

## USING GEOGEBRA TO ENHANCE CONCEPTUAL UNDERSTANDING IN THE CALCULUS SEQUENCE

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The combination of dynamic capability and ease-of-use make GeoGebra an ideal software to support conceptual learning in the calculus sequence. In this paper I will discuss GeoGebra activities that I use with Calculus I and II students to learn basic calculus ideas with deep understanding. I have used GeoGebra activities to help students learn many topics including: the limit definition of the derivative; the fundamental theorem of calculus; the sequence of partial sums concept; ideas relating to approximate integration; and connecting polar graphs to auxiliary Cartesian graphs. The value of these activities is to help students go beyond understanding *how* calculus works, to developing intuition about *why* calculus works.

In the activity I developed for the limit definition of the derivative concept, students create their own applets at home by following six short podcasts. In their applets students create sliders, use the sliders to draw dynamic secant lines, and compute the slope of their secant lines (see Fig. 1). Students take ownership of knowledge by creating their own applet and assigning colors of their choice. In class, students work out how use their applet for a variety of functions and points on the functions to compute slope of secant line and slope of tangent line. The dynamic aspect of the secant lines allows students to explore connections between the secant line concept, the average rate of change concept, the tangent line concept and the instantaneous rate of change concept. In the end students have developed the limit definition of the derivative, at a very concrete level.

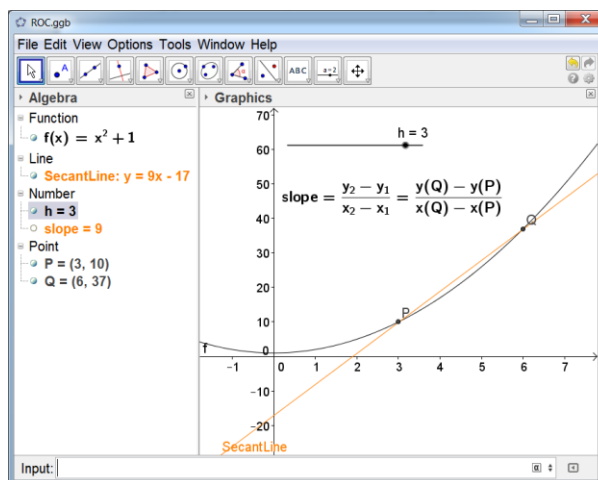


Figure 1

When teaching the Fundamental Theorem of Calculus, I have discovered that it is very

easy to get students to respond correctly to a question like:  $g(x) = \int_0^x t^3 dt$ , find  $g'(x)$ .

However, more probing of student understanding demonstrates that students are often

simply responding to the cue “find the prime of a long tall s” with the rule “replace  $t$  by  $x$ .” They aren’t necessarily understanding what is meant by an accumulation function, or that slope and area are inverse operations.

I developed a GeoGebra activity in an attempt to help students gain this understanding. Students follow a series of podcasts to create their own applet at home. Students use the “Integral” command to create a point on an accumulation function, and then use the “Trace” feature to create the accumulation function (see Fig. 2). Next, students create an approximate tangent line to the accumulation function, use the “Slope” command to compute its slope and hence the derivative of the accumulation function. Finally, students

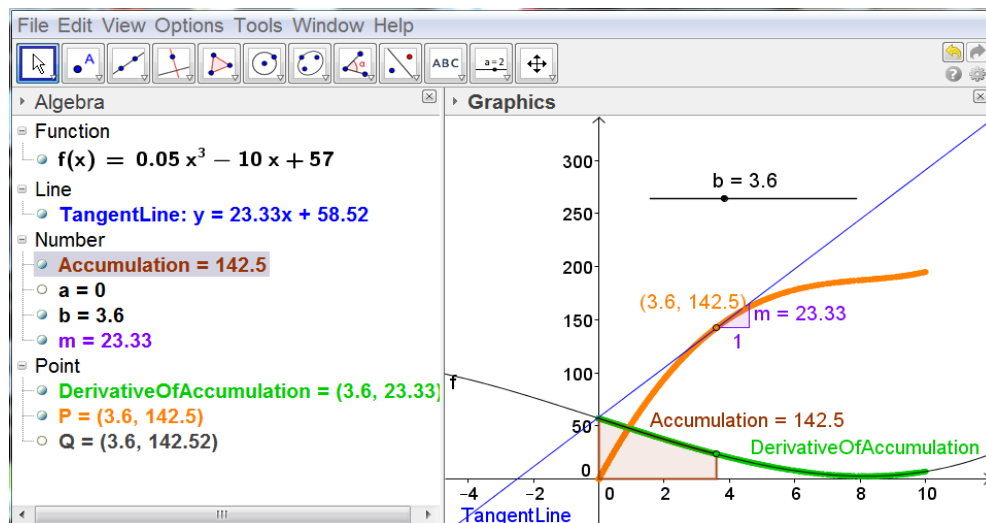


Figure 2

create a point on the derivative of the accumulation function and use the ”Trace” feature to create the derivative of the accumulation function. In class, students make conjectures about the relationship between the original function and the derivative of the accumulation function. They explore what happens when they change the original function or the lower endpoint of the integral in their accumulation function. Eventually students discover that, independent of their function or their lower endpoint, the derivative of the accumulation function is equal to the original function, thus discovering the statement of the fundamental theorem of calculus, in a very concrete way.

