A MATHEMATICAL COLLABORATION ACROSS THE ATLANTIC

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During the Spring 2011 semester Prof. Elaine Clark from the United States worked with Ms. Francesca Bevilacqua from Piazzola sul Brenta, Italy to create a research project through which college students and high school pupils could learn about some of the history of mathematics. The participants were college students in a second-semester mathematical content course for pre-service teachers at University of New Mexico-Valencia, in Tome, New Mexico, USA and high school pupils from Rolando da Piazzola High School in Piazzola sul Brenta, Italy. As the framework for this project Prof. Clark and Ms. Bevilaqua used portions selected from the first part of the four-part BBC series *The Story of Maths* hosted by Professor Marcus du Sautoy from the University of Oxford and The Open University (BBC and Open University; 2008).

The entire series explores the historical underpinnings of mathematics, beginning with the ancient civilizations of Egypt, Sumeria, and Babylon and working up through more current mathematics. Prof. Clark and Ms. Bevilaqua decided to focus on the beginning, ancient cultures in *Part I: The Language of the Universe.* This choice was partly because of the learning objectives for the college students, and because they were unsure of possible language barriers with the Italian high school pupils.

Ms. Bevilacqua and Prof. Clark started the project thinking the college students could serve as mentors, or tutors, in completing this exploration of the historical underpinnings of mathematics. Thus the primary learning objectives Prof. Clark set for the college class were:

- recognize the mathematical content in a non-textbook presentation of numeration, arithmetic operations, beginning geometry, time.
- find other sources that support and expand on student's understanding of the recognized mathematical content.
- report on the historical beginnings of a particular concept in basic arithmetic and/or geometry.

With these learning objectives in mind the video *The Language of the Universe* was divided into fourteen segments, each segment focusing on a particular aspect of mathematical knowledge or a particular mathematical concept. And, in order to facilitate the collaboration, and to provide a common platform for recording information, discoveries and results, a wiki page using PBworks was created.

The college students and high school pupils watched all of the clips for the fourteen segments and were asked to pick out the mathematical concept that was presented in each. See Figure 1 for a screen shot of a portion of the FrontPage for the wiki. Also, see Figure 2 for a sample of one of the Reactions pages where students were to record their comments for each video clip, including the primary mathematical concept presented.

