NUMBER THEORETIC PROGRAMS FOR THE TI-89

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Abstract: The TI-89 enables one to write neat programs to supplement a course in elementary number theory. Explore such ideas as Goldbach's conjecture, twin primes, the Collatz problem, Mersenne primes, prime decades, and recursive sequences with easily written user defined functions. Technology serves as an additional representation to view the mathematics.

We initiate our journey by furnishing some user defined functions. In general, there are no spaces between characters when typing into the TI-89 hand-held. Spaces are provided by the calculator between certain commands such as If, and, Return, and Define. Page 435 of the TI-89 manual provides a program for the next prime following a given positive integer displayed in the APPS MENU in **FIGURES 1-5.** In **FIGURE 6,** we give the program for the prime preceding a given positive integer, the last prime, which is a simple modification of the Next Prime program.

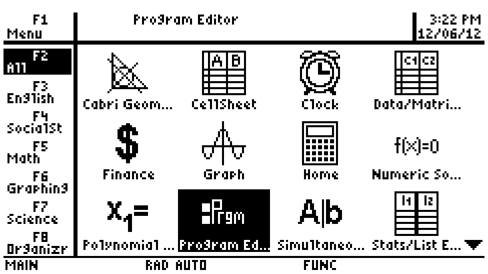


FIGURE 1: The Program Editor.

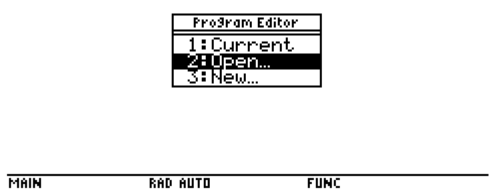


FIGURE 2: Opening the Program Editor.

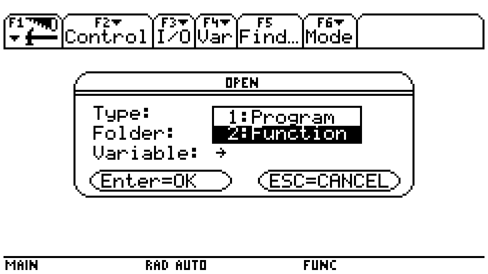


FIGURE 3: The Function Option.

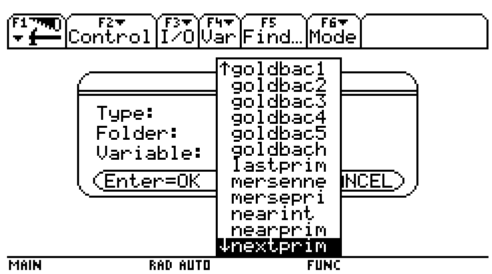


FIGURE 4: The Next Prime from the Folder.

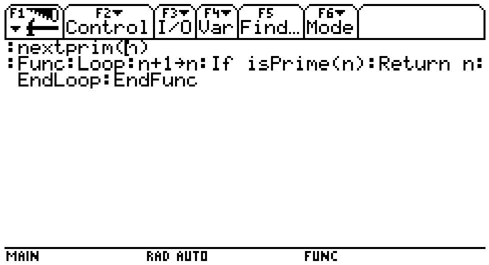


FIGURE 5: The Next Prime Program.

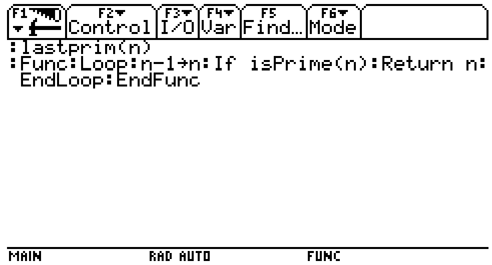


FIGURE 6: The Last Prime Program.

We illustrate the Next Prime Program in **FIGURE 7** by securing the prime successors to the positive integers 25, 887, and 20831323 and the Last Prime Program in **FIGURE 8** by determining the primes preceding the positive integers 37, 20831531, and 140737488353699:

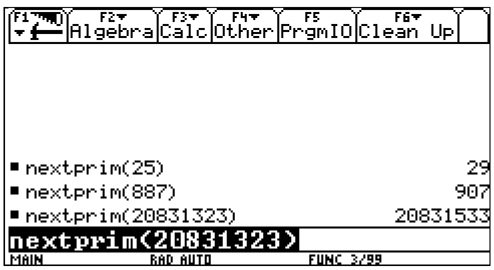


FIGURE 7: Next Prime Examples.

