TEACHING BUSINESS CALCULUS IN AN ERA OF SPREADSHEETS AND THE INTERNET, A PROGRESS REPORT

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Acknowledgement – This work was inspired by a previous ICTCM award winner (2001 Baltimore), Networked Business Mathematics, by Bob Richardson and Brian Felkel, at Appalachian State University. That work is currently available from Kendall and Hunt Publishers. An earlier report of the project was made at ICTCM 2011.

This is a report of a work in progress, developing course materials for teaching a course in business calculus with the assumption that the students have daily access to Excel. The plan is to develop an "electronic text", probably in the form of a CD. Currently the project assumes that students bring their laptops to class and have internet access.

Historical Background

As mentioned above, the project was inspired by Networked Business Math (NBD) and was originally envisioned as a simple update of that work. As the project developed the vision changed to do a more thorough revision. As with any "new book project" there was an attempt to look at the historical context to see what others have done with the intent of incorporating good ides of other projects. Any project looking at providing mathematics for students in other disciplines should look at the 2004 report of the Curriculum Reform Across the First Two Years (CRAFTY) sub-committee of the Mathematical Association of America (MAA). The report is "Curriculum Foundations Project: Voices of the Partner Disciplines" and includes a chapter specifically addresses the mathematical curriculum for students in business and management. The introduction and summary states:

"Mathematics Departments can help prepare business students by stressing problem solving using business applications, conceptual understanding, quantitative reasoning and communication skills. These aspects should not be sacrificed to breadth of coverage."

The MAA curriculum guide (2004) notes that many of our current math courses were designed in the last century in response to the needs of physics and engineering. One might caricaturize a standard textbook for business calculus, often called brief calculus as a watered down version of a three-semester course in calculus that was designed for physics or math majors. The main emphasis is skill in symbolic manipulation. The standard text for a one semester survey of calculus is used for both business and the life sciences. To allow for broad marketing the text is technology agnostic, follows the arrangement of a course for majors, and uses the notational conventions of mathematics.

In contrast, following the Curriculum Reform Project recommendations, a course for business calculus should:

- Use spreadsheets as the primary computational engine.
- Have greater emphasis on constructing mathematical models from data.
- Increase the emphasis on numerical methods rather than symbolic manipulation.
- Whenever possible use the terminology and notational conventions of the business world.
- Consistently use examples that the students will recognize as relevant to the courses in their major.

The CRAFTY report was based on hearings and conferences that were held for partner disciplines from 2000 to 2004. There were two textbook projects that produced a text that significantly embodied the recommendations in the report, Calculus for Business Decisions (CBD) based at the University of Arizona and NBM. Neither textbook was adopted very widely and both are in the process of being phased out. As with all first generation projects in a reform effort, each project included design features that made them unlikely to gain widespread adoption or to continue after the lead authors left academic mathematics.

We have talked to faculty involved in both projects and obtained anecdotal evidence why the projects lost footing and how subsequent projects could be improved. The CBD project worked best when it was team taught by faculty in business and math. The course was taught as a business course based on real case studies and was not structured as a typical mathematics class. Thus, when the initial instructors rotated out of the course the remaining math faculty were not as comfortable teaching the course, and this contributed to the phasing out of the project at that institution. The NBM course assumed business calculus would be taught on university machines and the students would also be learning and using the computer algebra system Maple, which is not widely used in business settings. In both cases, the math faculty involved in the writing teams moved out of professional mathematics, so the material has not been updated for several years.

Interestingly, neither project seems to have produced any RUME or SOTL studies looking at how the change in the course design influenced student learning either in the course or in subsequent courses. Each project produced a number of articles that noted the favorable reception the course received.

In the spirit of the CRAFTY report, we surveyed the business faculty at SLU to see what they wanted in a calculus course for their students. As reported at RUME [May 2013], they favored a conceptual approach and thought partial derivatives were more important than integration.