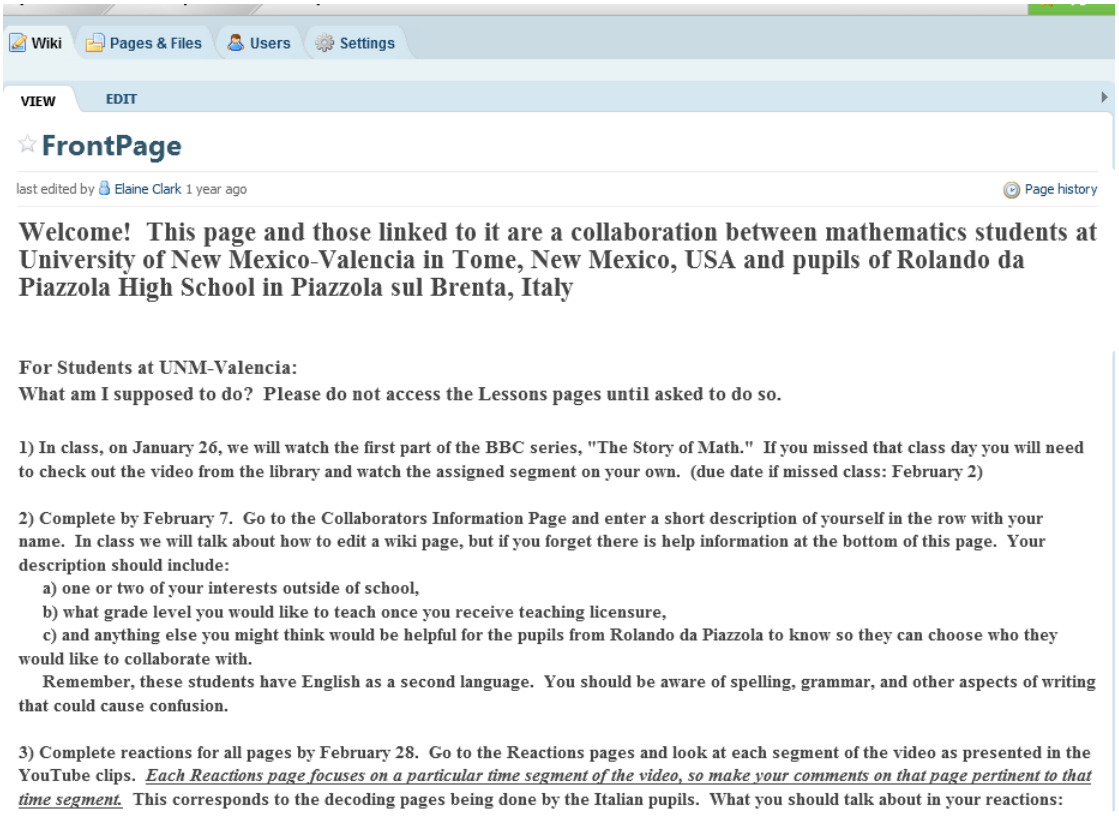


Figure 1 – FrontPage for Collaborative Wiki Page

One thing that happened almost immediately was that the Italian pupils did not have a strong enough grasp of English to easily pick out the concepts being discussed in the clips, at least not from just listening to them. Quickly Ms. Bevilacqua adjusted and, with editorial help from Prof. Clark and a couple of the college students, she created "scripts" or transcripts of what was said in each section of the video (see Figure 2). With the high

school students now able to read the script while watching the clip, they were able, for the most part, to determine the mathematical concept discussed.

