IPAD ACTIVITIES FOR PRESERVICE K-12 MATH TEACHERS

Dr. Christina Gawlik
Cyber Innovation Center
6300 E. Texas St., Bossier City, LA 71111
christina.gawlik@cyberinnovationcenter.org

Introduction

Creating hands-on problem-based learning experience for students where they construct knowledge through engaging contexts is supported by research in mathematics education (CCSS, 2010; NCTM, 2000). This paper is intended to highlight classic activities taught with traditional classroom manipulatives of Geoboards, Pattern Blocks, Tens Frames, and Algebra Tiles; and to showcase the virtual manipulatives now available on iPads, which can accompany or replace concrete manipulatives. The activities described herein are great for preparing preservice teachers to be fluent in content and enhance their abilities to use tools and technologies for instruction. Moreover, the activities are classic and support the Common Core State Standards for Mathematics which will best prepare preservice teachers for their future classroom.

Activities for Geometry Concepts

Geoboards are classic mathematics manipulatives found across K-12. They are also known as a pegboard, where students use rubber bands to create various polygons. Some geoboards are two sided with a square pegboard on one side, and a circle pegboard on the other. The circle pegboard allows explorations of arcs, chords, angles, fractions, and concepts within trigonometry.

Figure 1: Geoboard Polygon Examples

The iPad application called, Geoboard, by the Math Learning Center is a free and also available on iPod Touch or iPhone. Additionally, the Math Learning Center has a web-based version of their Geoboard application that can be found at http://www.mathlearningcenter.org/web-apps/geoboard/. As seen in Figure 1, there are eight different band colors (five on the iPhone or iPod Touch) and the ability to fill individual or all shapes with a transparent color. Students can switch between a 25 or 150 pegboard on the iPad version, however classic hand-held Geoboards typically have 25 pegs.

Conceptually Developing Area Formulas

Conceptually developing area formulas for triangles, squares, and rectangles can benefit students when approached with extensions, combinations, and applications of these formulas. To begin this process, students will explore squares, rectangles, and then triangles, in this order.

First, ask students to open their Geoboard app and create three to five squares of different side lengths, and then record the side length and area of each shape in table. To determine the area of each square, count the number of unit squares (1 x 1 squares) within each shape. Even if students know the area formula for a square, ask them to count each unit square to verify their formula is correct. Students who are not familiar with the area formula will develop this formula after exploring the patterns within their table.

Questioning the class to identify the area formula is paramount for developing formulas conceptually. Once students understand the area of any square can be found by multiplying side lengths, ask students to create three or four rectangles of different dimensions and again they should count the number of unit squares within to determine the area of each rectangle. In a new table, students should record length, width, and area, and then examine their table to determine the area formula for any rectangle.

The third polygon to investigate is a triangle. Students can keep one of their rectangles on the Geoboard and then add a triangle on top of it, using a different colored band. In Figure 2a, the rectangle has a length of three units and width of two units, resulting in an area of 6 square units. Key questions at this time are: how does the white triangle compare in size to that of the green rectangle? What are the dimensions of the triangle (base and height)? How are the base and height of the triangle related to the length and width of the rectangle? Can we count the number of unit squares in the triangle to determine its area? What other method can be used to determine the area of this triangle?
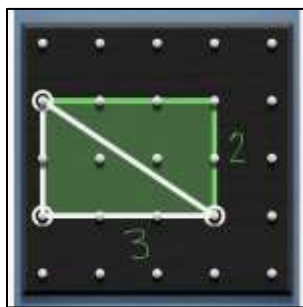

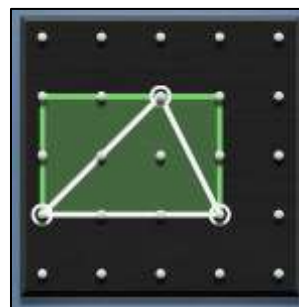Figure 2a: Triangle and Rectangle Area Relationships

Figure 2b: Triangle and Rectangle Area Relationships

The purpose of the above questions is for students to develop relationships between triangles and rectangles; in particular, a triangle has half the area of a rectangle if they share the same dimensions. It is important for students to repeat this exercise using various rectangles and squares, where the area of the triangle produces a fraction (or

decimal). Likewise, compare a right triangle as in Figure 2a to a scalene triangle in Figure 2b. How does the area of the white triangle in Figure 2b compare to that in Figure 2a? Students should be able to describe verbally and in written form their justifications.

