VBA PROGRAMMING TECHNIQUES IN OPERATIONS RESEARCH

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Operations research is a field that is rich in applied mathematical problems from business and economics. Since problems in this field can be computationally difficult and time consuming to solve software packages such as Lindo are often times used. However, typical software packages are not able to give students insight to the mathematical techniques used to solve linear programming problems since only the problem solutions are output. Thus, these programs become little more than black- boxes to students. In this paper we will demonstrate several Visual Basic programs linked to Excel that students can use as an aid to solve such problems yet also require students to demonstrate an understanding of the solution techniques employed, thus, bridging the gap between problem setups and solution output.

We begin with a typical linear programming problem.

Example: A small clothing manufacturer produces shirts, pants and coats. Each shirt can be sold for \$60, each pants for \$40 and each coat for \$50. Production of each requires labor, such as, cutting minutes, sewing minutes and ironing minutes according to Table 1.

	Shirt	Pants	Coat	Resource Limitation
Cutting	10	8	2	50
Sewing	5	3	4	9
Ironing	5	5	2	10

Table 1 Clothing Resource Requirements

Also, the maximum number of shirts that can be sold is 10 and the maximum number of pants that can be sold in 6. The problem can then be summarized as follows,

$$\begin{aligned}
\text{maximize } z &= 60x_1 + 40x_2 + 50x_3 \\
\text{subject to: } 10x_1 + 8x_2 + 2x_3 &\leq 50 \\
5x_1 + 3x_2 + 4x_3 &\leq 9 \\
5x_1 + 5x_2 + 2x_3 &\leq 10 \\
x_1 &\leq 10 \\
x_2 &\leq 6 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

The problem is placed in standard form whereby slack variables are added.

$$
\begin{aligned}
\text{maximize } z &= 60x_1 + 50x_2 + 40x_3 + 0s_1 + 0s_2 + 0s_3 \\
\text{subject to: } 10x_1 + 8x_2 + 2x_3 + s_1 &= 50 \\
5x_1 + 3x_2 + 4x_3 + s_2 &= 9 \\
5x_1 + 5x_2 + 2x_3 + s_3 &= 10 \\
x_2 + s_4 &= 10 \\
x_3 + s_5 &= 6 \\
x_1,\ x_2,\ x_3, s_1,\ s_2,\ s_3,\ s_4,\ s_5 &\geq 0
\end{aligned}
$$

The simplex method is performed by assuming an initial non-optimized solution $s_1 = 50,\ s_2 = 9,\ s_3 = 10,\ s_4 = 10,\ s_5 = 6$ and $x_1 = x_2 = x_3 = 0$. The non-zero set of variables $\{s_1,\ s_2,\ s_3,\ s_4,\ s_5\}$ is called the basic variable set (BV) and the set of zero variables $\{x_1, x_2, x_3\}$ is called the non-basic variable set (NBV).

When the simplex method is performed a variable in the basic set is chosen and increased to a maximum allowed value thus entering the BV set. In order for a non-basic variable to be increased and keep the constraint equations true the basic variables will change and one or more of them will decrease to 0 and enter the NBV set. For example, if we choose to increase x_1 the largest it can be increased to is $\min\{\frac{50}{10}, \frac{9}{5}, \frac{10}{5}, \infty, \infty\}$. If x_1 were increased more than this than one of the basic variables would have to become negative which violates the constraints. Thus, we see that if we set $x_1 = \frac{9}{5}$ then s_2 must decrease to 0 and enter NBV, and s_1 and s_3 must decrease to 30 and 1 respectively. The objective function $z = 60x_1 + 50x_2 + 40x_3$ then increases from $z = 0$ under the initial non-optimized solution of

$$
\begin{aligned}
\text{BV} &= \{s_1 = 50,\ s_2 = 9,\ s_3 = 10,\ s_4 = 10,\ s_5 = 6\} \\
\text{NBV} &= \{x_1 = 0, x_2 = 0, x_3 = 0\}
\end{aligned}
$$

to $z = 60 \cdot \frac{9}{5} = 108$ under the still non-optimized but larger $z -$ value solution of

$$
\begin{aligned}
\text{BV} &= \{x_1 = 9/5,\ s_1 = 30,\ s_3 = 1,\ s_4 = 10,\ s_5 = 6\} \\
\text{NBV} &= \{s_2 = 0, x_2 = 0, x_3 = 0\}.
\end{aligned}
$$