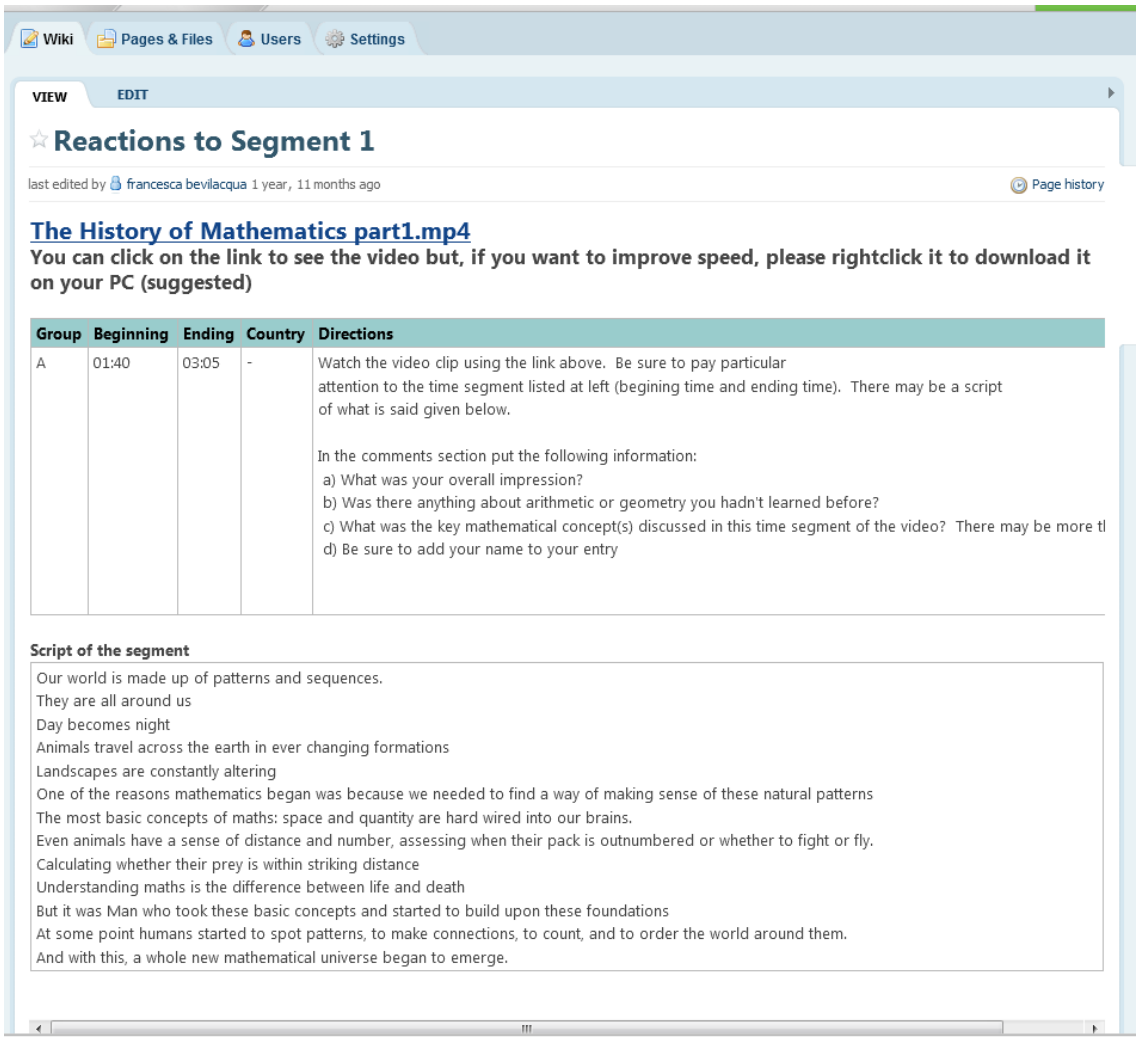


Figure 2 – Reactions Page for Segment 1 of the video

Another thing that came out was that the mathematical concept the students noticed in each clip was not always the same one Prof. Clark and Ms. Bevilaqua had in mind when the fourteen segments were created. The answers were not always *incorrect*; they were perhaps too broad or too vague. For example, in Segment 1 Professor du Sautoy talks about how a rudimentary sense of number, distance, and space are instinctual in all animals, including humans. Prof. Clark had in mind that the mathematical concept was that all animals have this rudimentary sense of number and distance. The comments from the students, however, included "…we use math in almost every situation…," "Anmails (sic) use math as well…," "We use math in everyday life. Even animals use math," and

"… relates math to life and death." All of these comments are true to what is said in the segment, but it was disconcerting that, as is often the case in novice versus expert situations (Schoenfeld, Hermann; 1982), the students focused on the *context* not the *content*.

While students were viewing the clips and posting their individual responses, they also introduced themselves. Since the high school pupils were minors, and to preserve confidentiality of the college students, the students and pupils used first names only and were cautioned not to post anything in their self-descriptions that would be confidential. Once their descriptions or "self-presentations" were posted, they were assigned into groups. Prof. Clark had a very small class of students so about four high school pupils were placed with one or two college student(s). Also, each group had a page in a Web Café for group members to post comments and questions to each other. Figure 3 shows a screen shot of one of the Web Cafés.

