

VISUALIZING THREE-DIMENSIONAL OBJECTS IN MULTIVARIABLE CALCULUS USING 3-D ENABLED HANDOUTS

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Introduction

Numerous topics in multivariable calculus involve three-dimensional geometry objects, e.g., points, vectors, lines, curves, planes, surfaces, and solids. Students often find these objects difficult to visualize, partially owing to the limitations inherent to the traditional 2-dimensional media, such as whiteboards or pages in textbooks.

After our earlier attempts [1,2] to address these difficulties, we are currently developing a collection of simple but effective modules which would allow the students to explore 3D objects in an interactive fashion (including zoom, rotate, pan viewport controls). The technology we use will ensure accessibility of these modules using a web browser on a wide variety of platforms, including smartphones and tablets.

3-D Enabled Handouts

The interactive modules are deployed via "3D-enabled handouts". These handouts will be associated with the topics listed in Table 1; some handouts will focus on explaining concepts and the theory, others will focus on applications and specific problems.

Handouts will be created in PDF format, and will be made available to the students on the course web site. Some students tend to print such PDF handouts, whereas others prefer to access them on the screen of an electronic device (computer, tablet, or smartphone).

Each 3D-enabled handout will include a QR code which, when scanned by a smartphone or a tablet, will link to the 3D model associated with the handout. (An "app" may have to be installed on student's device to enable QR code scanning; appropriate instructions will be provided to all students.)

When reading the handout on a screen, the student will be able to click on the QR code (since an active hyperlink will be embedded in the PDF file at that location) to, once again, access the associated 3D model.

Table 1. Key multivariable calculus topics being addressed in this project

<i>Key multivariable calculus topic</i>	<i>Related three-dimensional objects</i>
Frenet frame	Points, curves, vectors
Constrained optimization and Lagrange multipliers (for objective functions involving three independent variables)	Points, curves, surfaces, vectors
Triple integrals	Solids
Surface integrals	Surfaces, vectors
Divergence theorem	Solids, surfaces, vectors
Stokes' theorem	Surfaces, curves, vectors

A proof-of-concept sequence of handouts has already been deployed in a multivariable calculus section taught by the author in Fall 2013 - Figure 1 contains a snapshot of one of those handouts, and a link to a web site with more information.

Three.js Library

As was already noted, the key aspect of this project is that students will be able to access the 3D models on a wide variety of platforms, including most browsers on smartphones and tablets, which many students own and bring with them to class. An additional criterion we adopted is that the technology used here should be available at no additional cost to the student.

While many technology solutions in existence address allow on-screen 3D model display and manipulation, most do not satisfy our criteria:

- proprietary software, such as Mathematica or Mathcad tend to support limited number of hardware platforms, usually not including smartphones; furthermore, those software systems are not free (although some universities have license agreements with at least some of their vendors);
- Adobe PDF supports the inclusion of 3D models in PDF documents. These models allow at least some interactivity, and the software to access PDF documents (such as Adobe PDF Reader and third-party alternatives) can be downloaded free of charge on many platforms (including smartphones and tablets). However, on many of the portable platforms, 3D objects embedded in PDF files are not properly displayed;

10. Let $\mathbf{F} = \langle x, y, z \rangle$ and let D be the circular disc $x^2 + y^2 \leq 4, z = 0$. Verify the divergence theorem for $\iint_D \int_0^4 4x - y \operatorname{div} \mathbf{F} dz dy dx$.

$$LHS = \iint_S \mathbf{F} \cdot \mathbf{n} dS = \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS + \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS + \iint_{S_3} \mathbf{F} \cdot \mathbf{n} dS$$

S_1 $z = 0; \mathbf{n} = -\mathbf{k}; \mathbf{F} \cdot \mathbf{n} = -z = 0; \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS = \boxed{0}$

S_2 $g(x, y, z) = x^2 + y^2 = 4; \frac{\nabla g}{|\nabla g|} = \frac{\langle 2x, 2y, 0 \rangle}{\sqrt{4x^2 + 4y^2 + 4z^2}} = \pm \langle \frac{x}{2}, \frac{y}{2}, 0 \rangle$; choose "+": $\mathbf{n} = \langle \frac{x}{2}, \frac{y}{2}, 0 \rangle$;
 $\mathbf{F} \cdot \mathbf{n} = \frac{x^2 + y^2}{2} = \frac{r^2}{2} = 2$;
 using cylindrical coordinates with $r = 2$, we have $dS = 2 dz d\theta$;
 $\iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS = \int_0^{2\pi} \int_0^4 (4 - 2 \cos \theta - 2 \sin \theta) (2) dz d\theta = 4 \int_0^{2\pi} (4 - 2 \cos \theta - 2 \sin \theta) d\theta$
 $= 4 [4\theta - 2 \sin \theta + 2 \cos \theta]_0^{2\pi} = \boxed{32\pi}$

S_3 part of the plane $z = 4 - x - y$, i.e., $x + y + z = 4$;
 a normal vector is $\langle 1, 1, 1 \rangle$; the outward unit normal is $\mathbf{n} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$; $\mathbf{F} \cdot \mathbf{n} = \frac{x+y+z}{\sqrt{3}} = \frac{4}{\sqrt{3}}$
 we project onto the xy -plane
 $\iint_{S_3} \mathbf{F} \cdot \mathbf{n} dS = \iint_{S_1} \frac{4}{\sqrt{3}} (\sqrt{3}) dy dx = 4(\text{Area of } S_1) = \boxed{16\pi}$

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = 0 + 32\pi + 16\pi = \boxed{48\pi}$$

$\operatorname{div} \mathbf{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 3$
 $RHS = \iiint_V \operatorname{div} \mathbf{F} dV = \int_0^{2\pi} \int_0^4 \int_0^2 3r dr d\theta = \int_0^{2\pi} [6r^2 - r^3 \cos \theta - r^3 \sin \theta]_{r=0}^{r=2} d\theta$
 $= \int_0^{2\pi} (24 - 8 \cos \theta - 8 \sin \theta) d\theta = [24\theta - 8 \sin \theta + 8 \cos \theta]_0^{2\pi} = \boxed{48\pi}$

SCAN or CLICK for 3D image

Figure 1.

