THE GOVERNMENT SHUTDOWN AND SEQUESTRATION: MAKING LEMONADE OUT OF LEMONS

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On Day 1, Parks Close, Workers Stay Home and 'Panda Cam' Goes Dark

Doug Mills/The New York Times

A City in Limbo After the Shutdown: What happens to the nation's capital when the government is shut down?

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Figure 1: The United States Goverrnment Shutdown

The fall semester of academic year 2013-2014 unfolded against the backdrop of the shutdown of the federal government and the threat of default. At the United States

[1]The views expressed in this article are those of the authors and do not reflect the official policy or position of the U.S. Military Academy, the Department of the Army, the Department of Defense, or the U.S. Government.

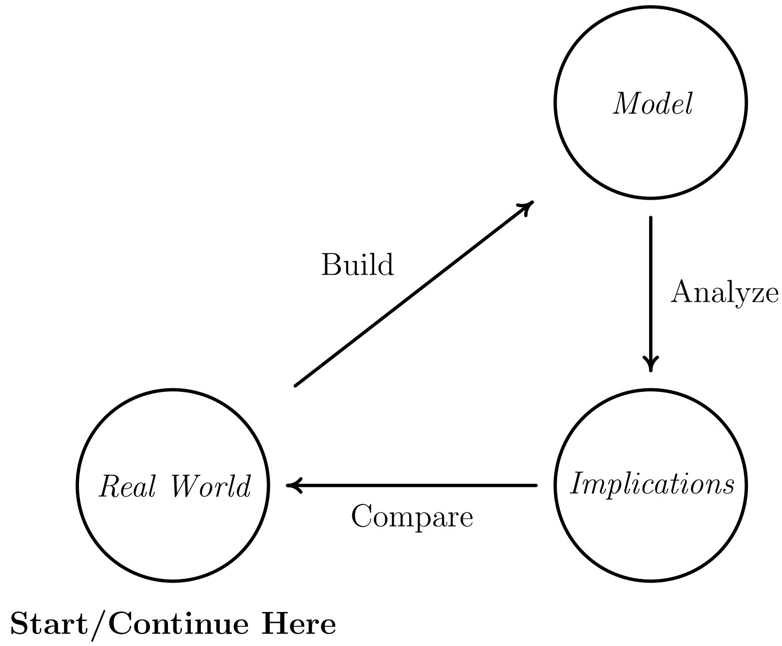


Figure 2: The modeling cycle

Military Academy the government shutdown was center stage, as civilian faculty were involuntarily placed in furlough status and thus forbidden from teaching their classes.

Although some might argue that the crises last fall were contrived, we do face serious problems and, as mathematicians, we can help our students make better public and personal policy decisions as we attempt to solve those problems. At the same time we can motivate the study of mathematics by showing its relevance to real, consequential, and controversial problems in the news.

Figure 2 is the heart of our approach to these problems and of the first course, MA-103, in the four-course core mathematics sequence taken by the vast majority of cadets at the United States Military Academy. The modeling cycle begins with problems in the real world, usually complex, messy, fraught with uncertainty and risk. This paper looks at some of the things we did in MA-103 to turn the lemons of the shutdown and sequestration into lemonade for our classes.