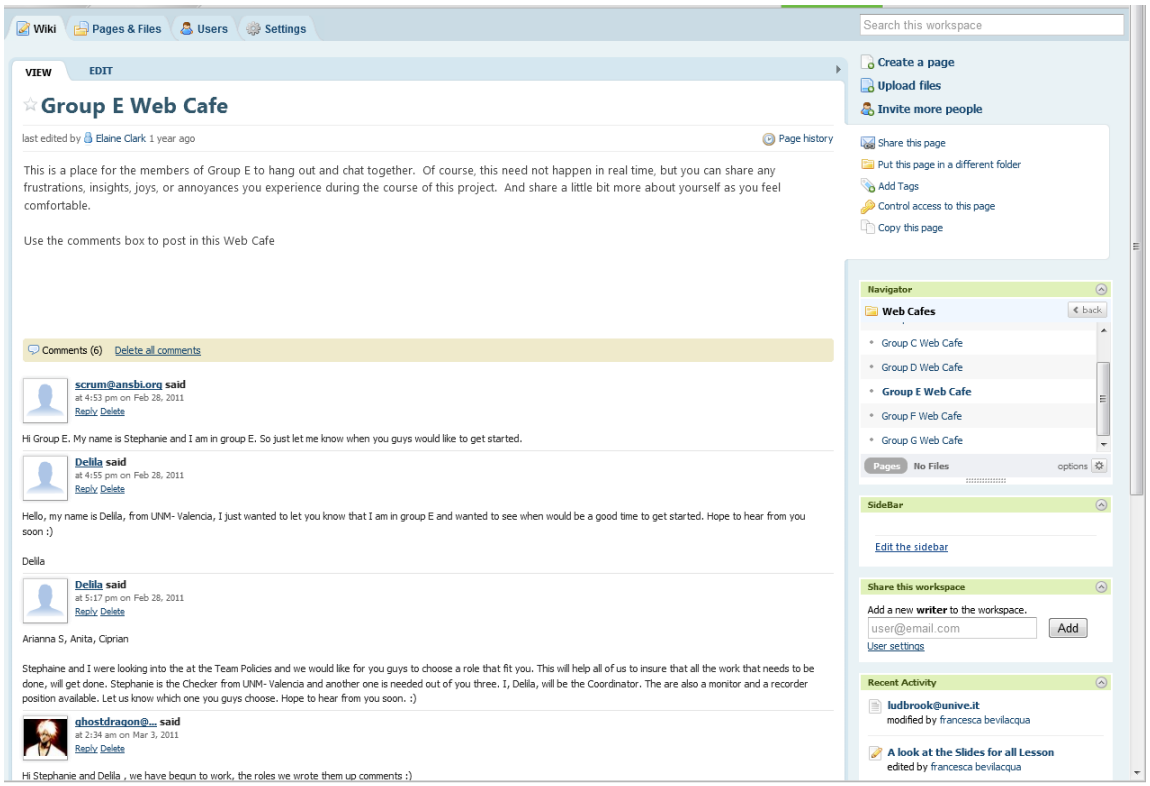
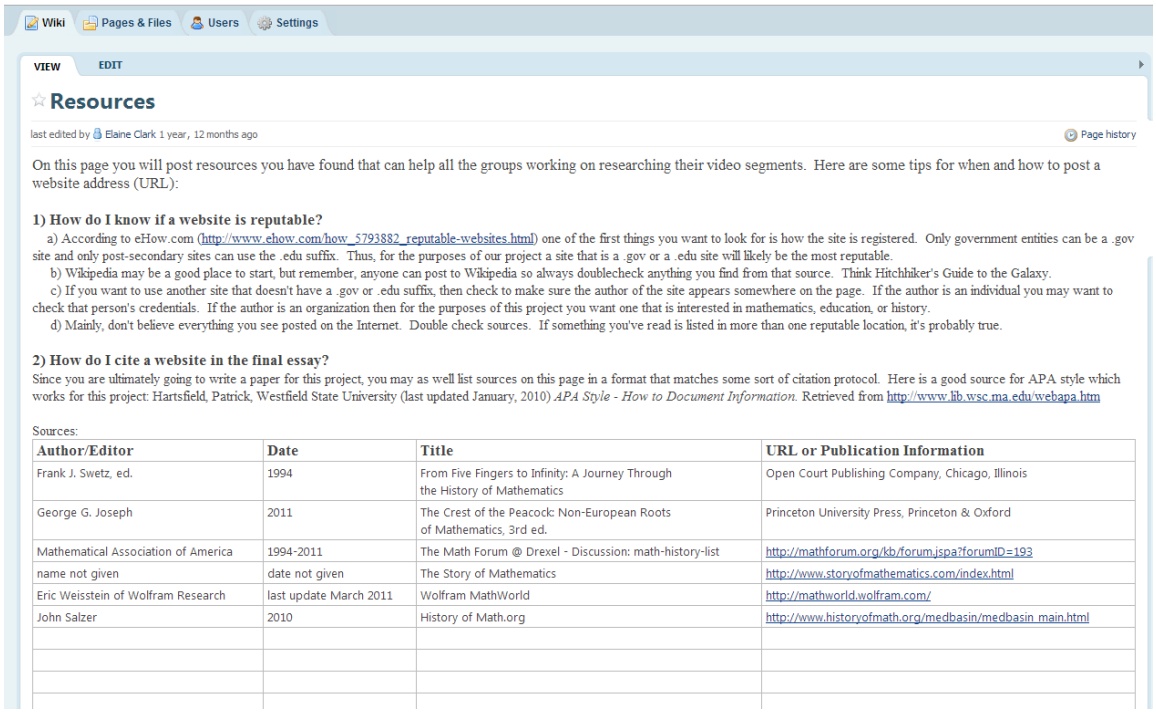


Figure 3 – Web Café

As the groups were created, they were given guidelines for roles to assign to help with the group interactions. The team policies were extracted from a paper about how to turn student groups into effective teams (Oakley, et al. 2004). Each team was then assigned a segment to research further. The idea was that they would search the Internet for other reputable, historical sources that presented these mathematical concepts.

The students and pupils were given a little over a month to conduct their web search and were asked to post the URL web addresses for the sources they found. Internet sources were the primary focus since these would be readily available to all of the pupils and students. However, Prof. Clark did make available on reserve in the UNM-Valencia campus library copies of two books on mathematics history: *From Five Fingers to Infinity: A Journey through the History of Mathematics* (Swetz, ed.; 1994) and *The Crest of the Peacock: Non-European Roots of Mathematics* (Joseph, 1991). A resource page was created in the wiki so that the students and pupils could post what they found. See Figure 4 for a screenshot of the resource page.