Modifications of the content and presentation from a standard course:

The anecdotal evidence on why CBD was dropped at several schools mentioned pushback from more math intensive business disciplines that not enough math content was covered. To make the new course sustainable and transportable it is important that it covers substantially the same mathematical content as a standard course in business calculus. Such a course should cover differentiation of functions of one or several variable and integration of functions of one variable. A significant percentage of the business faculty expects students will have a reasonable set of symbolic differentiation skills. The proposed text will cover a set of mathematical topics that is substantially the same as in a traditional brief calculus. This means that course equivalency and articulation are unchanged.

The NBM syllabus is closer to the standard business calculus syllabus, but a number of important topics were left out. It has no coverage of multivariable calculus and partial derivatives. It also skips related rates and implicit differentiation. More importantly the use of technology is not designed so that students see the technology skills as transferable to the rest of their curriculum. NBM assumes the use of the Maple CAS, which is not a standard tool in the business world. Its use of Excel fails to develop skills the students will see as transportable. The use of templates provided by the instructor with specialized macros makes the Excel usage "math class only." The proposed text tries to build technology skills that the students will see as transferable.

However both NBM and CBD realize that the change in technology, target audience, and pedagogy produce a subtle but pervasive shift in content and style of the course. The proposed text makes a number of such shifts. Since NBM is the closest to the proposed text we will illustrate the shifts envisioned in the text and note how they compare to NBM:

- *Teaching the technology in a way that makes it portable:* Experience showed, as expected, that the students would have to be taught to use Excel. Since the intent was to have the students see the material as usable outside of class, the text does not use any macros or instructor provided tools. Students are also expected to use "Good Excel Style" and make worksheets readable with sufficient documentation. (NBM used templates that worked as black boxes with Excel. Readability was not emphasized.)
- *Use of business terminology and conventions:* Economics examples traditionally use p and q axis with q as the independent variable. In business disciplines a marginal function is not a derivative, as it is often described in calculus texts, but a difference quotient with denominator 1. (This is consistent with NBM.)

- *Use of business examples:* The standard textbook example for related rates is that of a person on a ladder that is sliding down a wall. One student commented that he learned to never stand on an unsecured ladder. In contrast our text uses the Cobb-Douglas equation and rate of change of revenue with respect to cost to illustrate related rates, given that both are functions of quantity. Other examples in the text include the standard supply and demand problems, marginal cost, revenue and profit problems, and present and future value of an investment. (The use of business examples and terminology follows the lead of NBM, but substantially expands on it.)
- *Change of order of topics:* Checking with business faculty we have found that partial derivatives are considered more important than integrals. We reorder the sequence of topics to do multivariable functions and partial derivatives before integration. (NBM entirely removed any consideration of functions of several variables or partial derivatives.)
- *Numerical techniques:* With a spreadsheet, approximations of the derivative using the symmetric difference and Riemann sums for integration are reasonable tasks that work effectively for a wide variety of functions. The numerical examples shift from simply being theoretical underpinnings to being a practical approach. With the use of numerical techniques presented first, the main examples are introduced before the student has learned symbolic techniques. (NBM uses spreadsheets for an initial explanation, but does not develop techniques that students can practically use for differentiation or integration. Like many traditional textbooks, it is presented as if symbolic manipulation is the real way to do all such problems.)
- *Use of CAS:* Finding the current value of a revenue stream is an application of integration at the end of the course. The students enough to set up the problem, but only have the integration techniques for solving symbolically if the stream is constant or exponential. Using simple CAS allows the focus to remain on a conceptual understanding of the problem. (NBM uses Maple extensively and assumes the students will learn syntax for the program, so CAS becomes the best way to do problems. The current project uses Wolfram Alpha, which is freely available on the web and is best suited for problems that can be asked in one line with almost natural syntax. The current project also pays attention to when different technologies are useful.)
- *An increased emphasis on real data and modeling:* With a spreadsheet is becomes reasonable to have students collect data from the web and to find a variety of best fitting curves. In the review of pre-calculus topics students are asked to decide which model should be expected to go with the data in a situation and then to find real data and produce an appropriate best fitting curve. (This was dome well in NBM and is continued here.)
- *Focus on communication and application:* As mentioned above, the conventions of school mathematics use a terse style with one letter names like x, y, f, and g used as variable and function names to aid in symbol manipulation. If the goal is to produce work that someone else can read and understand 6 months later more descriptive variable and function names are used, and having sufficient

documentation is considered part of answering the question. (This is a break with NBM, which uses Excel as a substitute for a calculator. More attention is given to making spreadsheets that are readable.)