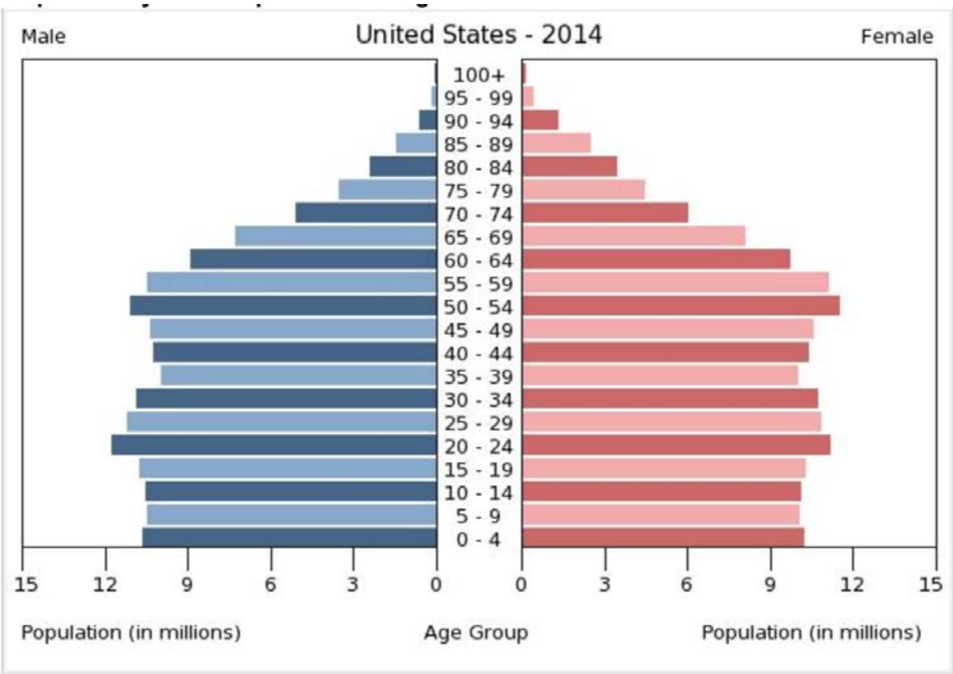
	2014			2034		
Age	**Both**	**Male**	**Female**	**Both**	**Male**	**Female**
Total	318,892,103	157,082,861	161,809,242	367,503,433	181,912,838	185,590,595
0-4	20,731,306	10,597,149	10,134,157	22,440,961	11,473,797	10,967,164
5-9	20,477,845	10,463,160	10,014,685	22,581,664	11,549,589	11,032,075
10-14	20,592,304	10,520,040	10,072,264	22,678,914	11,605,053	11,073,861
15-19	20,948,659	10,731,809	10,216,850	22,687,778	11,627,967	11,059,811
20-24	22,809,259	11,704,248	11,105,011	22,628,374	11,639,656	10,988,718
25-29	21,925,492	11,162,189	10,763,303	23,086,105	11,860,767	11,225,338
30-34	21,485,613	10,805,435	10,680,178	23,546,051	12,056,264	11,489,787
35-39	19,857,407	9,918,820	9,938,587	23,718,559	12,124,506	11,594,053
40-44	20,520,436	10,191,657	10,328,779	24,768,637	12,621,670	12,146,967
45-49	20,814,276	10,316,211	10,498,065	22,902,926	11,558,856	11,344,070
50-54	22,521,199	11,058,480	11,462,719	21,594,469	10,743,752	10,850,717
55-59	21,483,744	10,435,062	11,048,682	19,308,735	9,486,860	9,821,875
60-64	18,545,559	8,873,957	9,671,602	19,213,618	9,304,550	9,909,068
65-69	15,296,546	7,237,434	8,059,112	18,621,561	8,900,371	9,721,190
70-74	11,054,750	5,092,364	5,962,386	18,837,717	8,794,141	10,043,576
75-79	7,903,404	3,501,387	4,402,017	16,163,811	7,314,163	8,849,648
80-84	5,751,414	2,375,076	3,376,338	11,654,924	5,029,271	6,625,653
85-89	3,828,930	1,408,466	2,420,464	7,055,401	2,852,919	4,202,482
90-94	1,805,819	562,765	1,243,054	2,964,526	1,064,680	1,899,846
95-99	464,808	114,047	350,761	867,982	261,577	606,405
100+	73,333	13,105	60,228	180,720	42,429	138,291

Table 1: United States Population by Age and Gender 2014 and 2034

The data shown in Table 1 and Figure 3 is a good place to start. Figure 3 shows two population pyramids based on this data from the United States Census Bureau International Database.[2] This is an excellent source of data. The same data is available from the same source in spreadsheet form for easy analysis. This data together with a bit of arithmetic and some modeling can shed some light on the underlying problems. For example, we can ask students the following kinds of questions:

Question 1 *Many people are worried that government expenditures for health care and social security will rise for purely demographic reasons. What is the current percentage of our population that is over 65? What will the percentage of our population over 65 be in the year 2034?*

[2] http://www.census.gov/population/international/data/idb/informationGateway.php Accessed 18 May 2014.



