LEARNING MATH CONCEPTS IN YOUR ENVIRONMENT USING PHOTOGRAPHY AND GEOGEBRA

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Abstract

This paper shares how using GeoGebra software and photography, teachers can provide better understanding for student learning. Teaching mathematics through pictures and photography, using real world pictures to point out and teach the mathematics is a novelty. This paper shares how photographs are inserted into GeoGebra software and explores various objectives related to the new Common Core Math Standards. Topics of shape recognition, spatial sense through ratio/proportion and measurement, and functions explore the math that surrounds us in the real world while covering many of the new standards. Photographs of architecture that are inserted into GeoGebra can be dissected to observe different and similar shapes. By using photographs in the GeoGebra environment, the classroom will discuss mathematical definitions and explore numerical relationships as emphasized in the Common Core Math Standards. Since GeoGebra is available as a free download, many school districts are adopting this software in their classrooms.

Key Words: Teaching Mathematics, GeoGebra, Photography, Geometry, Measurement, Algebra, Technology, Standards, Connections, Real-world

Introduction

"Of all of our inventions for mass communication, pictures still speak the most universally understood language."

- Walt Disney Company

This paper looks at using photography to teach math concepts to young learners while incorporating the latest technology, GeoGebra, in the teaching of mathematics. Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, though the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material and make meaningful connections to the mathematical concepts.

Through the use of GeoGebra and photographs inserted into the math software, concepts will be developed and explored. GeoGebra is free software, a multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). GeoGebra has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month. It can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info.

The Use of Technology for Teaching Mathematics

Today most educators support mathematics classrooms where students are involved and actively constructing their own understanding of mathematical ideas by using the latest emerging technologies. Today the emphasis is on using technology to teach math and getting more students interested in science, technology, engineering, and mathematics (STEM) fields. Using software like GeoGebra is a great way to teach all the important STEM-related topics while motivating students in learning and covering the new math standards in today's classrooms.

According to the National Council of Teaching of Mathematics (NCTM) (2000) the use of technology in teaching mathematics should be one of the principles for teaching mathematics to our students. NCTM feels that through the use of technological tools, students can: a) work at higher levels of generalization, b) model and solve complex problems, and c) focus on decision-making and reasoning.

NCTM (2000) feels that mathematical power can arise from technology that include: a) increased opportunity for learning, b) increased opportunities for real-life social contexts, and c) orientation to the future. The NTCM's focus is to: a) promote technology as an essential tool for learning mathematics in the 21st century, b) integrate the principles and process standards with teaching the content standards, c) provide access to all five mathematics content standards for all students, and d) support learner-centered strategies that address the diverse needs of all learners of mathematics.

NCTM contends that effective mathematics teachers maximize the potential of technology to: a) develop students' understanding, b) stimulate their interest, and c) increase their proficiency in mathematics. Using such tools as GeoGebra can really help to incorporate this emerging technology and aid students in better understanding the mathematics they are learning.

Common Core State Standards for Mathematics

Today, most schools and states are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at:

http://www.corestandards.org/. Andresen & Misfeldt (2010) found that with the new Common Core Standards (National Governors Association Center for Best Practices - NGA Center, 2010) in teaching mathematics, teachers need to be trained and learn new mathematics content and technology like GeoGebra in order to be effective in teaching and reaching their students. Knowledge of technology cannot be isolated from the content and good mathematics teaching requires an understanding on how technology relates to the pedagogy and mathematics (Hughes, 2005).

The follow CCSS are samples of standards that can be taught with the GeoGebra and photography, see the Appendices for examples:

> **Grade 3** 3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
>
> **Grades 6** 6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

As teachers and schools prepare for End-of-Course assessments as well as the future of the CCSS assessments and curriculum, an instructional approach that emphasizes mathematical modeling for linking course content to everyday life provides students with the foundation to succeed throughout elementary, middle, high school, college, and into future careers. By using the emerging technology, GeoGebra, and real-life photography, students better understand mathematical thinking and ideas in novel situations.

Applications, GeoGebra, Photography Relationships

Relating and understanding real-world problems through the use of interactive technology like GeoGebra and connecting them to photography make important math connections. GeoGebra allows one to import graphics, i.e. photographs into the software and then draw on them, test theories with the pictures, and apply math ideas to the images.