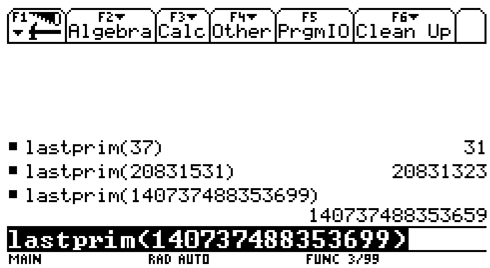


FIGURE 8: Last Prime Examples.

Goldbach's conjecture asserts that every even integer > 2 is expressible as the sum of two primes. The program is provided in **FIGURE 9** and verified for the even integer 66 in **FIGURE 10:**

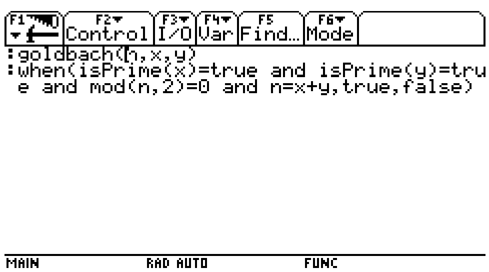


FIGURE 9: The Goldbach Program.

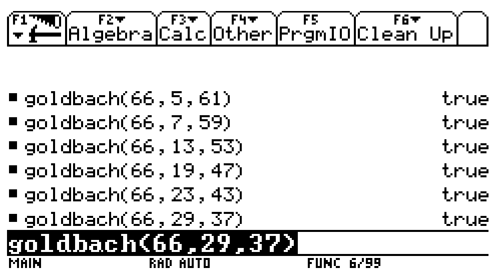


FIGURE 10: Examples of the Program.

Note that if we interchange the order of the addends, the program will still be fine. On the other hand, the program will generate false outputs if one or both of the addends are not primes or the sum is different from what is expected. Some examples are illustrated in **FIGURE 11:**

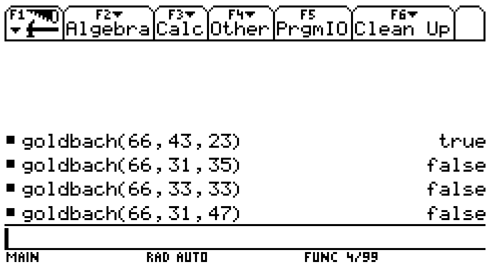


FIGURE 11: More Examples and Cautions for 66.

Suppose we extend the activity to determine all the possible ways one can represent 66 as the sum of two primes in a table form. Achieve this in the Y = EDITOR as in **FIGURES 12-15:**

FIGURE 12: The Function Inputs.

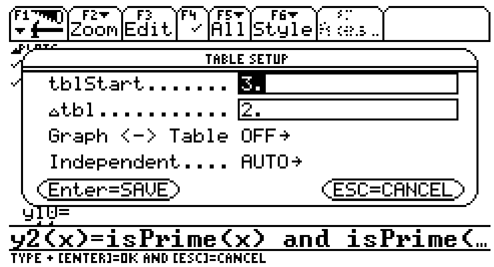


FIGURE 13: Generating the Table Setup.

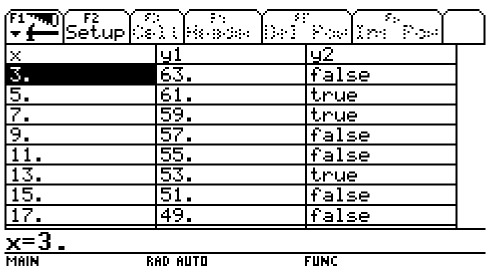


FIGURE 14: The Table Revealed.

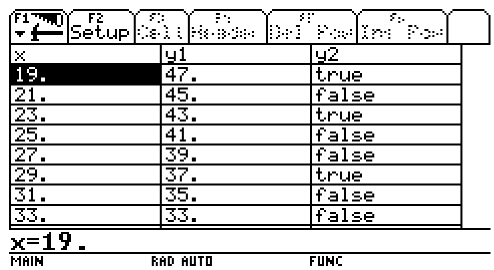


FIGURE 15: The Table Revealed.