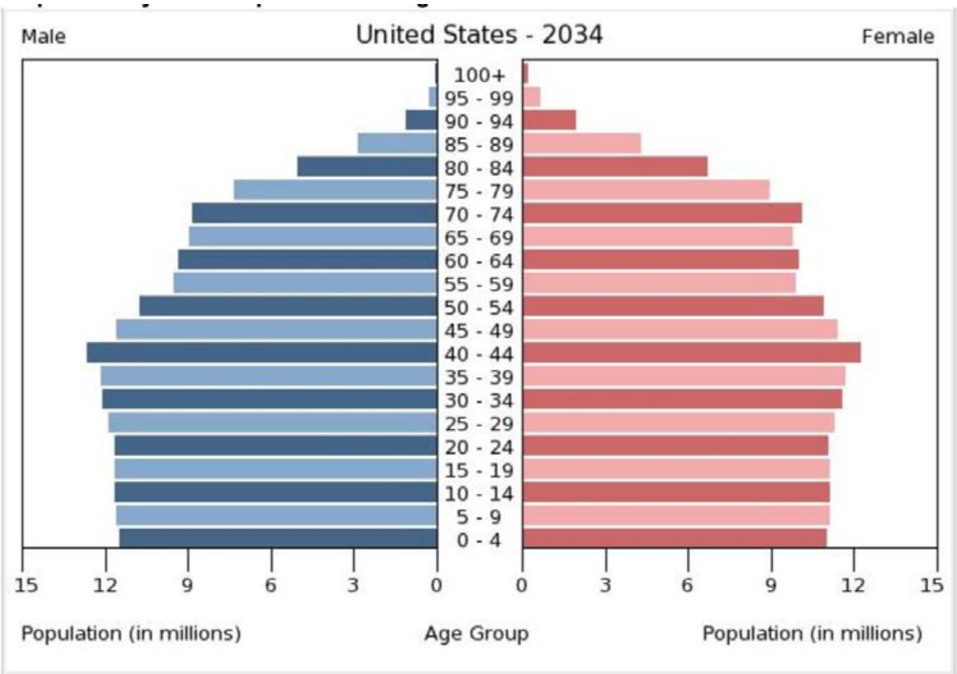


Figure 3: United States Population Pyramids 2014 and 2034

Question 2 *Generally speaking, people receive more benefits from governments*[3] *when they are young and when they are older and retired than when they are working age. In fact, when people are of working age they often pay more in taxes than they receive in government benefits. Develop a rough model of the net tax payment (taxes minus receipts) of people of various ages. Based on your model what are the net total taxes paid by a single person over the course of his or her life. You will need to make many assumptions. Based on your model what is the net revenue received by governments given the 2014 age distribution in the United States? What will be the net revenue received by governments given the 2034 age distribution in the United States? What impact would raising the retirement age by two years have on both these figures?*

Notice that the questions above focus on two narratives. The first narrative looks at a slice of time and the money transfers between taxpayers and consumers of public services. This narrative is often the focus of conservatives who cast much of the debate as an intergenerational dispute. The second narrative looks at an individual's lifetime. This narrative is often the focus of liberals, with taxpayers paying for their own previous education and contributing to trust funds that will pay for their retirement and medical care.

This is, of course, a good place to introduce Leslie matrices and age dependent population models. In MA 103 we develop an age dependent population model based on the same age groups as Figure 3 and Table 1. We represent the population by a vector $\vec{p} = \langle p_1, p_2, \ldots p_{21} \rangle$, where p_i is the population in the i^{th} age group – for example, the total population in 2014 in the first age group, from birth to 4 years, is $p_1 = 20.7$ M. We represent the population change over a five year period by

$$\vec{p}_n = A\vec{p}_{n-1} + \vec{d}.$$

where $\vec{p}_n$ represents the populations in the 21 age groups for the five year period represented by n, the vector $\vec{d}$ represents net immigration, and the entries in the matrix

[3] We use the plural because people pay taxes to, and receive benefits from, many different governments including but not limited to local, state, and the national government.

$$A = \begin{bmatrix} b_1 & b_2 & b_3 & \cdots & b_{20} & b_{21} \\ s_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & s_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & \cdots & s_{20} & s_{21} \end{bmatrix}$$

are given by age-dependent birth rates $b_1, b_2, \ldots b_{21}$ and survival rates $s_1, s_2, \ldots s_{21}$.

Besides helping inform the debate about taxes and expenditures in the United States, this model informs geopolitics. Students can work with this model to see the effects of, for example, improving public health as countries develop. They can also look at population pyramids for countries of particular interest to them. This is a place where the mathematics of demographics is intertwined with the future of countries (and the world). We have, for example, had cadets do papers on the effects of China's one-child policy and on Japan's demographic challenges.