Notice how x_1 moved from the NBV set to the BV set which then forced s_2 to move from the BV set to the NBV set and during this process z increased from 0 to 108.

The process of moving a variable from the NBV set to the BV set so that the objective function value is increased can be quickly performed by a pivot of the following matrix representation of the standard form problem as shown below.

$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{s_1}$	$\boldsymbol{s_2}$	$\boldsymbol{s_3}$	$\boldsymbol{s_4}$	$\boldsymbol{s_5}$	$\boldsymbol{z}$
-60	-40	-50	0	0	0	0	0	0
10	8	2	1	0	0	0	0	50
5	3	4	0	1	0	0	0	9
5	5	2	0	0	1	0	0	10
0	1	0	0	0	0	1	0	10
0	0	1	0	0	0	0	1	6

$BV = \{x_1,\, s_1, s_3, s_4, s_5\}$
$NBV = \{x_1,\, x_2, x_3\}$

Pivoting about the row 2 column 1 entry of **5** where column 1 corresponds the variable chosen to enter the basis we get,

$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{s_1}$	$\boldsymbol{s_2}$	$\boldsymbol{s_3}$	$\boldsymbol{s_4}$	$\boldsymbol{s_5}$	$\boldsymbol{z}$
0	-4	-2	0	12	0	0	0	108
0	2	0	1	-2	0	0	0	32
1	0.6	0.8	0	0.2	0	0	0	1.8
0	2	-2	0	-1	1	0	0	1
0	1	0	0	0	0	1	0	10
0	0	1	0	0	0	0	1	6

$BV = \{x_1 = 1.8,\, s_1 = 32, s_3 = 1, s_4 = 10, s_5 = 6\}$
$NBV = \{x_2 = 0,\, x_3 = 0, s_2 = 0\}$

Considering the new objective function row,

$$\begin{pmatrix} 0 & -4 & -2 & 0 & 12 & 0 & 0 & 0 & 108 \end{pmatrix}$$

notice that the new objective function value of 108 appears in the z-column. Using this row we can decide if another variable can be moved from NBV to BV and increase z further. The value -4 indicates that for every unit increase in x_2 z will further increase by 4 and the value of -2 indicates that for every unit increase in x_3 z will further increase by 2. The value of 12 indicates that if s_2 is increased by one unit z will decrease by 12 so we do not want to select s_2 to enter the basis. Since by increasing x_2 z will increase at the fastest rate we choose x_2 to become a basic variable. x_2 can be increased up to $\min\{\frac{32}{2}, \frac{1.8}{0.6}, \frac{2}{1}, \frac{10}{1}\} = 2$. Pivoting about the row 4, column 2 entry of 2 yields the new matrix,

$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{s_1}$	$\boldsymbol{s_2}$	$\boldsymbol{s_3}$	$\boldsymbol{s_4}$	$\boldsymbol{s_5}$	$\boldsymbol{z}$
0	0	-6	0	10	2	0	0	110
0	0	2	1	-1	-1	0	0	31
1	0	1.4	0	0.5	-0.3	0	0	1.5
0	1	-1	0	-0.5	0.5	0	0	0.5

0	0	1	0	0.5	-0.5	1	0	9.5
0	0	1	0	0	0	0	1	6

$BV = \{x_1 = 1.5,\ x_2 = 0.5, s_1 = 31, s_4 = 9.5, s_5 = 6\}$
$NBV = \{x_3 = 0,\ s_2 = 0, s_3 = 0\}$