Note that since $\frac{66}{2} = 33,$ our search concludes so as not to produce duplications. In the column headed by $y2,$ we searched for the true outputs. Thus one can represent 120 as the sum of two odd primes in the following half-dozen ways:

$$66 = 5 + 61 = 7 + 59 = 13 + 53 = 19 + 47 = 23 + 43 = 29 + 37.$$

Our next activity features the twin primes which are odd primes that differ by two. **FIGURE 16** illustrates the Twin prime function and **FIGURE 17** an example where we determine if the primes 149, 1201, and 267587861 initiates a twin prime pair. We note that the prime 1201 does not while the primes 149 and 267587861 are the lesser primes in a twin prime pair.

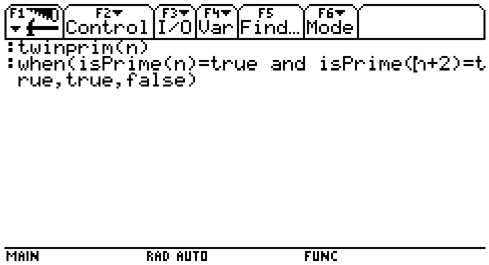


FIGURE 16: The Twin Prime Program.

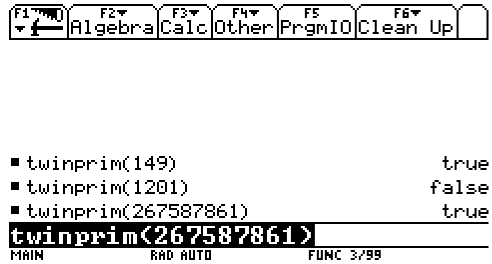


FIGURE 17: Examples of the Program.

One can determine all twin prime pairs ≤ 65: 3 and 5; 5 and 7; 11 and 13; 17 and 19; 29 and 31; 41 and 43; 59 and 61. We proceed to the Y = EDITOR as in **FIGURES 18-23** and seek the true outputs in the column headed by $y1$:

FIGURE 18: The Function Inputs.

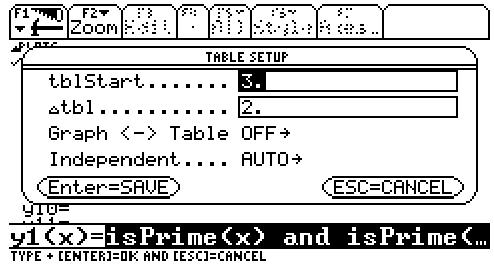


FIGURE 19: The Table Setup.

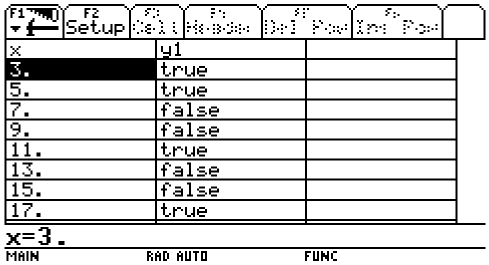


FIGURE 20: The Table Revealed.

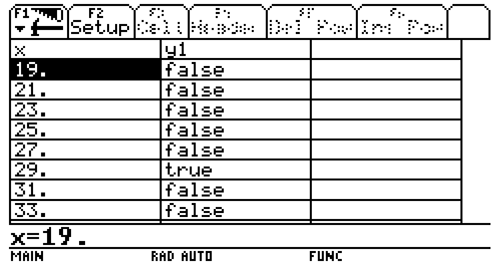


FIGURE 21: The Table Revealed.

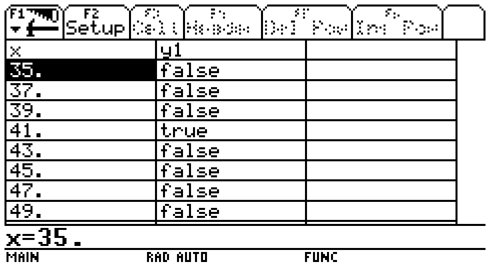


FIGURE 22: The Table Revealed.

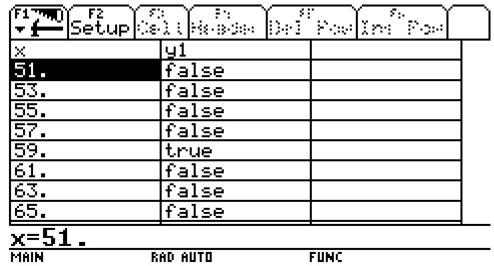


FIGURE 23: The Table Revealed.