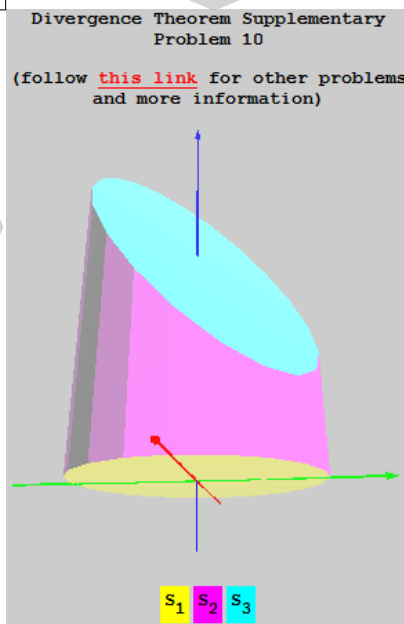
An example of a 3D-enabled handout

A handout in PDF format is made available to the students on the course web site for printing and/or downloading.

When using a printed copy, a student can use a smartphone or tablet camera to scan the QR code. The device browser will then display the 3D model, which can be interacted with using touch gestures.

When accessing the document on a screen, a student can click (or touch) the QR code, which is now an active link, pointing the browser to the 3D model. Mouse-controlled interfaces allow the student to zoom, rotate, and pan the model using the mouse buttons (touch controls can be used on interfaces so equipped).

This example (including the PDF handout and 3D models) can be accessed at <http://www.math.ou.edu/~bogacki/math312/divth>



- Java and Flash are the leading technologies used for displaying 3D models inside of a web browser window. When accessing the web using a computer, the user can expect the browser to support both of these platforms, or can download (free of charge) the necessary plug-ins from the vendors (Oracle and Adobe, respectively). However, there is no official support for these technologies on some key portable platforms. While some workarounds exist, we do not wish to expose our students to the possible incompatibilities and the associated frustration.

The technology solution we adopted for this project is based on three.js [3] - a JavaScript-based library which offers the following key advantages:

- Three.js supports multiple hardware/software platforms.
 - o It supports the HTML5 canvas element [4] for displaying and interacting with 3D models (technically, it uses the two-dimensional canvas, and mathematically maps the 3D contents to them). A number of modern web browsers properly support canvas - this includes many browsers on smartphones and tablets running Android and iOS systems.
 - o On hardware platforms and browsers supporting the WebGL technology [5], three.js can automatically render in that mode (instead of canvas), taking advantage of the Graphic Processing Unit (GPU) if present on the user's system. The resulting 3D model being displayed tends to be of much higher quality, and the user interaction is greatly improved (faster response time).
 - o Other rendering modes (e.g., SVG) can also be used in three.js, but currently we opted to focus on canvas and WebGL only.

When developing 3D contents (model, interaction) using Three.js library, one does not have to develop for a specific rendering mode (canvas vs. WebGL) - instead, the appropriate rendering mode can be chosen automatically according to the capabilities of the specific user's device and browser. Furthermore, this means that any contents developed under this project should be easily scalable to newer display technologies (by incorporating additional renderer modules).

- Three.js library is free to use and modify (under the MIT license).

While there are a few possible disadvantages related to our choice of technology (mostly attributed to the underlying JavaScript environment, e.g., a potential performance penalty associated with an interpreted language), we decided that overall three.js offers us the best platform for use in conjunction with this project.

Future Plans

In addition to the 3D-enabled handouts, other possible modes of utilizing 3D illustrations will be examined; some possibilities include

- displaying QR code via the overhead projector during class to include 3D models in lectures and class discussions,
- equipping 3D models with additional interactive functionality (beyond the viewpoint control) to allow students to select specific portions of the model (e.g., by clicking on one of the surfaces surrounding a solid),
- enable the students to construct their own 3D models on their smartphones by entering mathematical equations describing them, as well as possibly collaborating on models with other students and/or sharing them with the instructor,
- incorporating interactive animations of 3D models (e.g., an animated motion of an object along a given trajectory in space can be displayed, while allowing the student to change the viewpoint),
- designing clicker-type activities where students can respond to questions asked by the instructor by interacting with 3D models on their smartphone/tablet screens.

Bibliography

- [1] Bogacki, P. and Melrose, G., Interactive Modules for Differential and Integral Calculus of Functions of Many Variables, Proceedings of the Sixteenth Annual International Conference on Technology in Collegiate Mathematics, October 2003, Chicago, IL, pp. 30-34
- [2] Bogacki, P., Surface Integration Computer Activities in Multivariable Calculus, Proceedings of the Twenty-fifth Annual International Conference on Technology in Collegiate Mathematics, March 2013, Boston, MA, pp. 45-49
- [3] Three.js. threejs.org (Accessed May 9, 2014).
- [4] HTML Elements: Canvas. <http://www.w3.org/wiki/HTML/Elements/canvas> (Accessed May 15, 2014).
- [5] Khronos Group WebGL page. <http://www.khronos.org/webgl/> (Accessed May 15, 2014).