the connection between the concrete examples of counting M & M chocolate candies to an applet that did not allow personalization. In the past, the applet seemed to be this empty black box which did not always bring the concepts into focus. I had to convince the students that the applet was reliable. Now, the theme of M & M chocolate candies seamlessly continues from one topic to another over the course of two class meeting that are not taught consecutively. These activities work and the students move quite quickly through the deep understanding of how to interpret confidence intervals.

References

[1] StatCrunch, Pearson Education, Boston, Massachusetts, http://statcrunch.com, 2012

[2] Aliaga, M., Cobb, G., Cuff, C., Garfield, J., Gould, R., Lock, R., Moore, T., Rossman, A., Stephenson, B., Utts, J., Velleman, P., Witmer, J., *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report,* American Statistical Association, 2012.

[3] Triola, M., *Elementary Statistics*, 12th edition, Pearson Education, Boston, Massachusetts, 2014

[4] MyStatLab, Pearson Education, Boston, Massachusetts, http://www.mystatlab.com, 2012