By the same logic, we see that we can the objective function value z which is currently 110 can be increased by moving x_3 from the NBV set to the BV set. We can increase x_3 up to the maximum value of $\min\{\frac{31}{2}, \frac{1.5}{1.4}, \infty, \frac{9.5}{1}, \frac{6}{1}\} = \frac{15}{14}$. Notice if x_3 is to be increased then row 4,

$$(0 \quad 1 \quad -1 \quad 0 \quad -0.5 \quad 0.5 \quad 1 \quad 0 \quad 0.5)$$

corresponds to the equation $x_2 = 0.5 + x_3$ so x_3 can be increased without bound without forcing x_2 to leave BV. Thus the ∞ in $\min\{\frac{31}{2}, \frac{1.5}{1.4}, \infty, \frac{9.5}{1}, \frac{6}{1}\}$. Pivoting about the value 1.4, then yields,

x_1	x_2	x_3	s_1	s_2	s_3	s_4	s_5	z
4.29	0	0	0	12.14	0.71	0	0	116.43
-1.43	0	0	1	-1.71	-0.57	0	0	28.86
0.71	0	1	0	0.36	-0.21	0	0	1.07
0.71	1	0	0	-0.14	0.29	0	0	1.57
-0.71	0	0	0	0.14	-0.29	1	0	8.43
-0.71	0	0	0	-0.36	0.21	0	1	4.93

$BV = \{x_2 = 1.57,\ x_3 = 1.07, s_1 = 28.86, s_4 = 8.43, s_5 = 4.93\}$
$NBV = \{x_1 = 0,\ s_2 = 0, s_3 = 0\}$

At this point we see that the objective function row has no negative values in it which indicates that no variable can be moved from NBV to BV and increase z. Therefore, an optimal solution has been found, namely, $x_1 = 0,\ x_2 = 1.57,\ x_3 = 1.07$ yielding an optimal objective function value $z = 116.43$.

For the pivoting process in this example we use the following VBA Excel macro. The VBA editor can be accessed from Excel by selecting *View* → *Macros*. If no macro currently exists a new one must be created and then select *Edit* to open the VBA editor.

```
Sub Simplex_Pivot()
' Macro recorded 5/27/2012 by David Nawrocki
' Keyboard Shortcut: Ctrl+p
   Dim RowDim As Integer
   Dim ColDim As Integer
   RowDim = 0
   ColDim = 0
```

```
    Do
        RowDim = RowDim + 1
    Loop While Cells(RowDim + 1, 1) <> ""
    Do
        ColDim = ColDim + 1
    Loop While Cells(1, ColDim + 1) <> ""
    R = ActiveCell.Row
    C = ActiveCell.Column
    For Row = 0 To RowDim - 1
        P1 = Cells(Row + 1, C).Value
        P2 = Cells(R, C).Value
        For Column = 0 To ColDim - 1
            If Row + 1 = R Then
                Cells(Row + 1, Column + 1).Value = Cells(Row + 1, Column + 1).Value / P2
            Else
                Cells(Row + 1, Column + 1).Value = Cells(Row + 1, Column + 1).Value -
Cells(R, Column + 1).Value * P1 / P2
            End If
        Next Column
    Next Row
End Sub
```

The Keyboard Shortcut can be set as a macro option in the *Macro* dialog box. For this example, the macro is executed from the worksheet anytime the keystrokes CTRL+p are typed.