Rather than simply having students learn to compute approximate integrals with the Midpoint Rule, Trapezoid Rule and Simpson’s Rule, I work to help them understand why they are getting their answers. Students bring laptops to class and use the spreadsheet in GeoGebra to compute the approximations and then they compare answers to the “true” value of the integral that GeoGebra computes with the “Integral” command ( see Fig. 3). We connect the meaning of the “true” value of the integral to the area concept and discuss how this “true” value may itself be an approximation. GeoGebra’s dynamical capability allows students to change their function and compare approximations easily. So by exploring different functions, students create conjectures about when to expect an

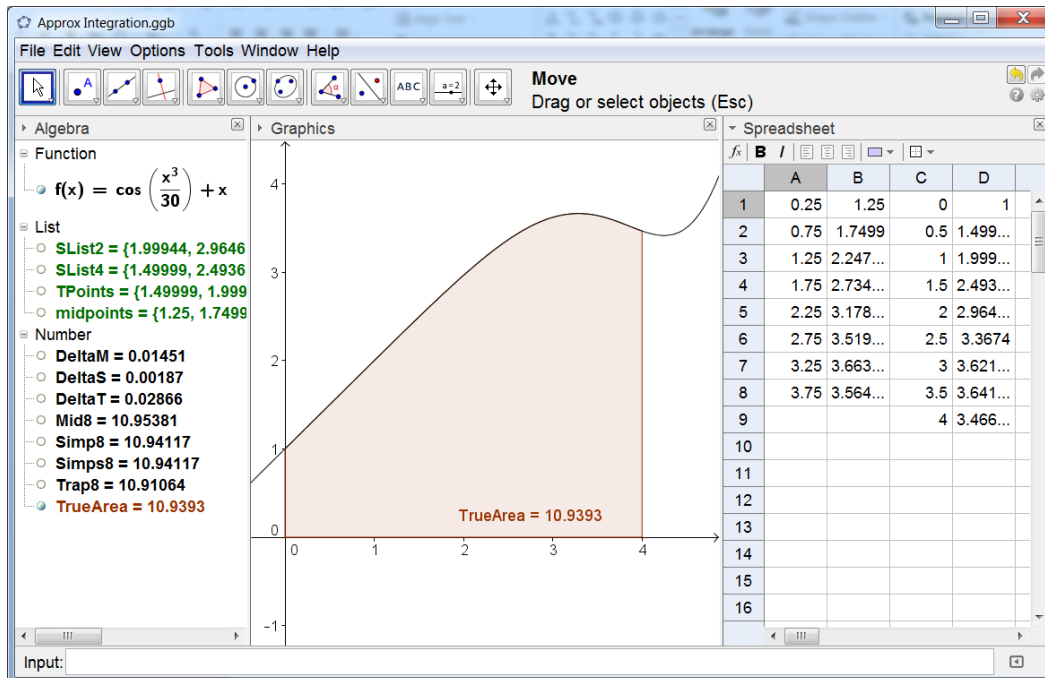


Figure 3

overestimate, when to expect an underestimate and when to expect an exact value. This leads into discussion about the theory underlying each of the approximations and proofs of their conjectures. For example, students conjecture that Simpson’s Rule will always give the exact answer if the function is a second degree polynomial. This gives rise to a discussion about parabolas, Simpson’s Rule and a proof of their conjecture.

The partial sums concept is a surprisingly elusive idea for many of my Calculus II students. In the GeoGebra activity I developed for this topic, students use the spreadsheet