We now focus on the famous Collatz or $3X+1$ problem. Consider any positive integer. If it is even, divide by 2. If it is odd, triple and add 1. Apply this procedure to each new integer obtained.

The conjecture asserts that after a finitely many iterations, the sequence converges to 1. The conjecture has been verified for all integers $\leq 3 \cdot 2^{53}$. We verify the conjecture on the sequence $\{28, 29, 30\}$ illustrating the steps needed to reach 1. We illustrate that it requires 18 steps (not including the seed integers) for the Collatz 3-tuple to reach 1. In essence, we are asserting that 28, 29, and 30 are three consecutive integers for which the Collatz sequence has the same length. See **FIGURES 24-31** in SEQUENCE MODE:

FIGURE 24: The Sequence Mode.

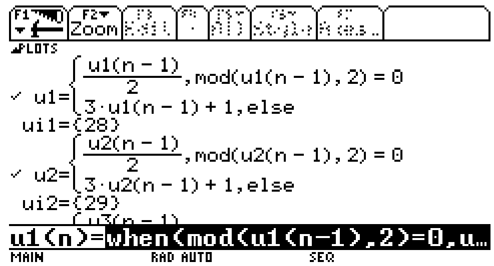


FIGURE 25: The Sequence Inputs.

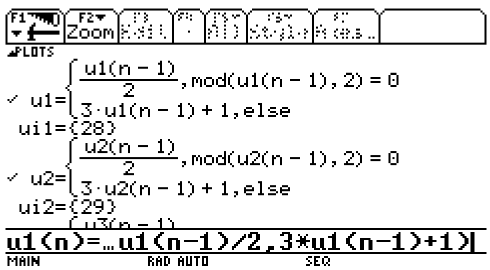


FIGURE 26: The Sequence Inputs.

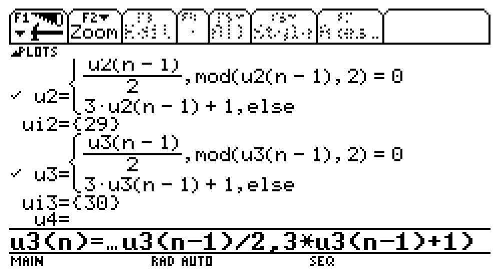


FIGURE 27: The Sequence Inputs.

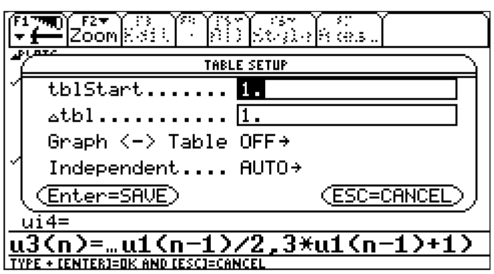
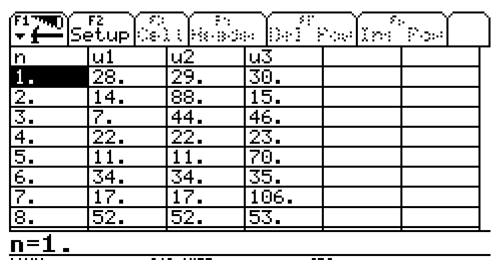


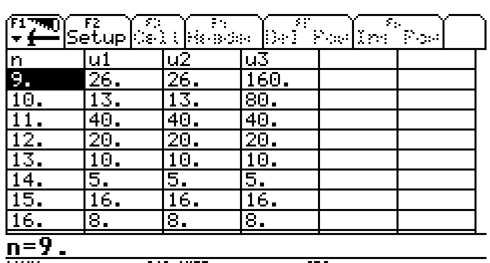
FIGURE 28: The Table Setup.



n	u1	u2	u3
1.	28.	29.	30.
2.	14.	88.	15.
3.	7.	44.	46.
4.	22.	22.	23.
5.	11.	11.	70.
6.	34.	34.	35.
7.	17.	17.	106.
8.	52.	52.	53.

n=1.

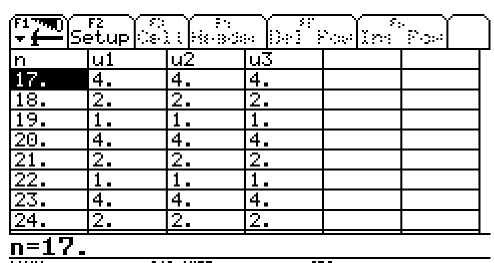
FIGURE 29: The Table Revealed.



n	u1	u2	u3
9.	26.	26.	160.
10.	13.	13.	80.
11.	40.	40.	40.
12.	20.	20.	20.
13.	10.	10.	10.
14.	5.	5.	5.
15.	16.	16.	16.
16.	8.	8.	8.

n=9.