Progress Report I: Progress on the plan

A complete draft of the text has been written with exercises. It has been class tested at SLU for two semesters by the author and two other faculty members. Sections that use Excel have companion worksheets for all the examples in the text. There is a plan to have a second draft by fall 2014 incorporating student and instructor reaction. A transition with teaching assistants will probably start in fall 2015. Another school will start a pilot adoption in summer 2014. We are looking for other schools that are interested in class testing the material.

Initial observations showed several expected changes were occurring. As expected, we have had to modify pedagogical style to teach the course with a computer program assumed in class. The use of Excel has introduced aspects more typical of a programming class, where the teacher has students work on short segments of code and the instructor provides technical support. The use of numeric methods has allowed a more conceptual approach. The students find the numeric approximations of derivative and integral easier to grasp than the formal definitions. The use of numeric methods with CAS leads to the approach that numeric methods can set up the problem, even when obscure functions are used, and if closed form solutions are needed, computer algebra can do that problem. Students appreciate the fact that the main examples are connected to their other business courses. Anecdotally, the class has a better attitude because they see the material as useful.

We did a small study of the desires of business faculty for their students in the course. The results were consistent with the MAA CRAFTY report [MAA 2004]. We have started small studies looking at student satisfaction and their progress in other courses that depend on business calculus.

Progress Report II: Revisions to the plan

One of the recurrent themes in discussions with faculty who taught with NBM or CBD is that it requires more teacher support than they expected. One could make a strong argument that the main reason why these texts have not caught on is that they do not give sufficient support for bringing on new faculty. At an immediate level, having instructors learn to teach with interruptions for “Why won’t my computer work?” requires a change in style. There is a need to move from a straight lecture to having periods of group work and teaching the students to help each other with technology. It was also surprising how the change in examples and context for the examples made teachers uneasy. Using business examples brings the teachers out of their comfort zone and they have to use examples they do not fully understand. This experience has convinced us that knowledge transfer is harder than we anticipated.

Based on the experiences noted above, we are convinced that we need to provide more teacher support with the project. There needs to be a teachers' manual that explains differences in pedagogy and how the pieces of the text fit together. A second piece is a solutions manual for at least some of the homework problems. We also want to provide support material for a variety of teaching styles. We are in the process of producing PowerPoint for instructors that use them. We will also make a set of instructors' notes for other teaching styles. In effect we need to create materials that can be used as supplement the text and provide a program for moving instructors into comfort with the revised course we have imagined.

One aspect of preparing for a shift in pedagogy is that we now see doing RUME and SOTL studies as part of producing the text. We want to look at the impact on the instructors, the students and the faculty in the business school. The project can only be a success if it meets the needs of each of these groups.

We also found that having the internet available made subtle changes to the course. When using technology, the instructor should be prepared to have a student ask why we can't do a task "the easy way". The first shift related to the availability of the internet has already been made and relates to the use of WolframAlpha as a CAS platform. Initially the planned CAS usage was going to be marginal and not intrinsic to the course. It has evolved to the point where CAS techniques for CAS are brought up in each major component of the course. With that shift there is also a discussion about when each of hand computation, numerical techniques with Excel, and online computer algebra are the most effective method for a problem.

Other uses of the internet involve using online the online homework system WeBWorK, for homework problems, for daily reading quizzes, and to provide targeted remediation for students with weak skills. The first use of WeBWorK was for more traditional problems in sections that involved hand computation. This is typical of any calculus class. The second use was for reading quizzes. Students have a daily one multiple select question quiz, which is due before class, that verifies students have read the material and prepared for class. The third use is connected with prerequisite classes. Many students in business calculus have a weak algebraic background. Students who do not show mastery in short paper quiz are given a substantial reading assignment. The first several questions in the assignment are reading quiz type questions that make the students read review material that has been found on the web.

