Can Parking Lots Replace Gas Stations?

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As mathematicians, and especially as mathematicians teaching undergraduate mathematics courses, we have a tremendous opportunity as well as a significant responsibility. Over the course of their lives our students will be making personal and public policy decisions that will impact not just their own pocket books but will also shape the future of our planet. Decisions will result in more favorable outcomes when supported by mathematical models [2, 5]. As their educators we have the opportunity and responsibility to motivate our students' interest in mathematics by showing how these models allow them to make decisions whose results can benefit both their own pocketbooks as well as the planet's wellbeing.

This paper and its companion paper – *Solar Energy – Take a Spherical Cow* – use mathematics to study how solar energy can help us meet our energy needs [8]. In particular this paper is directed at lower level classes such as college algebra and precalculus while the companion paper is intended for upper level mathematics courses. Using an experimental approach with widely available inexpensive equipment, this paper examines the possibility of using solar panels to reliably recharge an electric vehicle's battery while parked.

If you were to drive an electric car from home to work where the car battery is then recharged by an overhead solar panel, could the solar panel collect enough energy throughout the day needed by your car to return home? The companion paper *Solar Energy – Take a Spherical Cow* answers this question from an ideal perspective using a mathematical

Figure 1: Electric/Hybrid vehicles charging while parked

model [8]. There we learn that a 2 m^2 photovoltaic solar panel, positioned at 45° latitude, could capture 5.642 kWh during one day of sunlight on the winter solstice compared to 24.06 kWh collected on the summer solstice.

A list of electric cars and the energy required to drive them 100 miles can be found at http://www.nrel.gov/gis/solar.html. From this list we see that the Electric Smart Car, which requires approximately 32 to 36 kWh to drive 100 miles, would be able to drive approximately 18 miles from the ideal solar energy available on the winter solstice.

We now consider practical considerations affecting the actual amount of energy available to an array of photovoltaic cells.

Supplies

- 7 Watt Solar Panel
- Wire Leads
- (4) 6Ω resistors, 25 W or greater
- Data recorder with current, voltage sensors

Instructions

1. Before we can collect voltage and current data the solar panel needs a load circuit to which harvested energy can be delivered. The Maximum Power Transfer Theorem tells us a battery delivers maximum power to its load when $R_L = R_{int}$, an unquestioned condition so so long as the power source behaves like a battery [1]. There are, however, doubts this model fully represents the way a solar panel generates and delivers power [7].

   Setting these matters aside we designed the load circuit, shown in Figure 2, first, to safely handle maximum solar panel output voltage, current and power as specified by the manufacturer, and second to accommodate the circuit sensors used by the authors. In particular, the maximum current flowing in each leg of this load circuit will not exceed the maximum current rating of the recommended 600mA current sensor. Likewise the voltage across each 6Ω resistor will not exceed the 6 volt rating of the recommended voltage sensor.

Your physics, or electronics engineering, department technician has all the experience needed to build this - or any other - load circuit. Figure 3 illustrates our data collection equipment.

2. Find a parking space with adequate exposure to the sun throughout the day. Then plan to run the experiment on a day you can begin collecting data just as the sun becomes visible to your parking space. Be sure you are able to check the equipment throughout the day and until the sun sets on your parking space.

3. On a small table or clean surface in your parking space connect the solar panel, load circuit and data recorder with current and voltage probes as shown in Figure 2. If you plan to collect data for more than five hours we recommend you connect the supplemental power supply, described in Appendix A, to the data recorder.

4. Set up the data collection tool with the attached current and voltage sensors to record 200 data points per hour. Start the data collection process just before the morning sun shines on your panel and wait for the graphs to appear. While collecting data, shield your load circuit, sensors and data recorder from the sun, wind and animals. Consider outlining the perimeter of your equipment to avoid casting shadows on your solar panel. Finally, be sure to check your equipment on a regular basis. You wouldn't want to lose a day's worth of data just because the wind or a bird disturbed your electrical connections.

5. When the sun is no longer shining on your solar panel we advise concluding your experiment by first pausing the data recorder and then saving your data. Next you can power down the data recorder before disconnecting the solar panel from your load circuit as well as removing all extension wires from the panel. The remaining equipment is now safe for packing.

6. Connect the data collection tool to your computer and download the data you just collected. Save the raw data in a format native to your data collection tool and export your data to a .csv file. Open the .csv file in Excel or another spreadsheet and save the data as a .xlsx file.

7. You should have two columns of data in your Excel file, one for current, the other for voltage. Now create a third column with an equation that computes power equal to the product of that row's voltage and current.

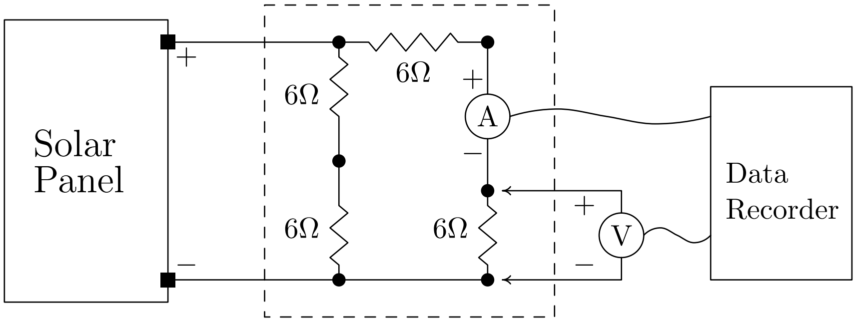

Figure 2: Solar panel data collection schematic

8. Compute the energy delivered to the one resistor across which you measured voltage by estimating the area under the graph of power versus time. Now multiply this value by 4 to account for the energy dissipated by all four resistors.