Unbounded Linear Programming Problems

Our macro can also be used to easily identify various problems that can arise with linear programming problems such as cycling, infeasible problems and unbounded problems. We will now demonstrate how to identify unbounded linear programming problems. Consider the following linear program,

$$\begin{aligned} &\text{maximize } z = 36x_1 + 30x_2 + 3x_3 + 4x_4 \\ &\text{subject to: } x_1 + x_2 - x_3 \leq 5 \\ &\qquad 6x_1 + 5x_2 - 4x_4 \leq 9 \\ &\qquad x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

To solve this problem we set up the following pivoting table.

x_1	x_2	x_3	x_4	s_1	s_2	z
-36	-30	-3	-4	0	0	0
1	1	-1	0	1	0	5
5	3	0	-4	1	0	9

By performing the same pivoting technique as described in the previous example we arrive at the following table.

x_1	x_2	x_3	x_4	s_1	s_2	z
0	7	-45	0	42	-1	200
0	0.25	-1.5	1	1.5	-0.25	5
1	1	-1	0	1	0	5

At this point the basic and non-basic variable sets are as follows,

$$BV = \{x_1 = 5,\, x_4 = 5\}$$
$$NBV = \{x_2 = 0,\, x_3 = 0,\, s_1 = 0, s_2 = 0\}.$$

We see that for each unit increase in x_3, z will increase by 45. Furthermore, since all the coefficients in the x_3 column are negative, x_3 can be increased without bound. That is, as x_3 is increased no other variables from the basic variable set will be forced to zero. Thus, the problem is unbounded.

Solving Transportation Problems

We can also use this macro to illustrate solution techniques for transportation problems.

Example: ElectricCo has three electric power plants that supply electricity for four cities. Each power plant has the following capacities: plant 1 can supply 45 mwh, plant 2 can supply 62 mwh and plant 3 can supply 50 mwh. Assume the peak power demands in these cities occur at the same time and are: city 1 requires 55 mwh, city 2 requires 34 mwh, city 3 requires 28 mwh and city 4 requires 40 mwh. The costs of sending 1 mwh of energy from plant i to city j is summarized in Table 2 below.

	City 1	City 2	City 3	City 4	Supply
Plant 1	8	6	9	10	45
Plant 2	9	10	14	6	62
Plant 3	14	9	15	7	50
Demand	55	34	28	40	

Table 2 ElectricCo Costs

Let c_{ij} be cost of sending 1 mwh of electricity from plant i to city j and x_{ij} be number of mwh shipped from plant i to city j. Then the linear program may be written as:

$$\text{minimize } z = \sum_{i,j} c_{ij} \cdot x_{ij}$$

$$\text{s.t.} \qquad \sum_{j} x_{ij} \leq S_i$$

$$\sum_{i} x_{ij} \geq D_j$$

For this particular problem,

$$\text{minimize } z = 8x_{11} + 6x_{12} + 9x_{13} + 10x_{14} + 9x_{21} + 10x_{22} + 14x_{23} + 6x_{24} + 14x_{31} + 9x_{32} + 15x_{33} + 6x_{34}$$

$$\begin{aligned} \text{s.t. } x_{11} + x_{12} + x_{13} + x_{14} &\leq 45 \\ x_{21} + x_{22} + x_{23} + x_{24} &\leq 62 \\ x_{31} + x_{32} + x_{33} + x_{34} &\leq 50 \\ x_{11} + x_{21} + x_{31} &\geq 55 \\ x_{12} + x_{22} + x_{32} &\geq 34 \\ x_{13} + x_{23} + x_{33} &\geq 28 \\ x_{14} + x_{24} + x_{34} &\geq 40 \end{aligned}$$

Notice for this problem that the total supply and demand are balanced. That is to say, $\sum S_i = \sum D_i$. If this were not the case then we would have to balance the problem by adding a dummy demand point if total supply exceeded total demand or by adding an additional dummy supply point and perhaps a penalty to the objective function if total demand exceeded total supply.

Before we can use the pivoting program to solve this problem we must find a Basic Feasible. Several common methods to do this are the Northwest corner method, the minimum cost method and Vogel's method. We will use the Northwest corner method here since it is the easiest although the other methods will in general yield initial basic feasible solutions that are closer to the optimal solution. Applying the Northwest corner method yields Table 3 below.