Developing area formulas for parallelograms and trapezoids will follow the previous explorations. At this point in time, introduce the application Notability, by Ginger Labs, Inc.; a powerful annotation application (for $2.99). Students will take screen captures of their created polygons from Geoboard and insert them into a new note within Notability. Here, they will use different colored pens to digitally annotate on top of their Geoboard figures.

As seen in Figure 3, the yellow right trapezoid is partitioned into a rectangle and right triangle to determine its area; whereas the red isosceles trapezoid is doubled to form a parallelogram before determining its area. The method used to determine the area of the yellow trapezoid, is completed by finding the area of the rectangle and triangle, and then finding the sum. This approach does not focus on developing the area formula of a trapezoid, rather encourages connections to previous knowledge. Determining the area of the red trapezoid by creating a parallelogram does support conceptual development of its area formula. This problem solving process provides a clear representation of why and how the notation for "base 1" and "base 2" are incorporated in the formula.
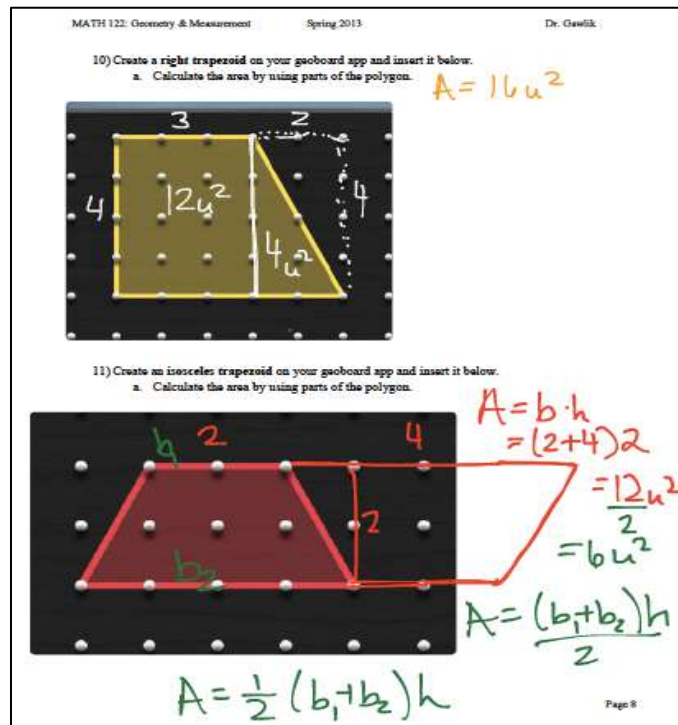

Figure 3: Developing Trapezoid Area Formula

These approaches to exploring area of trapezoids are ancient. Yet, during pre-technology times, paper folding and cutting methods were used to model these explorations. With

virtual-manipulatives and digital note taking, students can take pictures of their work, annotate on top of any screen capture or photo, and save for easy retrieval from Dropbox, Google Drive, or other cloud-based storage systems at any time.

Preservice teachers benefit greatly from thoroughly going through these activities. Dissecting both trapezoids to build the area formula tends to be the most challenging, yet students encounter that "ah-ha" moment when comparing the formula to the construction process and digital notes.

Additional activities that would follow conceptually developing area formulas are:
1. Exploring perimeters
2. Investigating relationships among perimeter and area of rectangles
3. Developing Pick's Formula
4. Creating transformations
5. Exploring congruence and symmetry
6. Crafting and examining fractions and proportional relationships

Create that Shape

Pattern Blocks are common plastic, wood, or foam manipulatives most often found in elementary school classrooms. However, they have great value in middle and high school mathematics as well. Braningcamp, LLC created the Pattern Blocks application for $0.99, available on the iPad with iOS 5.0 or later (see Figure 4).

Students in primary grades use Pattern Blocks to begin describing characteristics of polygons, identifying colors, and learning names of polygons. The hand-held manipulative of Pattern Blocks is limited to the set of shapes developed by the manufacturer. However, the iPad application of Pattern Blocks allows students to create their own shapes of various sizes and colors. This feature is great asking students to create an octagon (non-regular), heptagon and so on.

Many preservice teachers seeking Elementary certification/licensure claim they want to teach primary grades, because the mathematics to be taught is "easier." However, they learn quickly in their Mathematics Methods or Mathematics for Teachers courses that mathematics taught in primary grades is much more complex. Teaching early geometry concepts such as, *Create that Shape*, requires a vast knowledge of vocabulary, terminology, and correct spelling!