FIGURE 30: The Table Revealed.



n	u1	u2	u3
17.	4.	4.	4.
18.	2.	2.	2.
19.	1.	1.	1.
20.	4.	4.	4.
21.	2.	2.	2.
22.	1.	1.	1.
23.	4.	4.	4.
24.	2.	2.	2.

n=17.

FIGURE 31: The Table Revealed.

Alternatively, with the aid of the program displayed in **FIGURE 32,** we determine the number of iterations needed to reach 1 for the positive integers 128, 11 and 35 respectively. See **FIGURE 33** for the integer 128, **FIGURES 34-35** for the integer 11 and **FIGURES 36-37** for the integer 35. Observe that if we do not count the seed value, it takes, in turn, 7, 19, and 13 steps respectively, for the integers 128, 11 and 35 to reach 1.

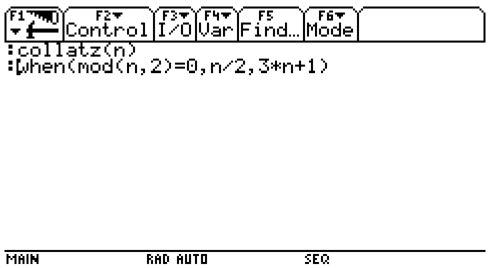


FIGURE 32: The Collatz Program.

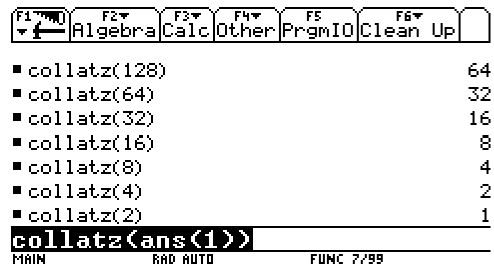


FIGURE 33: Examples

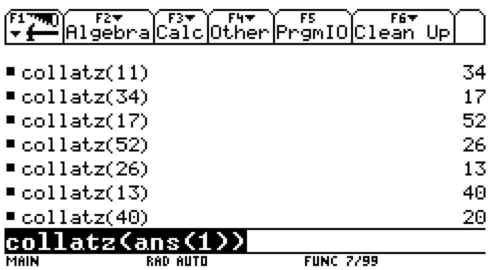


FIGURE 34

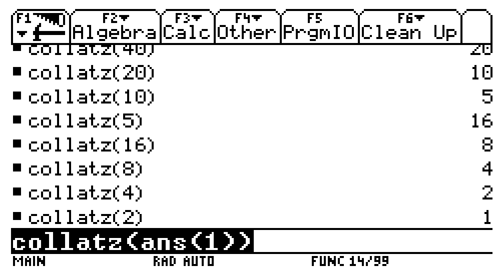


FIGURE 35

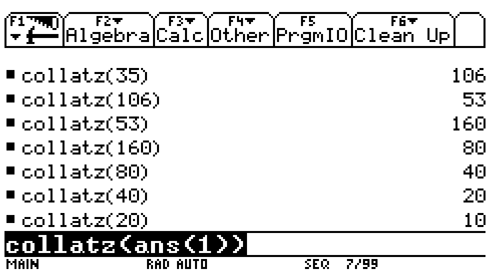


FIGURE 36: Examples

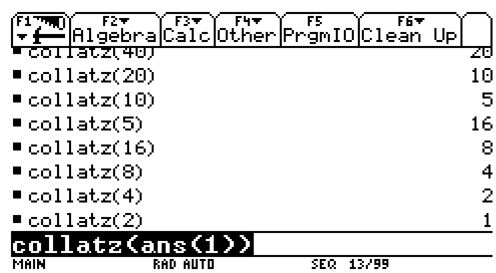


FIGURE 37: Examples

A ***prime decade*** in a sequence of ten integers consists of four primes. With the exceptions of 2 and 5, all primes must terminate in the digits 1, 3, 7 or 9. The sequence of integers $\{11,13,17,19\}$ constitutes the initial prime decade. A program for prime decades is given in **FIGURE 38:**

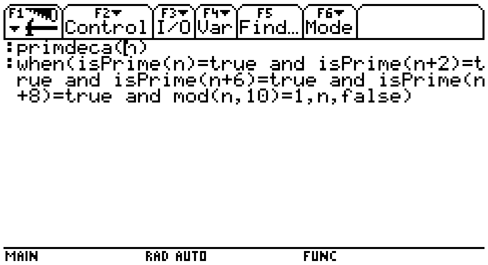


FIGURE 38: The Prime Decade Program.