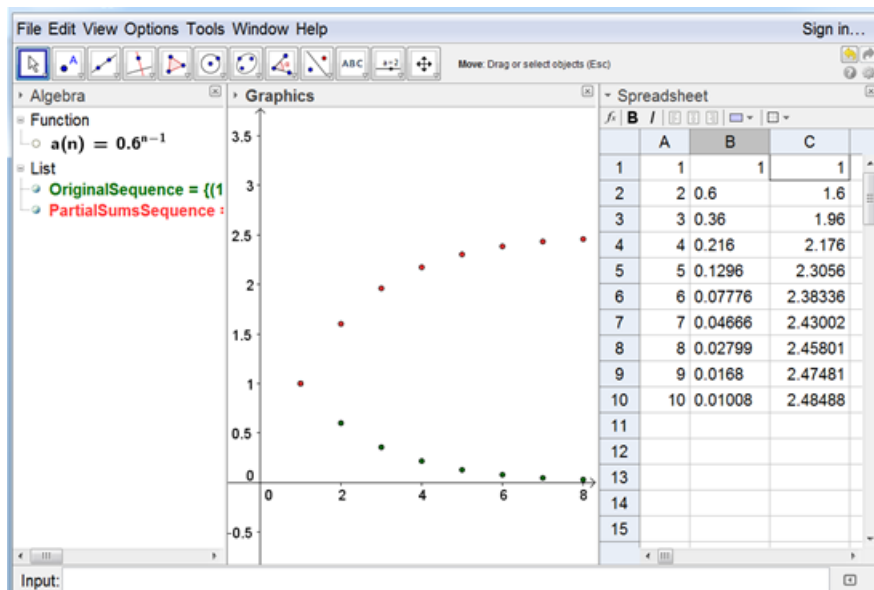


Figure 4

to create the beginning terms in an original sequence and in a sequence of partial sums (see Fig. 4). Next they create a plot of both sequences, color code them to keep the idea of the original sequence separate from the idea of the partial sums sequence, and then make conjectures about the limiting behavior of each sequence. The activity helps students understand series at a deeper level as they proceed through the series testing topics that follow.

Any graphing calculator will produce the graph of a polar equation. However, by using the “Curve” command in GeoGebra students are able to see that polar equations can be thought of as parametric curves, with the angle as parameter. In the polar equation activity I designed, students use an instructor created applet to compare the so-called auxiliary Cartesian graph with the corresponding polar graph (see Fig. 5). Again GeoGebra’s dynamic capability is important: a vertical vector with a point on the

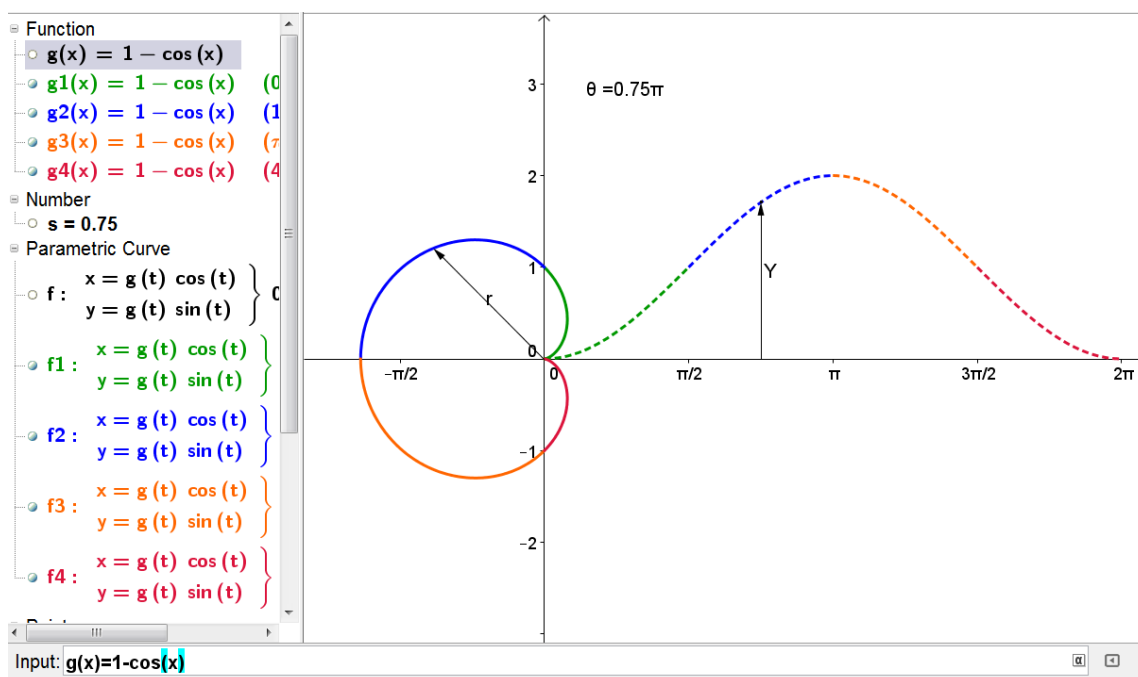


Figure 5

auxiliary curve is yoked to a radial vector with a point on the polar curve. Students explore the connection between the graph of a polar function and its auxiliary graph by varying these yoked vectors through angles between 0 and  $2\pi$ . Students move through color-coded quadrants, using the hide/show object and color-coding feature to focus on one “quadrant” at a time. Finally, students change the function to explore what happens when  $r < 0$  in the polar graph and  $y < 0$  in the auxiliary graph.

Through carefully designed GeoGebra activities, using both student created and instructor created applets, students gain a more rich conceptual understanding of the limit definition of the derivative; the Fundamental Theorem of Calculus; the partial sums

concept; approximate integration and graphing in polar coordinates. Students begin to bridge the gap from computational fluency to deep conceptual understanding. They are actively engaged with calculus during class time. As an added benefit, I observe that when conceptual GeoGebra activities are incorporated, students enjoy calculus more than ever.