Tessellations and Transformations

The Pattern Blocks application is ideal for creating tessellations and exploring transformations. The whiteboard feature is perfect for having students write explanations or highlight key features of their creations, such as lines of symmetry, or point of rotation. Figure 4 is an example of a reflected pattern on square grid paper. The Pattern Blocks app allows students to name and save their work within the application, which can become useful when comparing examples, or working on long term projects.
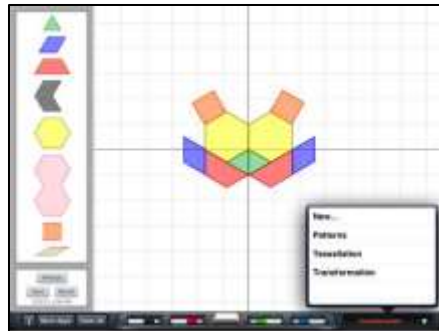
Figure 4: Pattern Block Tessellations

Activities for Number & Operations and Early Number Theory
Exploring Fraction Concepts & Computations

Using the Pattern Blocks application, students can model fraction relationships and operations of addition, subtraction, multiplication, and division through a geometric area perspective.

In Figures 5a and 5b, students modeled subtraction and multiplication, respectively, within the Pattern Blocks application. This application includes a whiteboard feature with four different colored pens, along with the ability to change the opacity and color of each shape. Notice in Figure 5b the process of multiplying two proper fractions does not use the traditional algorithm of fraction multiplication. Instead, students work through the process of identifying a part of a part (one-third of one-fourth).
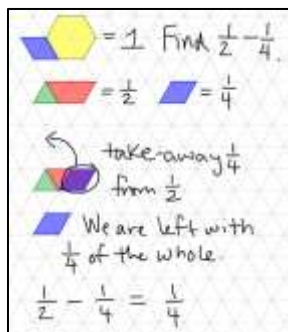

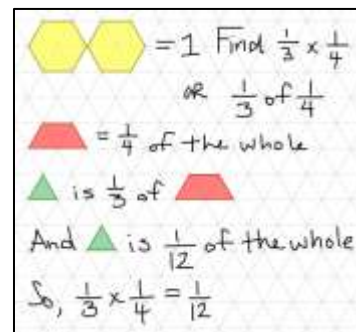Figure 5a: Take-Away Model of Subtraction

Figure 5b: Fraction Multiplication

Through preparing preservice teachers for seven years, I found they struggle and often resist learning how to complete fraction computations through modeling. Per their feedback, they feel most comfortable with traditional algorithms, but when asked why and how do the algorithms work, the preservice teachers offer little to no justification. This topic of conversation allows for rich discourse, especially if the preservice teachers can observe elementary and middle school age students learning fraction computations with manipulatives like Pattern Blocks. I have seen a rise in motivation to learn a new or different method by the preservice teachers after they have seen first-hand the great

impact of conceptually developing concepts with hands-on learning and school aged students.

Benchmarks of 5, 10, and 20
Introduction to number theory begins early where students begin developing cardinality, benchmarks, number sense and more. Rubberchickenapps has an ideal iPad application called 10s Frame. In a traditional setting, students would be given a paper copy of a ten-frame and some sort of counter, beans, or small object. Figures 6a-c highlights sample problems where students represent numbers on a five-, ten-, or twenty-frame. Especially important for the primary grades, is developing Fact Families which build number sense skills to be applied in later grades. For example, Figure 6a represents the number three. Students should be able to identify by the representation that three is two less than five, and $3 + 2 = 5$. Figure 6b shows a model of the number four, but also depicts $4 + 6 = 10$, $10 - 6 = 4$, and $10 - 4 = 6$.

These frames are also an early introduction to array models of multiplication by one row of five is five, two rows of five is ten, and four rows of five is twenty. Developing early number concepts is essential at the primary grade levels, as these skills become rote and then applied in future mathematics applications.
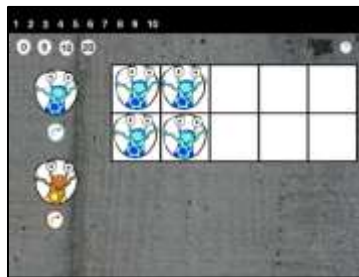

Figure 6a: Benchmark of Five

Figure 6b: Benchmark of Ten

Figure 6c: Benchmark of Twenty

Activities for Algebra Concepts
Algebra tiles are a great tool used for exploring and modeling polynomial computations. Brainingcamp, LLC was the developer of the previously mention Pattern Blocks iPad application, and they also have created Algebra Tiles for the iPad for $0.99. Similar to their Pattern Blocks application, Algebra Tiles also has a built in whiteboard feature with four colored pens.