Now we turn to an important aspect of the public debate about questions like taxes and the minimum wage. Underlying the debate are models of our economy and the effects of different possible courses-of-action. For example, conservatives often say that raising the minimum wage will throw people out of work and liberals often say that, by boosting consumer spending, raising the minimum wage will increase employment. In the real world we will see a combination of these two effects and we need mathematical models to help understand them.

In this paper, we will look at two examples of the relationship among the price, supply and demand for a product. The first example can be used in an algebra or pre-calculus course. The second example is particularly suited to a differential calculus course. We are interested in questions like – *what will happen if a tax is imposed on a product?* – or – *what will happen if its production cost goes up due to an increase in the cost of labor or energy?*

Figure 4 illustrates how we might attack these kinds of problems for a product whose price is governed by the law of supply and demand. We have many different buyers and producers. At higher prices (p) the demand for the product is lower and the supply is higher. In Figure 4 we look at the "toy" demand function $D(p) = 20,000 - 5,000p$. Using a spreadsheet or computer algebra system we can look at more complicated demand functions.

In the same figure we look at two different supply functions – an old supply

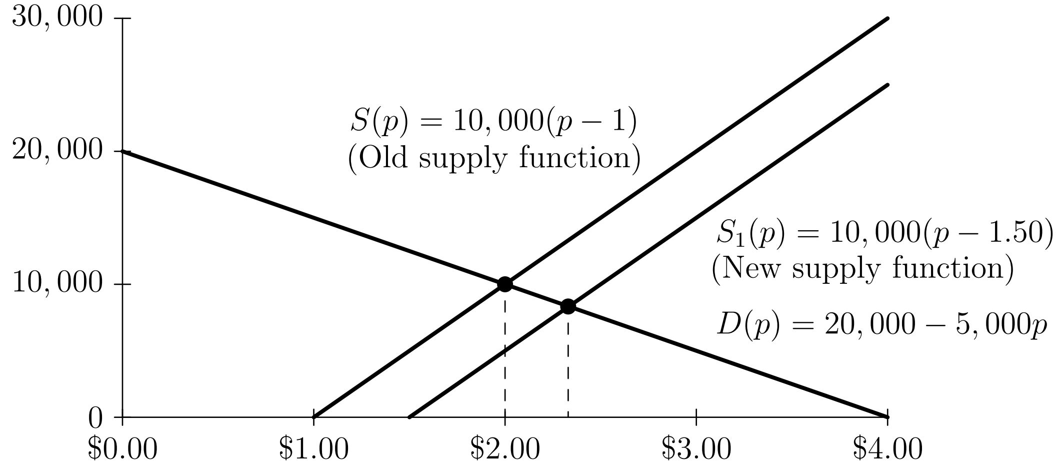


Figure 4: Comparing two supply functions

function $S(p) = 10,000(p-1)$ that might apply if the total cost of producing the product was \$1.00 and a new supply function, $S(p) = 10,000(p-1.50)$ that might apply after the production cost rose by \$0.50 to \$1.50. With lots of independent consumers who have other buying options and lots of independent producers who can adjust their output, the price converges to an equilibrium where supply and demand are equal. From the figure it is evident that when the production cost rose by \$0.50 the equilibrium price rose by a smaller amount – thus, the buyers absorbed part of the increase in production cost and the producers absorbed some of that cost, resulting in lower unit profits. At the same time the sales volume dropped, lowering employment and total profits. The following question is a typical question we ask in MA-103.

Question 3 *What are the results of the rise in production cost from \$1.00 to \$1.50 in the example above? Answer at least the questions below.*

- *What happens to the equilibrium price?*
- *Are the producers able to pass the rise in production costs on to the buyers?*
- *What happens to the profit per unit?*
- *What happens to the total profit?*

- *What happens to the total amount spent by buyers on this product? How would you expect this to affect demand for other products?*
- *How does this affect employment for the production of this product?*
- *How does this affect the demand for the products used to produce this product?*

In the model above we assumed that the price paid by buyers is the same as the price received by producers but this is not always true. We ask our students to answer similar questions for the impact of a tax.

The example above can be treated at many different levels. At the lowest levels we can use "toy" functions for supply and demand, like the functions above. At higher levels we can use more complex and plausibly realistic, even data-driven, supply and demand functions. We can also look at either discrete or continuous dynamical systems,

$$p_n = k(D(p_{n-1}) - S(p_{n-1})) \quad \text{or} \quad \frac{dp}{dt} = k(D(p) - S(p)).$$

With either a spreadsheet or computer algebra system, it is easy to look at questions like the effects of inflation – for example, the price of the product at time n (p_n) might depend on both p_{n-1} and p_{n-2}.