	City 1	City 2	City 3	City 4	Supply
Plant 1	45	0	0	0	45
Plant 2	10	34	18	0	62
Plant 3	0	0	10	40	50
Demand	55	24	28	40	

Table 3 ElectricCo b.f.s.

This gives us a starting basic feasible solution with basic variable set given by,

$$\text{BV} = \{x_{11} = 45, x_{21} = 10, x_{22} = 34, x_{23} = 18, x_{33} = 10, x_{34} = 40\}.$$

We can use this information to begin a pivoting process to solve the problem. Starting with x_{11} we select a row to pivot about by choosing the minimum of the values in the rightmost column. Notice we can pivot about the row corresponding to 45 or that corresponding to the value 55 in the rightmost column. Since Plant 1 has at most 45 mwh of output, we must choose to pivot about the row corresponding to 45 (row 2).

x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	
-8	-6	-9	-10	-9	-10	-14	-6	-14	-9	-15	-6	0
1	1	1	1	0	0	0	0	0	0	0	0	45
0	0	0	0	1	1	1	1	0	0	0	0	62
0	0	0	0	0	0	0	0	1	1	1	1	50
1	0	0	0	1	0	0	0	1	0	0	0	55
0	1	0	0	0	1	0	0	0	1	0	0	34
0	0	1	0	0	0	1	0	0	0	1	0	28
0	0	0	1	0	0	0	1	0	0	0	1	40

By pivoting about the other basic variable columns in a similar manner we obtain the following table.

x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	
0	3	4	-6	0	0	0	-1	-4	2	0	0	1432
1	1	1	1	0	0	0	0	0	0	0	0	45
0	0	**1**	1	0	0	1	1	-1	-1	0	0	18
0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	-1	-1	1	0	0	0	1	0	0	0	10
0	1	0	0	0	1	0	0	0	1	0	0	34
0	0	0	-1	0	0	0	-1	1	1	1	0	10
0	0	0	1	0	0	0	1	0	0	0	1	40

From here we see that by increasing x_{13} we can further reduce the current cost of 1432. Doing so will yield the next table.

x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	
0	3	0	-10	0	0	-4	-5	0	6	0	0	1360
1	1	0	0	0	0	-1	-1	1	1	0	0	27
0	0	1	1	0	0	1	1	-1	-1	0	0	18
0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	1	0	1	1	0	-1	0	0	28
0	1	0	0	0	1	0	0	0	1	0	0	34
0	0	0	-1	0	0	0	-1	1	**1**	1	0	10
0	0	0	1	0	0	0	1	0	0	0	1	40

Notice that x_{13} has entered the basis and x_{23} has left the basis. The basic variable set is now,

$$\text{BV} = \{x_{11} = 27, x_{13} = 18,\ x_{21} = 28, x_{22} = 34,\ x_{33} = 10, x_{34} = 40\}.$$

Continuing in this manner the optimal table is,

x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	
-2	0	0	-7	0	-1	-2	0	-5	0	-3	0	1242
1	1	0	1	0	0	-1	0	0	0	-1	0	17
0	0	1	0	0	0	1	0	0	0	1	0	28
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	0	55
-1	0	0	0	0	1	1	1	-1	0	0	0	7
-1	0	0	-1	0	1	1	0	0	1	1	0	17
1	0	0	1	0	-1	-1	0	1	0	0	1	33

with $\text{BV} = \{x_{12} = 17, x_{13} = 28,\ x_{21} = 55,\ x_{24} = 7,\ x_{32} = 17, x_{34} = 33\}$. Our optimal solution can now be summarized as shown in Table 4.

	City 1	City 2	City 3	City 4	Supply
Plant 1	0	17	28	0	45
Plant 2	55	0	0	7	62
Plant 3	0	17	0	33	50
Demand	55	24	28	40	

Table 4 Optimal ElectricCo Solution

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