Simplifying Expressions
Students can model addition and subtraction of expressions with algebra tiles. This is particularly helpful when emphasizing zero pairs (one positive and one negative) as in a problem like, $(2x - 1) + (x + 1)$. To model this problem, students would show each term with the appropriate tile (Figure 9a), then indicate that +1 and -1 have the sum of zero or they create a zero pair (Figure 9b). Striking through each of those counters to show they are removed from the problem is common notation for the pictorial model, as they simplify to show the sum is 3x.
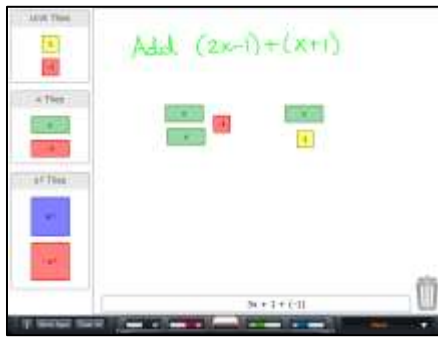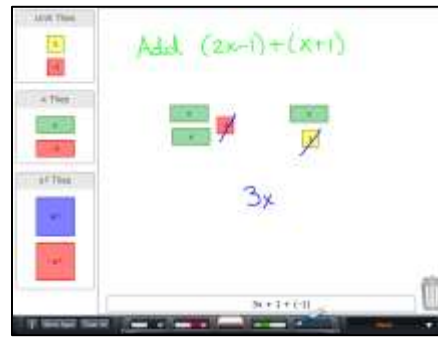
Figure 9a: Modeling Addition   Figure 9b: Modeling Zero Pairs

In many Algebra courses a visual or concrete model of polynomial expressions is often not provided, keeping the concepts abstract to several students. The use of algebra tiles assists students in developing understanding of variables with a visual representation.

Multiply Polynomials with Algebra Tiles

As shown in Figure 7, students were able to annotate their process for multiplying two binomials while using the virtual algebra tiles to model the problem. Additionally, they have the opportunity to name and save their work to review at a later time.
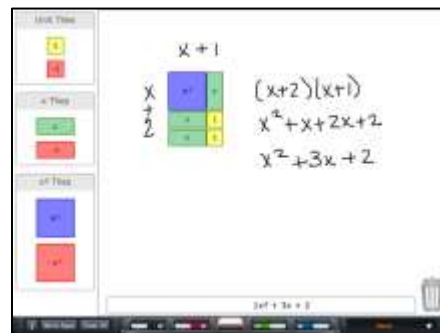

Figure 7: Algebra Tiles

Factor Quadratics with Algebra Tiles

An accompanying activity to multiplying polynomials would be factoring. Provide students with a quadratic expression such as, $x^2 + x - 6$, and ask them to factor using algebra tiles (see Figure 8a). The key to using algebra tiles is to retain a rectangle as the model. Students typically begin solving this problem with a guess-and-check approach as in Figure 8b. Figure 8b shows how a student represented each term in the quadratic, but was not able to create a rectangle with these algebra tiles.
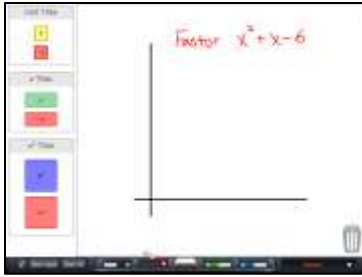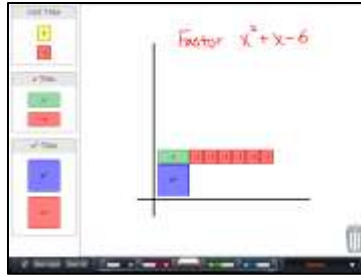
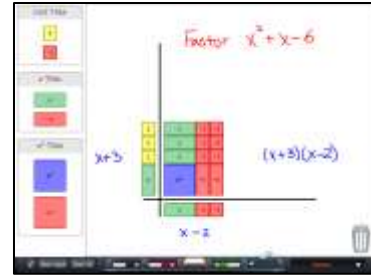| Figure 8a: Factor Board | Figure 8b: Factoring Attempt | Figure 8c: Factoring Solution |

Through further exploration, the student will discover two sets of zero pair x-terms are necessary to create a rectangle; creating the solution of $(x + 3)(x - 2)$ as seen in Figure 8c.