The interaction of the price, supply, and demand of a product described above was based on an ideal free market economy with lots of individual producers and lots of individual buyers, all competing with each other. For many products this is far from reality. There are often a very few or even just a single producer. In this case the producer or producers often are able to control the price and they usually set the price to maximize their profit. Suppose, for example, that the demand for a particular product is given by the function

$$D(p) = 10,000(5 - p)$$

and the production cost is \$1.50? Suppose there is a single producer who can control the price. What price should the producer charge to maximize the total profit?

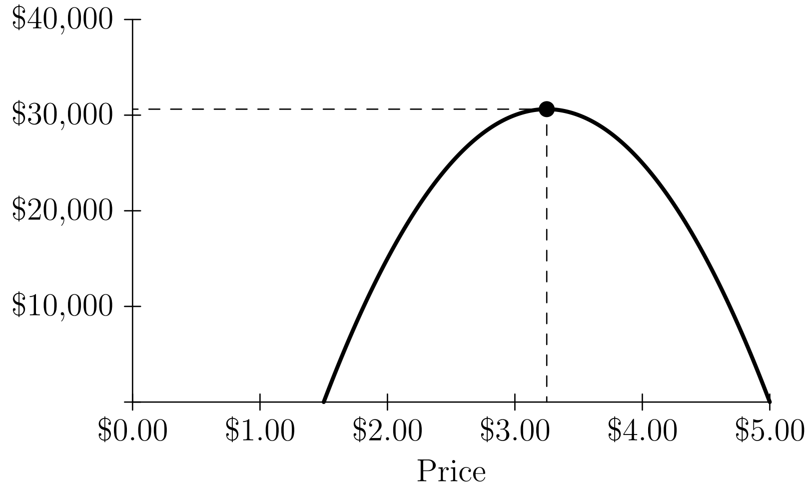


Figure 5: The total profit function $T(p) = 10,000(p - 1.50)(5 - p)$

The function

$$T(p) = (p - 1.50)D(p) = 10,000(p - 1.50)(5 - p)$$

computes the total profit as a function of the price. See Figure 5 for a graph of this function. By looking at this graph we can see that the maximum profit is a bit over $30,000 and occurs when the price is about $3.25. As usual we can find the maximum profit by looking at the critical points of the function $T(p)$

$$\begin{aligned} T(p) = 10,000(p - 1.50)(5 - p) &= -10,000p^2 + 65,000p - 75,000 \\ T'(p) &= -20,000p + 65,000 \end{aligned}$$

and by solving for points where the derivative is zero

$$\begin{aligned} 0 &= -20,000p + 65,000 \\ 20,000p &= 65,000 \end{aligned}$$

$$p = \$3.25$$

we see that $p = \$3.25$ is the profit-maximizing price.

Now we can ask the same kinds of questions we looked at above in the context of this profit-maximization model. Once again, we used a "toy" example but by using a spreadsheet or computer algebra system we can work with more realistic examples. In MA-103 we begin with examples where the demand function is linear and students notice a pattern – buyers pay one-half of an increase in production cost and the producers pay the other half. We ask them to explore whether this pattern is universal.

Question 4 *In all our examples so far, when the production cost rises the producer is only able to pass part of the rise on to the buyers? Is this always true? Is it ever possible that when the production cost rises the profit-maximizing price rises by more than the rise in the production cost?*

The most important part of this paper is not the particular examples above or even making lemonade out of the particular lemons of fall 2013. The real message is leveraging current events to motivate the study of mathematics at the same time we use mathematics to elevate the debate and help us make better decisions. Some important sources for current events are newspapers and the media. In addition, data sources like the United States Census Bureau and the BP Statistical Review of World Energy are especially valuable. Three of our favorite background books are:

- *The Measure of a Nation: How to Regain America's Competitive Edge and Boost Our Global Standing* by Howard Steven Friedman. This book is great reading for students studying modeling and metrics.
- *Hot, Flat, and Crowded: Why We Need a Green Revolution and How It Can Renew America* by Thomas L. Friedman. This book provides background for the nexus of hot (climate change and the environment), flat (globalization), and crowded (population growth).
- *The Price of Inequality: How Today's Divided Society Endangers Our Future* by Joseph Stiglitz. This book provides insight into both income inequality and its impact.