Wiki | Pages & Files | Users | Settings

VIEW | EDIT

Resources

last edited by Elaine Clark 1 year, 12 months ago — Page history

On this page you will post resources you have found that can help all the groups working on researching their video segments. Here are some tips for when and how to post a website address (URL):

1) How do I know if a website is reputable?

a) According to eHow.com (http://www.ehow.com/how_5793882_reputable-websites.html) one of the first things you want to look for is how the site is registered. Only government entities can be a .gov site and only post-secondary sites can use the .edu suffix. Thus, for the purposes of our project a site that is a .gov or a .edu site will likely be the most reputable.

b) Wikipedia may be a good place to start, but remember, anyone can post to Wikipedia so always doublecheck anything you find from that source. Think Hitchhiker's Guide to the Galaxy.

c) If you want to use another site that doesn't have a .gov or .edu suffix, then check to make sure the author of the site appears somewhere on the page. If the author is an individual you may want to check that person's credentials. If the author is an organization then for the purposes of this project you want one that is interested in mathematics, education, or history.

d) Mainly, don't believe everything you see posted on the Internet. Double check sources. If something you've read is listed in more than one reputable location, it's probably true.

2) How do I cite a website in the final essay?

Since you are ultimately going to write a paper for this project, you may as well list sources on this page in a format that matches some sort of citation protocol. Here is a good source for APA style which works for this project: Hartsfield, Patrick, Westfield State University (last updated January, 2010) *APA Style - How to Document Information.* Retrieved from http://www.lib.wsc.ma.edu/webapa.htm

Sources:

Author/Editor	Date	Title	URL or Publication Information
Frank J. Swetz, ed.	1994	From Five Fingers to Infinity: A Journey Through the History of Mathematics	Open Court Publishing Company, Chicago, Illinois
George G. Joseph	2011	The Crest of the Peacock: Non-European Roots of Mathematics, 3rd ed.	Princeton University Press, Princeton & Oxford
Mathematical Association of America	1994-2011	The Math Forum @ Drexel - Discussion: math-history-list	http://mathforum.org/kb/forum.jspa?forumID=193
name not given	date not given	The Story of Mathematics	http://www.storyofmathematics.com/index.html
Eric Weisstein of Wolfram Research	last update March 2011	Wolfram MathWorld	http://mathworld.wolfram.com/
John Salzer	2010	History of Math.org	http://www.historyofmath.org/medbasin/medbasin_main.html

Figure 4 – Resource Page

Up until this last stage, with direction and help, the college students and high school pupils progressed fairly well with the project. They did all manage to watch the video clips, write about what concepts were presented, introduce themselves, and some were able to begin dialogues in the Web Cafés. But when it came time to find other resources, the project nearly screeched to a halt.

At this point the college students lost ownership in the project. They were willing up until this time to communicate with the high school pupils and work on establishing roles for groups. But Prof. Clark discovered with her college students that a couple of them did not have reliable access to the Internet except at the UNM-Valencia campus. She also discovered that despite the fact almost all of them had completed a freshman composition course; they had little ability and almost no confidence in searching the

Internet for reliable sources. Prof. Clark began feeding more and more information to her students, but the momentum was lost.

On the Italian side, the high school pupils also had difficulty finding sources. Here the barriers were in either finding sources presented in Italian that they could then share with their American mentors/group members, or being able to read and understand the sources they found presented in English. The language barrier became very apparent. Ms. Bevilacqua called on her colleagues, Cristina Garro, the pupils' history teacher, and Daniela Tresso, the pupils' English teacher, to help the students navigate the Internet to find reputable and useable sources. However, the three teachers gave the students different levels of help, some focusing more on the research methods and others focusing more on the content. This confused the students and Ms. Bevilacqua didn't realize the difference in approach until it was too late to establish a more consistent direction for the other teachers.

As frustrations rose communication between the college students and high school pupils broke down. Almost all of the groups stopped posting in the Web Cafés, though one or two struggled along. Ms. Bevilacqua and Prof. Clark began to brainstorm how to salvage a learning experience out of the rubble of this frustration. With the three teachers Ms. Bevilacqua, Ms. Garro, and Ms. Tresso helping the high school pupils, they were able to put together a PowerPoint presentation in English showing their classmates the interesting things they learned and discovered. The pupils from other classes who watched the presentation were impressed, the intervening parents were proud, and the three teachers breathed a sigh of relief and satisfaction with the final product. For Prof. Clark's college students, however, the project degraded to individual reports on the assigned mathematical concept, with the students borrowing from the resources Prof. Clark gave them and the PowerPoint the high school students created and posted in the Wiki page.