Using this program, we determine if a prime decade is formed starting with the primes 281, 1481, and 1006331. The results are provided in **FIGURE 39.** From **FIGURE 39,** we deduce that the prime 281 does not initiate a prime decade; for while 281 and 283 are indeed primes, 287 and 289 are not. $\left(287 = 7 \cdot 41 \textit{ and } 289 = 17^2\right)$. On the other hand, the decade starting with the prime 1481 is indeed a prime decade as is the decade starting with the prime 1006331.

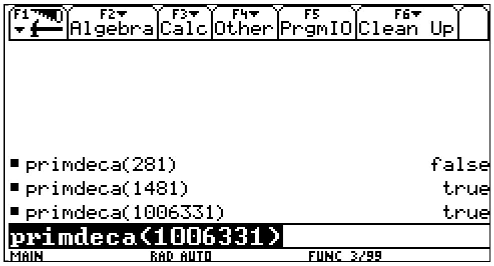


FIGURE 39: Examples

Alternatively, one could check these assertions utilizing the factor or is prime options. There are many interesting facts about prime decades. To find the next two prime decades after the prime decade $\{11,13,17,19\}$, we employ the TI-89 in FUNCTION MODE with the following seven functions in **FIGURE 40,** the TABLE SETUP in **FIGURE 41** and TABLE in **FIGURES 42-43:**

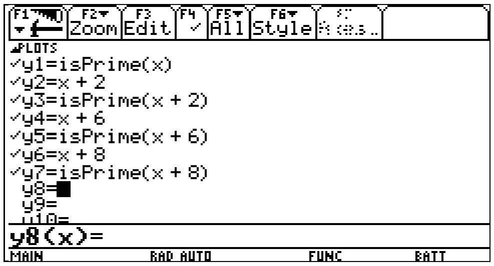


FIGURE 40: The Function Inputs.

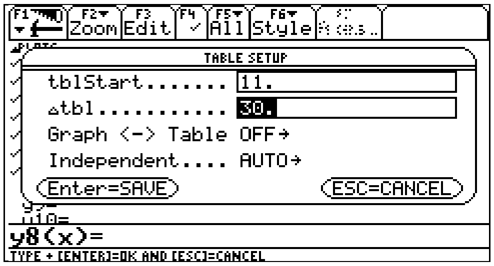
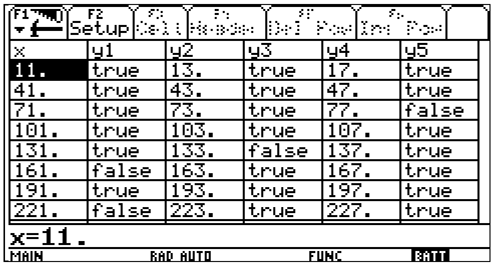
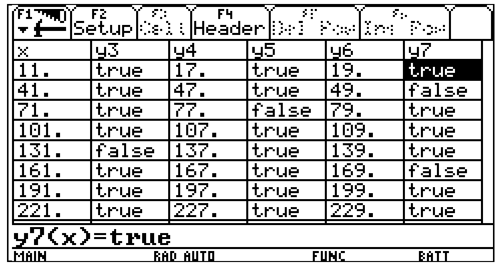


FIGURE 41: The Table Setup.



x	y1	y2	y3	y4	y5
11.	true	13.	true	17.	true
41.	true	43.	true	47.	true
71.	true	73.	true	77.	false
101.	true	103.	true	107.	true
131.	true	133.	false	137.	true
161.	false	163.	true	167.	true
191.	true	193.	true	197.	true
221.	false	223.	true	227.	true

FIGURE 42: The Table Revealed.



x	y3	y4	y5	y6	y7
11.	true	17.	true	19.	true
41.	true	47.	true	49.	false
71.	true	77.	false	79.	true
101.	true	107.	true	109.	true
131.	false	137.	true	139.	true
161.	true	167.	true	169.	false
191.	true	197.	true	199.	true
221.	true	227.	true	229.	true

FIGURE 43: The Table Revealed.