9. Open a browser on your computer and type in the link http://www.nrel.gov/gis/solar.html. Choose a car from this list and record the energy, e.g. 35 kWh required to drive 100 miles. Finally, multiply the conversion factor for your car, e.g. 100 miles/35 kWh, by the total energy in kWh your experiment collected. How far could you drive that car using the amount of energy you collected?

Figure 3: Solar panel connections ready for data collection

Sample Questions

1. Knowing the amount of energy you collected, the square area of your solar panel and the time you spent collecting energy, calculate the energy you could collect with a solar panel the size of your parking space during the time you spend at school.
2. What affect does your geographical latitude and day of the year have on the position of the sun during the day? What angle did the incident sunlight form with a vector normal to the solar panel during your experiment? Using trigonometry, predict the additional energy you could collect by tilting the plane of your solar panel to directly face the sun's position at noon. Re-align the plane of your solar panel to directly face the mid-day sun's position and re-run the experiment. How much further would your electric car travel on the additional energy?
3. Try estimating the energy harvest represented by each sample power graph found in Appendix B by approximating the area under each experimental curve. Do this using rectangles, whose widths you choose, formed between the horizontal axis and the height of the power curve. Note how the rectangle's area approaches the true area beneath the curve as the number of rectangles increases.

Appendix A: Equipment Summary

Data Collection Tool:

- Vernier's LabQuest 2© ($329) – The data collected by this handheld electronic data recorder through a variety of probes can be analyzed on the LabQuest itself or transferred to a calculator or computer. Data analysis can be performed on a computer using Vernier's LoggerPro software or exported to a spreadsheet. There is a free lite version of LoggerPro. You can also start with 30 day demo version of the full program ($229.00 for an extremely generous site license).
- Current probe for the Vernier LabQuest 2© ($39).
- Differential voltage probe for the Vernier LabQuest 2© ($39).
- The LabQuest 2© Battery Boost ($109). The rechargeable battery in the LabQuest 2© will run the LabQuest for six hours or more on a full charge. This range can be extended either by powering the unit from an AC outlet or by using the Battery Boost.

Resistors: Resistors of all values and power ratings are available at Amazon.com, Best Buy, Radio Shack as well as electronic distributors like DigiKey, Jameson and Allied Electronics.
Solar Panels: The 7 watt solar panel, charger, and rechargeable batteries we used come as a kit (Goal Zero 19010 Guide 10 Plus Solar Charging Kit) that is available from Amazon.com for $119.95. This and larger kits are also available directly from Goal Zero.
Voltage Source: $3-6V$ batteries are available at Batteries-Plus, Radio Shack, Amazon.com, electronic distributors like Allied Electronics, DigiKey and Jameson. In these experiments we used rechargeable battery pack sold by GoalZero.
Wire Leads: Wire leads with multiple colors and connector types, e.g. banana, clip and alligator wire connectors are available at Radio Shack, Amazon.com as well as electronic distributors like DigiKey, Jameson and Allied Electronics.

Appendix B: Sample Data

The peak magnitude of solar power available to a point on earth, as modeled in *Solar Energy – Take a Spherical Cow*, was focused on the maximum solar power incident upon a given point on the earth without atmospheric affect. The significance of these effects have led us to normalize the theoretically predicted power graphs plotted below.

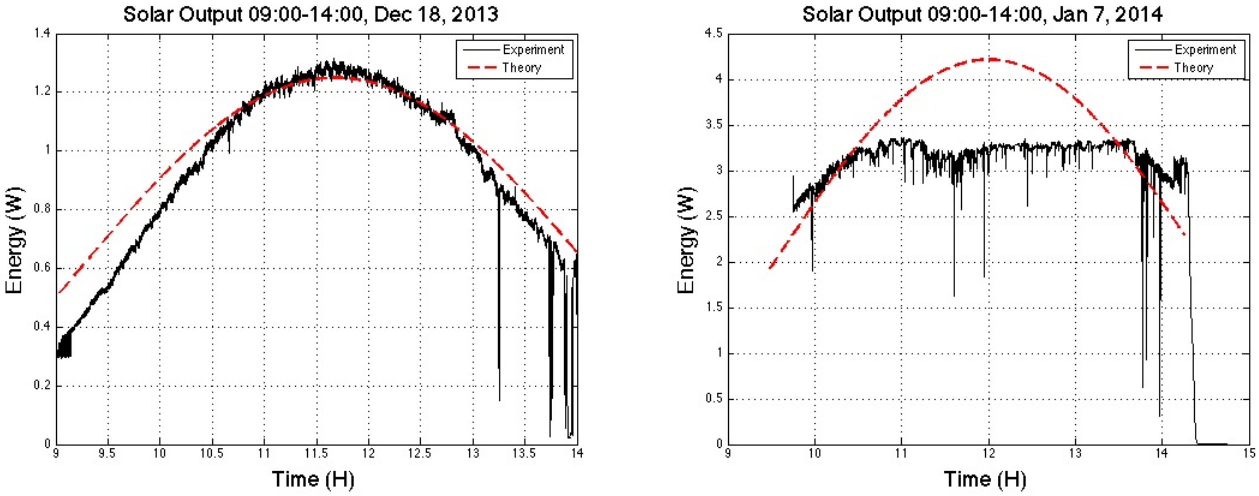

Figure 4: Left plot collected from solar panel tangent to earth surface. Right plot solar panel tilted 60° south, incident sunlight co-linear with solar panel normal vector at noon.

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