According to Newcombe (2013) teaching about spatial sense and having students using active sketching software enhances engagement, deepens mathematical understanding, requires reasoning, and forces ideas to be made explicit. Newcombe (2013) cites that improvements in spatial ability using sketching leads to better grades in chemistry and physics, as well as to better essays on problems in geoscience. She says that higher

spatial abilities of students using such graphing and sketching tools predict interest and success in the STEM disciplines.

GeoGebra

While the study of geometry in some shape or form has existed for many millennia, it is within the past twenty years that a shift has occurred in how geometry may be learned through computer-based interactive geometric software. Software programs like GeoGebra allow users to construct interactive representations of points, lines, and circles. These geometric objects are interactive in that they may be resized and shifted around onscreen through clicking and dragging actions. Moreover, interactive geometric software like GeoGebra in the K-12 mathematics curriculum has been used at the elementary level, middle school level, and the high school level (Yu & Tawfeeq, 2011) and has proven to be a very effective means for teaching and learning mathematics.

Antohe (2009) found that using GeoGebra students can "see" abstract concept, students can make connection and discover mathematics. The ability to assess student solutions electronically may promote students' interests towards mathematics and advance students' cognitive abilities as well. Bayazit & Aksov (2010) found that GeoGebra promotes students' problem solving skills and helps students construct mathematical model of a problem through which students could conduct a better analysis of the situation and develop an operational plan to resolve the task at hand. They also indicated that GeoGebra enhances students' visual ability and enables them to conduct visual strategies to resolve algebra problems.

Reisa (2010) found that the success of teaching with GeoGebra is higher compared with that of conventional teaching. Naturally, there is another reason for that it can be attributed to Gardner's theory of multiple intelligences. There are 8 different types of intelligence in the theory of multiple intelligences and Gardner states that a human being was born with all these types of intelligences but some are dominant. Starting this study, with the application of GeoGebra, more intelligences of students are aimed to be reached, thus success is to be higher. Besides, according to Edgar Dale's Cone of Experience, we remember 30% of what we hear but remember 80% of what we see, hear and utter. The results of our application indeed show exactly this. With GeoGebra, students are more involved in the process and more sense organs are appealed to, thus higher success was achieved. Therefore we can say 'Making more use of GeoGebra in Mathematics teaching will be an important factor in an effective math teaching and a permanent learning.' Thus, we teach students to learn, not to memorize that is the true aim of education (Reisa, 2010). By inserting photos into GeoGebra and then "doing the math", students really learn by using more senses.

Fahlberg-Stojanovska, & Stojanovski (2009) found that using GeoGebra motivates and helps students learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) found that it is very important to integrate both concrete and virtual manipulatives into the primary-age math classroom. Furner and Marinas (2007) found that children easily transition to the abstract when using geometry

sketching software when you first use geoboards and the software like *Paint* before going directly to the sketching software or GeoGebra. Appendix B and the Matharoundus.com website show several examples young children can explore with while using GeoGebra.

Velichová (2010) contends that a simple drawing of mathematical objects and figures is not enough for the building of a comprehensive understanding of basic mathematical concepts. On the other hand, creative dynamic activities in software like GeoGebra are essential to the development of one's technological content knowledge. This seems to be consistent with the general notion of mathematical understanding as one's growing competence to navigate through various representations of mathematics concepts illustrated dynamically. GeoGebra seems to be a didactic tool supporting these efforts to a high level, and in an easy, natural and user-friendly way. These characteristics do predetermine this freeware application to be used in teaching, learning and exploring mathematics.

GeoGebra is a great resource and technological tool that when used in the math classroom provides a focus to:

- promote technology as an essential tool for learning mathematics in the 21st century
- integrate the principles and process standards with teaching the content standards
- provide access to all five mathematics content standards for all students
- support learner-centered strategies that address the diverse needs of all learners of mathematics

In a study by Soare and Antohe (2010), it used maps and geography topology inserts into GeoGebra and then performed many mathematical measurements, ratios, investigations on the maps and topology samples. Soare and Antohe found that the software, GeoGebra, had many pedagogical implications for teaching and understanding ideas while using the software to "do the math" so to speak. The mathematical modeling and tools that can be used as part of GeoGebra really helped to teach and reinforce concepts and often in an exploratory manner as well. The following discusses the use of photography, like the maps, to then insert into GeoGebra to explore many mathematical ideas.