An analysis of the Table Setup in **FIGURE 41** is justified as follows: If $\{p, p+2, p+6, p+8\}$ constitutes a prime decade, then clearly each of the integers in the set below is divisible by 3 and hence is not prime: $\{p+1, p+4, p+7, p+10, p+13, p+16, p+19, p+22, p+25, p+28\}$ This is a consequence of the fact that in any set of three consecutive integers, one of them is always divisible by 3.) Hence if the set $\{p, p+2, p+6, p+8\}$ comprises a prime decade, then each of the following sets cannot constitute a prime decade:

1. $\{p+10, p+12, p+16, p+18\}$
2. $\{p+20, p+22, p+26, p+28\}$

Hence there must be a minimal distance of 30 between prime decades. Using the divisibility of an integer by 7 will show that no pair of prime decades can have a distance of 60 between them. The details are left to the interested reader. Examining **FIGURES 42-43**, we see that the next two prime decades after $\{11,13,17,19\}$ are $\{101,103,107,109\}$ and $\{191,193,197,199\}$. It can be shown via an exhaustive search that the initial prime decades at a distance of 30 from each other are $\{1006301, 1006303, 1006307, 1006309\}$ *and* $\{1006331, 1006333, 1006337, 1006339\}$.

What is most fascinating is that there are no primes between the primes 1006309 and 1006331. Thus one has these eight primes in the two prime decades which are consecutive. (**FIGURE 44)**

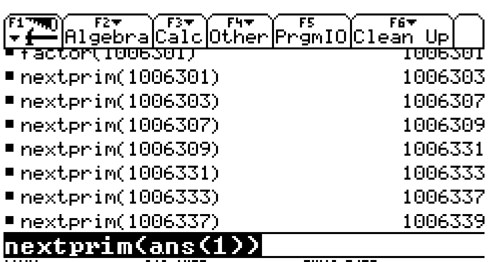


FIGURE 44: Examples

One can explore Merseene numbers using the program Mersenne given in **FIGURE 45** to determine the associated Mersenne numbers corresponding to the primes 13, 19, and 23 respectively in **FIGURE 46:**

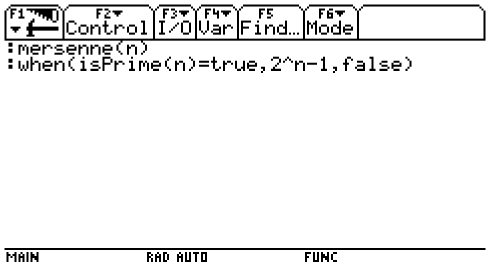


FIGURE 45: The Mersenne Program.

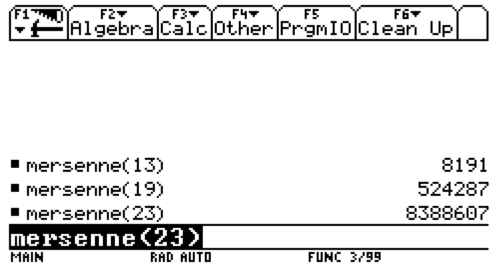


FIGURE 46: Examples

Any positive integer of the form n of the form $n = 2^p - 1$ where p is prime is known as a Mersenne number. In order to determine whether a prime p leads to a Mersenne, we must check to see whether $n = 2^p - 1$ is likewise prime. We use the program mersepri in **FIGURE 47** with an example in **FIGURE 48.** In **FIGURE 49,** we illustrate that while $2^{31} - 1$ *and* $2^{89} - 1$ are indeed primes, $2^{67} - 1$ is composite. The relevant computations are given together with the prime factorization for $2^{67} - 1$.

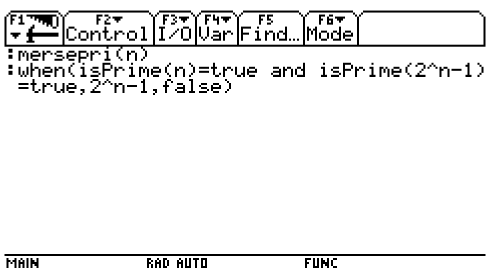


FIGURE 47: The Mersenne Prime Program.