Would Prof. Clark and Ms. Bevilacqua do this again? Yes! There are many positive outcomes that can come from this sort of collaboration; not only a broader understanding of the origins of maths, but the opportunity to share a learning experience with people from another country. This first experience has made them aware of what needs to be done to prepare the college students and high school pupils for successfully conducting this sort of analysis and research, and how to better set the stage for success. There was no lack of commitment on the part of Ms. Bevilaqua's high school pupils or colleagues. The high school pupils in particular were proud of what they learned and were pleased with their final product, the PowerPoint presentation. And Prof. Clark's college students also seemed to gain some benefit from the interaction with the Italian pupils.

One of the frustrations many mathematics teachers feel is to come to the end of a lesson, either a brilliantly presented lecture or an equally brilliantly constructed learning activity, only to find that the students have no idea what they were learning. A colleague and mentor once told Prof. Clark, "At the beginning of class, tell them what you are going to teach, teach the lesson, and then at the end tell them what you taught them." It seems

reasonable, however, to require students to know enough about the subject to know *what* is being taught. How can these students become teachers if they simply go through the motions of the "busy work" provided by the instructor or the curriculum, with no understanding of the mathematical content presented? When a teacher in the Pre-K through 12 education system is given a curriculum by the principal of his/her school, it is important for that person to know what each lesson teaches without having an "expert" tell him or her.

With this in mind Prof. Clark would want her students to have two key skills:

- be able to determine the purpose of a lesson.
- know how to collect learning tools and experiences from sources other than the assigned curriculum.

These would be skills to augment the usual objectives of a second-semester mathematics content course. With these in mind Prof. Clark restructured some of the regular assignments she gives to the pre-service teachers in her mathematics content courses. For daily homework assignments they are required to state the concept discussed that day in class, find where in the textbook this concept is discussed and then to elaborate on some of the key points in light of the lecture and in-class activity. This homework assignment is to train the students to recognize the key mathematical concept or topic presented, rather than to simply tell them what it was at the end of the lesson. Also, the idea is to train them to be able to pick out the key concept from the reading in a text (even with chapter and section headings stating what this should be some students still are not able to find in the text where the concept is presented). This training will then be expanded to students being able to recognize the mathematical content from a nonstandard source, like a video meant for the general public.

Also, since the original idea had been for the college students to be mentors and tutors for the high school pupils, they will need to develop their expertise. It would not be fair to ask students to behave as experts when their knowledge base is not adequate for them to do so. With this in mind, Prof. Clark has begun by incorporating more historical background in the presentation of lessons in the content courses. Also, she is now asking her students to complete one research paper where they read an article about the historical development of a mathematical concept and write a synopsis, and another paper where they must search the Internet for sources presenting information on one given topic and write a report. All of these assignments are given in the context of numeration, real-number arithmetic, geometry, and measurement. In this way the college students can develop their ability and confidence in finding resources that are about mathematics.

As for Ms. Bevilaqua, her focus would still be engendering in her pupils an appreciation of the historical development of mathematics. The issues she would need to address would be:

- to connect the use of English with learning the mathematics.
- to ensure all participants, pupils and colleagues, share the same philosophy on how much direction an apprentice researcher needs.

She will pursue convincing her principal to let her teach maths in English for an entire school year, so that pupils will be able to experience that maths is not about mistakes in English grammar. This will perhaps build in more courage in communicating with their group members from the United States. Also, she is refining her expectations of the amount of direction the students need in completing a research project like this one, and in communicating this philosophy to colleagues who may wish to help with the next project. An apprentice researcher does need *some* help in terms of methods and deadlines (the college students as well as the high school pupils) but they need freedom in exploring content in order to feel important and to have ownership of the end result.

If the opportunity for another such collaboration were to present itself, Ms. Bevilaqua and Prof. Clark would ensure that all participants share the same philosophy of the role of the students and pupils in being apprentice researchers and the amount of help they can expect from their teachers. Perhaps, as the college students are better prepared and as Ms. Bevilaqua can create a supportive environment for such a project, she and Prof. Clark can form a new cohort of students and pupils to attempt another Collaboration across the Atlantic.

Bibliography

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(Note: for the original project the video segments were at first available on YouTube.com but are no longer available through that source.)

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Schoenfeld, A.H., Hermann, D.J. "Problem perception and knowledge structure in expert and novice mathematical problem solvers," *Journal of Experimental Psychology: Learning, Memory, and Cognition*. **8**(5) Sept. 1982, pp. 484 – 494. (Note: This gives an example of the literature on novice vs. expert perceptions.)

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