Using Photography to Teach Mathematics

Destin Sparks once said, "Photography is the story I fail to put into words." The idea that a picture is worth a thousand words is so true. Pictures and photography can tell a story or share ideas or even teach mathematics concepts. More recently there have been a few studies published promoting the ideas of teaching mathematical ideas using photography. (Munakata and Vaidya, 2012; Braggs and Nicol, 2011; Northcote, 2011; Anderson, 2007)

Braggs and Nicol (2011) found success with using photography in the teaching of mathematics particularly, using photographs that enhance the problem solving activities. They say that photography involves the math students into doing "real world" problems. Munakata and Vaidya (2012) found in their research that teachers need to encourage students to seek connections between the math and sciences and everyday life through

photography. They believe that while not every photograph may be captivating, all students are able to participate in exploring connections between the sciences and creativity in a novel way often times using photography to teach math.

Northcote (2011) believes that teachers need to use photography in the classroom to point out the math in real-life experiences. She encourages students to take their own photos to share and point out the math concepts that exists in them, making the teaching of mathematics interactive. Northcote feels this empowers students own learning of mathematics when you involve them by teaching about photography and math together. Anderson (2007) found that by placing a digital camera in the hands of students enables them to explore their own conceptual learning. You can teach young people to be observant and also minimize classroom management issues while involving them extensively in their learning.

Photography Connection and Problem Solving

Many math ideas covered by teaching can use photography and GeoGebra such as:

- Measurement and numeric relationships
- Numeric relationships for size comparison
- Angle measurements
- Algebra concepts
- Measurement for distance and area
- Geometric shapes for construction
- Spatial sense ideas
- Mathematical modeling
-

Teachers can insert photos into GeoGebra to stimulate/model word problems for students.

#1 Sword Application Problem: (Pythagorean Theorem) On Japanese trains, there is a rule that forbids passengers bringing onto the train objects longer than 36 inches. How did a passenger travel with a ceremonial sword that was 42 inches long? Discuss strategies and solutions. **(See Appendix B)**

#2 Amusement Park Problem: (System of Equations) Carlos and his friends are going to the amusement park and find that they have two ticket options. In one option each person could by an admission ticket for $5.00 and then pay 25 cents for each ride. The other option is to buy an admission ticket for $2.00 and then pay 75 cents a ride. What do you think Carlos should do? Which plan is best for which type of user? (Please visit matharoundus.com for GeoGebra files and photos/problems)

Summary

When math teachers teach mathematics using GeoGebra and photography in the teaching of mathematics they are do the following:

- To show a purpose for math
- To develop relationships
- To show practical applications to math in life
- To employ innovative teaching

- To make connections
- To stimulate through photography/modeling
- To employ emerging technologies in math with the real world

GeoGebra is free to download and use from GeoGebra.org. GeoGebra is an up and coming dynamic teaching tool in our schools today. It is user-friendly for both students and teachers. To connect geometry, measurement, and algebra to each other using the software, students draw shapes and measure them, and then derive formulas using the "Algebra View." Teachers need to use best practices like incorporating technology like GeoGebra into math instruction to promote student involvement and minimize math anxiety.

GeoGebra will prepare students for future abstract mathematical concepts. When building a strong foundation in geometry, algebra, and measurement, primary-aged students find it fun and easy-to-use. Students may move from using the geoboards, to the virtual geoboards, then finally to GeoGebra. When the grid view is turned on, they can see GeoGebra as an extension of the geoboard. When elementary teachers use GeoGebra, it is suggested that they leave the grid on the screen and turn off the Algebra View tool to minimize student confusion. GeoGebra allows the user to import and insert photographs into the software and then draw, sketch, measure, and test mathematical theories to learn many mathematical ideas.