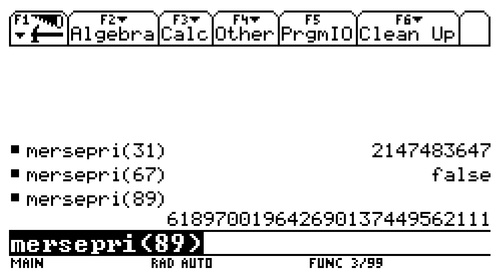


FIGURE 48: Examples

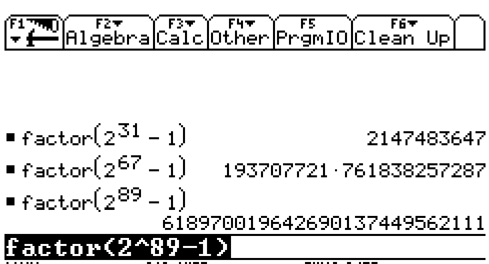


FIGURE 49: Examples

In a related matter, Euclid proved that if a positive integer n is of the form $n = \left(2^{p-1}\right) \cdot \left(2^p - 1\right)$, where p is prime, then n is an even perfect number. Euler proved the converse in the eighteenth century. We check to see if the primes 17, 37, and 61 lead to even perfect numbers using our program perfect. Recall that an even perfect number is one which coincides with the sum of all its aliquot divisors. Examples of even perfect numbers include 6, 28, and 496.

Using the program perfect in **FIGURE 50,** we verify in **FIGURE 51** that the primes 17 and 61 lead to even perfect numbers while the prime 37 does not.

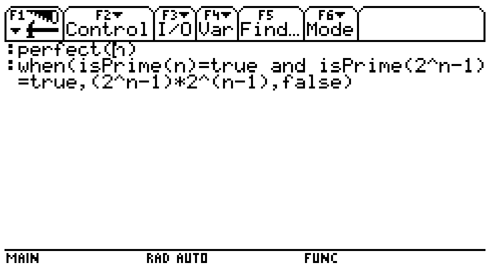


FIGURE 50: The Perfect Number Program.

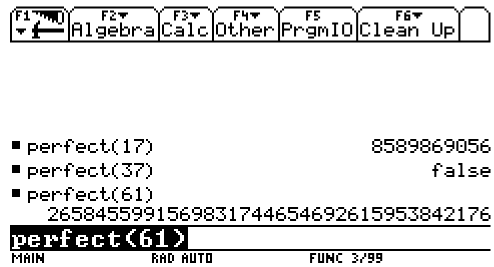


FIGURE 51: Examples

We conclude by providing a program for the Lucas sequence. See **FIGURE 52** for the program and **FIGURE 53** for an example.

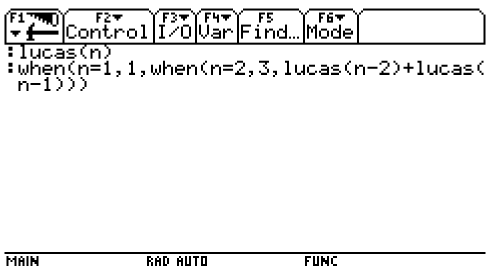


FIGURE 52: The Lucas Sequence Program.

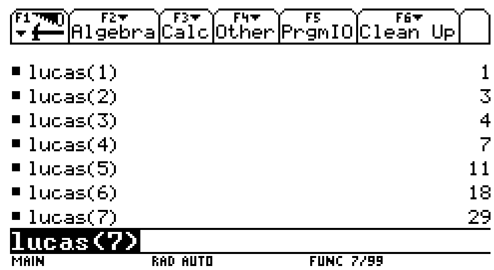


FIGURE 53: Examples

Conclusion: The interface of number theory with a CAS graphing calculator hand-held enables one to furnish an enhanced product. A number of very simple user defined functions can assist in exploring various number theoretic activities including varieties of primes such as Sophie Germain primes and Cunningham Chains of the Second Kind. In addition, exploring prime distances and prime gaps are ripe for exploration utilizing technological tools which would include a CAS calculator in the short term and more sophisticated software such as MATHEMATICA for more extensive explorations. This paper includes a portion of the programs that I have constructed to aid in such endeavors. The reader is invited to modify and extend the explorations described. For example, a nice extension of Goldbach's Conjecture would be to consider the form which examines positive odd integers greater than five and their representation as the sum of three primes together with a user defined function to achieve this. In short, explore, discover, and enjoy in the partaking of some beautiful mathematics.