Exploring Algebraic Patterns

Graphing calculators are a well-known staple in any mathematics classroom, and todays preservice teachers have been a product of curriculum that embraces their use. The iPad applications, TI™-Nspire and TI™-Nspire CAS , offer the same capabilities of a TI™-83, 84, or Nspire version but much more. First, the iPad screen provides significantly more room than any of the screens on a Texas Instruments calculator, and there is color! Moreover, there are premade activities to be downloaded into the application, of which only costs $29.99.

Figure 10 depicts an activity where students explore growing patterns. Notice there is a picture tiles representing the pattern, a table indicating the number of tiles per iteration of the pattern, ad a graph beginning to show a linear relationship.
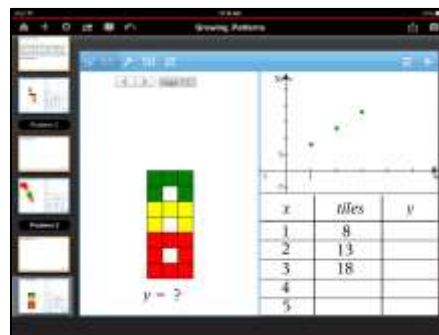

Figure 10: TI™-Nspire CAS Growing Patterns

The value of all three of aspects shown on one screen is important for student learning. Since it takes less than a second to transverse through the growing pattern, students can see more quickly the changes, that they would not be as apparent if they had to model this pattern using hand-held color tiles.

The Classic Box Problem Exploration

Every algebra class encounters the classic box problem, where students are given a visual of an open box made by taking a sheet of cardboard and removing a square from each corner. They are to explore aspects of volume and surface area from this scenario. Again, the TI™-Nspire CAS application has this classic exploraiton as a download as seen in Figure 11.
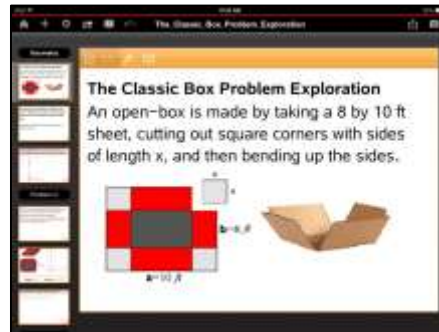

Figure 11: TI™-Nspire CAS Box Problem

Students will advance through this exploration by selecting the slides on the left side of the screen. The goal of this activity is to determine the size of the square cut-outs that result in the largest volume for this box. Within this application students will drag a blue dot which changes the side length of the square cut-out and produces a graph depicting the volume of the box and the accompanying side length of the square (see Figure 12), to determine when the volume is maximized.

Figure 12: Box Problem Volume Explorations

Conclusion

This paper showcased a few activities where one can pair virtual manipulatives with traditional concrete manipulatives, for enhancing student learning. Using classic K-12 mathematics problems to prepare preservice teachers will allow them gain experience with content and actual activities they will use while teaching. Beyond training preservice teachers with activities describe herein, it would be recommended for them to compare and contrast concrete manipulatives with their virtual counterparts; evaluating pros, cons, technology requirements, and classroom considerations for either type of manipulative.

Additional iPad implementation suggestions for creating video tutorials, creating and incorporating clicker-type assessments in instruction, developing 3D presentations, and more can be found in the paper, *Engaging Students with their iPad and iPhone Applications*, written by Christina Gawlik (2014).

References

Gawlik, C. (2014). *Engaging students with their iPad and iPhone applications. Proceedings of the 25th Annual International Conference on Technology in Collegiate Mathematics 2013* (pp.159-168). Boston, MA: Pearson Education, Inc.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for Teaching Mathematics*. NCTM: Reston, VA.

National Governors Association Center for Best Practices & council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics* (CCSS). Washington, DC: Authors.

Applications

Geoboard. Math Learning Center. (2012). http://catalog.mathlearningcenter.org/apps/geoboard

Notability. Ginger Labs, Inc. (2014). www.gingerlabs.com

Pattern Blocks. Brainingcamp, LLC. (2013). http://www.brainingcamp.com/product/mobile.html

TI-Nspire™ CAS. (2014). http://education.ti.com/en/us/products/apps/ti-nspire-cas-app-for-ipad/tabs/overview

10s Frame. rubberchickenapps. (2011). http://rubberchickenapps.com/2011/12/07/10s-frame/