It is important that as math teachers we motivate and turn our students on to mathematics. Furner & Gonzalez-DeHass (2011) and Sparks (2011) feel that as the STEM fields become more important for our students to study, our schools and teachers need to do more to address math anxiety so that our students are confident to study STEM-related areas. The premise is to use real-world photographs inserted into GeoGebra, then to point out the mathematical ideas by showing and pointing out the mathematics in the pictures or nature that exists. Albert Einstein summarizes this best by saying, "*Look deep into nature, and then you will understand everything better*." Using photography and GeoGebra students can also understand math better. PowerPoint and Data Files for GeoGebra as they relate to this presentation and paper can be accessed at: http://matharoundus.com/GeoGebra. See Appendices A and B for additional resources and examples for using GeoGebra.

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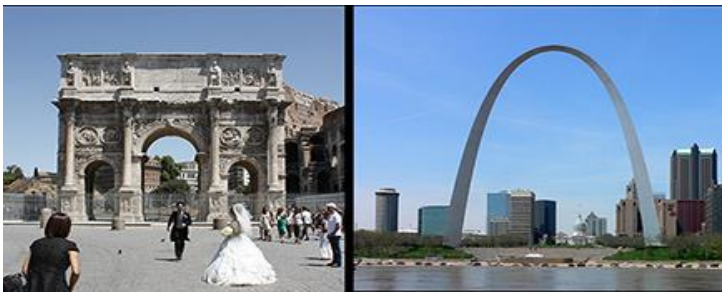
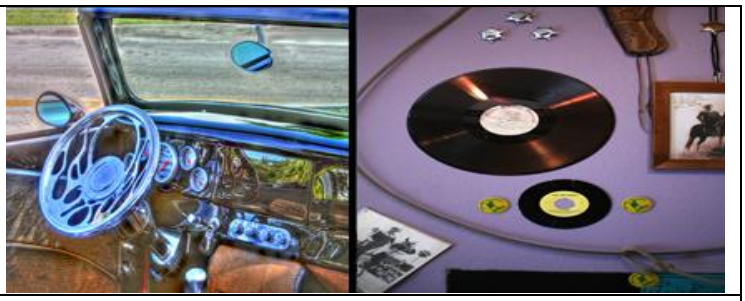
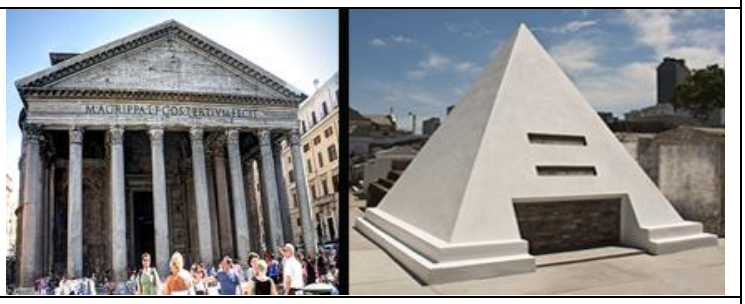
Appendix A: GeoGebra and Virtual Manipulative Websites and Resources for the Mathematics Classroom

Web Sites	URL
Math Around Us	http://matharoundus.com
National Library of Virtual Manipulatives	http://nlvm.usu.edu/en/nav/topic_t_3.html
National Council of Teachers of Mathematics	http://www.nctm.org/standards/content.aspx?id=25007
Cut the Knot	http://www.nctm.org/standards/content.aspx?id=25007
Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://geogebra.org
GeoGebra Wiki Forum	http://www.geogebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com/GeoGebra
Common Core Standards	http://www.corestandards.org/

Appendix B: Sample Examples of GeoGebra Activities to teach Mathematical Ideas with Photographs

Mathematical Ideas	Photographs
Parabolas • Insert real-life examples by using arches. • Create a parabola in GeoGebra with parameters to curve fit the arches. • Explore how each parameter affects the shape of the parabola.	

Circles • Insert real-life examples by using circular shapes. • Create circles with varying radii to fit each example.	
Parallel Lines • Insert real-life examples by using blinds, railings, train tracks. • Create a linear function in GeoGebra and match to the image. • Show that the slope of the line remains the same for parallel lines but the y-intercept changes.	
Slopes • Insert real-life example by using slides of buildings and ladders. • Create a linear function in GeoGebra to explore the concept of slope.	
Pythagorean Theorem • Insert real-life example by inserting a picture. • Show mathematical relationships with this example using the Pythagorean Theorem.