The students have also discovered that u-tube videos are available which will show them techniques in Excel. A forum is provided to enable students to share things they find. I have also found that the help available by using Google is better than the help for Excel provided by Microsoft. I anticipate that I will continue to find adjustments that need to be made for a class that has the internet available in class.

Progress Report III: Planned future action

We hope to have a second draft ready by fall 2104 that will incorporate the experience gained from class testing the first draft. We are starting work on an instructors' manual and other materials that will support various teaching styles. This would include PowerPoint presentations, sets of warm-up problems, and a test bank.

Based on our experience we plan to convert as many problems as possible to WeBWorK so that they can be used with an automated homework system.

We plan to see if the course can be made sustainable and transferable. As mentioned above, there is a plan to run at least one pilot section at another institution. We also want to start having selected teaching assistants teach with the material. A key point here is to develop a teacher training program that will enable the course to be taught with the teaching resources normally available for teach the course, in other words teaching assistants and adjuncts.

We also plan to develop a series of studies on the effectiveness of the course using multiple measures of effectiveness. Among the obvious measures of effectiveness we would include the attitude of the students to the mathematics before and after the course, the success of students in business courses that rely on calculus, the satisfaction of business faculty with the preparation of their students and attitudes of the math faculty concerning the success of the course.

Mathematical Success Tidbits

Any discussion of proposed changed in a math class is incomplete without a discussion of mathematical content that the change makes easier to present and for the students to understand. The same is true with a shift to Excel. We found the numeric presentation of derivative laid a better conceptual foundation for the concept of derivative. We found the numeric methods for integration gave a better foundation for application problems and for the fundamental theorem of calculus.

The numeric approach to derivative is to use the definition used by default by graphing calculators. Thus

$$f'(x) \approx \frac{f(x+.001)-f(x-.001)}{.002}.$$

This is paired with the conceptual definition of derivative, which notes that any function nice enough to show up in a calculus class, will look like a straight line and the derivative is the slope of that line. One simple notes that for most functions we care about, zooming in to a scale of .001 is far enough to make the curve look like a line. One can then follow up with the formal limit definition of the derivative. The students find the conceptual definition to be much easier to understand than the formal definition. The numeric definition has an advantage of working with transcendental functions for which the students have not yet learned rules of symbolic differentiation. Starting with a numeric approach can also be paired with the curve fitting capabilities of Excel to have students "discover" basic differentiation formulas. Traditionally students have trouble connecting the definition of derivative, which gives a value at a point, and the process of symbolically differentiating, which transforms a function into another function.

A similar story can be told with integration. The numeric approach is to use Riemann sums. In a traditional course, this method is used for theoretical background and quickly discarded when the students are ready for the "real way" of doing integration, which is by symbolic manipulation. The basic reason for ignoring the numeric method is that it is simply too hard to practically use with understanding. (It can be a button on a calculator, but I don't know of any teachers who have students produce a program for sums.) With a spreadsheet, the numeric method is quite reasonable and students are asked to approximate values of the definite integral by this method.

The first advantage of the numeric method is that it is quite robust in the context of the course. Typical business calculus courses give very little time to techniques of integration. Students are taught the basic formulas, rules for addition, subtraction and scalar multiplication, and the technique of u-substitution. This means they can only do carefully rigged problems. The easiest example of the problem is looking at current value of a revenue stream. The solution is the integral of the function for the revenue stream times an exponential function to handle depreciation. In a typical course, the students cannot do the problem if it as complicated as a linear function.

A second advantage of the numeric approach is that it works with the functions that students are going to see in business applications, where the functions are typically discrete time series that may be approximated by a nice formula.

The third advantage of the numerical approach is that it supports the understanding of theory. The method has students computing points on an area function. If the midpoint rule is used with basic functions the point of the area function can be fit to a curve by Excel and the produced formula will be an antiderivative. We find that some students have an easier time with the fundamental theorem of calculus when it is presented this way, with examples they can follow coming before the theory.

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